# **ONE APPROACH OF ADEQUATE MATHEMATICAL DESCRIPTION CONSTRUCTION**

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## ABSTRACT

Mathematical description of physical process consists from the mathematical model of physical process and vector function of external load. In this work mathematical descriptions of physical processes which are being described by linear systems of ordinary differential equations are investigated. The algorithms of construction of the mathematical description of real process (for example the motion of some dynamic system) are considered which allows receive adequate results of mathematical simulation. Two basic approaches to this problem are selected. Within the framework of one of these approaches some algorithms are offered.

Keywords: adequate mathematical description, inverse problems, regularization.

### 1. INTRODUCTION

The typical situation which arises by the analysis of new processes or phenomena is investigated. It is supposed that real physical process is observed and the records of experimental measurements of some characteristics of this process are given. It is necessary to develop adequate mathematical description of this process for further use.

The similar questions and problems considered in many other works [1,2,3,4,5].

In the given work the mathematical models of physical processes described only by the system of the ordinary differential equations will be examined [6,7,8,9]. Such idealization of real processes or dynamic systems is widely used in various areas for the description of control systems, as well as of mechanical systems with the concentrated parameters, as well as economic processes, biological and ecological processes etc. It is shown that with the help of such systems in some works even human emotions are simulated [10].

As a concrete example, the dynamic system which describes motion of the main mechanical line of the rolling mill is used.

We will assume also, that the process is open, i.e. it interacts with other neighbouring processes (in the closed processes there is no such the interaction).

For the simplicity, we select the dynamic process with mathematical models in the form of linear system of ordinary differential equations:

$$\dot{x}(t) = \tilde{C} x(t) + \tilde{D} z(t), \qquad (1)$$
with the equation of observation

$$y(t) = \tilde{F} x(t), \qquad (2)$$

where  $x(t) = (x_1(t), x_2(t), ..., x_n(t))^T$  is vector-function variables characterized the state of process,  $z(t) = (z_1(t), z_2(t), ..., z_l(t))^T$  is vector-function of unknown external loads,  $y(t) = (y_1(t), y_2(t), ..., y_m(t))^T$ ;  $\tilde{C}, \tilde{D}, \tilde{F}$  are matrices of the appropriate dimension with constant coefficients which are given approximately,  $\tilde{F}$ is nonsingular matrix dimension  $m \times n$  and rang  $\tilde{F} = m$ ,  $((.)^T$  is a mark of transposition).

If the part of external loads of real process is known, this case can be reduced to one which is examined earlier with using the linearity of initial dynamic system.

We assume that state variables  $x_i(t), 1 \le i \le n$  of system (1) correspond to some real characteristics  $\tilde{x}_i(t), 1 \le i \le n$  of process which are being investigated and that the vector function  $\tilde{y}(t) = \tilde{F} \tilde{x}(t)$  is obtained from experimental measurements.

The problem of synthesis of adequate mathematical description with the use of system (1) can be formulated as follows: it is necessary to find unknown vector function of external loads z(t) in such a way that the vector function y(t), which are obtained from system (1), (2) under this external load z(t), coincides with experimental data  $\tilde{y}(t)$  with a given accuracy of experimental measurements in chosen functional metrics.

Let us consider questions what prospects of adequate mathematical descriptions are valid for further use and what goals should be selected at the creation of adequate mathematical descriptions.

It will be useful to address to classical works in this area. In work [11] the following statement was done: "...the goal of the imitation simulation is the creation of experimental and applied methodology which aimed at

the use of its for a prediction of the future behaviour of system".

So the adequate mathematical descriptions first of all are aimed at the forecast of behaviour of real processes. With the help of adequate mathematical description it is possible to predict of behaviour of real process in new conditions of operation. For example, it is possible to test more intensive mode of operations of the real machine without risk of its destruction. Such tool (adequate mathematical description) allows to simulate the characteristics of process in the unconventional modes of operations, and also to determine optimum parameters of real process.

The considered situation requires the formation of some uniform methodological approach to this problem, creation of general algorithms and common criteria of adequacy evaluation [11,12,13].

## 2. STATEMENT OF A PROBLEM

The main problem of mathematical simulation is the construction (synthesis) of mathematical model (MM) of motion of real dynamic system which in aggregate with model of external load (MEL) gives the adequate to experimental observations the results of mathematical simulation [14].

There exist two approaches to problem of construction of adequate mathematical description [12,13,14]:

1) Mathematical model of process of type (1) is given a priori with inexact parameters and then the models of external loads were determined for which the results of mathematical simulation coincide with experiment [12,13,14];

2) Some models of external loads are given a priori and then mathematical model of process of type (1) is chosen for which the results of mathematical simulation coincide with experiment [15,16,17].

Now we will consider the synthesis of adequate mathematical description in the frame of first approach analysing the process with the concentrated parameters, for which the motion is described by ordinary differential equations of n-order (1).

We assume that vector function  $\tilde{y}(t)$  in system (2) is obtained from experiment and presented by graphics. Besides, we suppose that some of functions of external loads, for example,

 $z(t) = (z_1(t), z_2(t), ..., z_l(t))^T, l \le m$  are unknown.

According to first approach, it is necessary to develop the construction of vector function of external loads which define such the functions of state  $x(t) = (x_1(t), x_2(t), ..., x_n(t))^T$  of mathematical model (1), are coincide with experimental measurements  $\tilde{y}(t)$  with inaccuracy of initial data in given metrics. Such mathematical model of process behaviour together with obtained vector function of external loads z(t) can be considered as *adequate mathematical description of process*.

Such method of obtaining of mathematical model of external loads (function  $z(t) = (z_1(t), z_2(t), ..., z_l(t))^T$ ,  $l \le m$ ) is determined in literature as a method of external loads identification [18,19]. By the way, physical reasons of occurrence of such external loads are not being taken into account. They are only functions which in combination with mathematical model (1) provide results of mathematical simulation, which coincide with experiment with the given accuracy.

Such coincidence is being attained by synthesis of "correct" mathematical model (MM) of the dynamical system and the choice of "good" model of external load (MEL). MM of object the motion of which coincides with experimental measurements with acceptable accuracy under action of MEL which corresponds to real EL ("good" model) is understood as "correct" model. Thus the degree of "correctness" of MM depends directly on the chosen MEL and required accuracy of the coincidence with experiment.

Now we consider the synthesis of model of external loads by a method of identification [12,13].

Let's assume that external load z(t) is unknown and vector function  $\tilde{y}(t)$  in the equations (2) is measured by an experimental way.

The part of state variables  $\tilde{x}_1(t), \tilde{x}_2(t), ..., \tilde{x}_m(t)$  can be obtained by an inverse of equation (2) with function  $\tilde{y}(t)$  as  $\tilde{F}$  is nonsingular matrix:

$$\widetilde{x}_k(t) = N_k(\widetilde{y}(t)), \ 1 \leq k \leq m,$$

where  $N_k(\tilde{y}(t))$  is known functions.

Let's consider known state variable  $\tilde{x}_k(t), 1 \le k \le m$  as two known internal loads  $d_k \tilde{x}_k(t)$  and  $-d_k \tilde{x}_k(t)$ , where  $d_k, 1 \le k \le m$  is known constants. Such interpretation of state variables  $\tilde{x}_k(t)$  allows to simplify initial system. Such transformation will be determined as "k-section" of initial system [12,13].

In some cases after lines of "k-sections" the initial system (1) will be transformed to some subsystem at which one state variable is known, for example,  $\tilde{x}_1(t)$  and at which all external loads  $z_k(t)$ , k = 2,...,m, except  $z_1(t)$ , for example, are known. For the system of type (1) with the help of a number of "sections" can be obtained the subsystem of initial system which movement is described by the differential equations

$$\dot{x}(t) = A_1 x(t) + B_1 z_1(t) , \qquad (3)$$

with the equation of observation

$$\widetilde{y}(t) = c_1 \,\widetilde{x}_1(t) \,, \tag{4}$$

where  $A_1$  is matrix with constant coefficients of the appropriate dimension;  $B_1$  is vector column,  $c_1$  is const. So, it is supposed that the subsystem of initial system has one unknown external load  $z_1(t)$  and one known variable of state  $\tilde{x}_1(t)$  which is obtained by experimental way.

After simple transformations it can be obtain an integral equation for the unknown function of external load  $z_1(t)$ :

$$\int_{t_0}^{t} K_1(t-\tau) \, z_1(\tau) \, d\tau = P(t) \tag{5}$$

where  $K_1(t-\tau)$ , P(t) are known functions.

If the initial system (1),(2) does not satisfy the condition, as have been specified above, then this system can be reduced to system (3),(4) by aid of additional measurements [12,13].

With the use of similar transformations it is possible to receive the other integral equations for all unknown functions of external loads  $z_k(t)$ ,  $k \le m$ . After the solving of the integral equations, such as (5), all unknown functions of external loads of system of the equations (1) will be obtained.

The model z(t) which was obtained with the use of such method depends on chosen mathematical model (1).

Further, the model of external loads which are found in such a way together with mathematical model of process (1) gives the adequate mathematical description of process.

Function P(t) in equation (5) was obtained with use of experimental measurements  $\tilde{x}_1(t), \tilde{x}_2(t), ..., \tilde{x}_m(t)$ . So the function P(t) is function which is given as graphic.

Let us present now (5) as

$$A_p z = u_g = B_p x_g \tag{6}$$

where  $A_p$  is linear operator,  $A_p: Z \to U, z \in Z$ ,  $x_g \in X, u_g \in U$ ,  $x_g$  is initial experimental data (graphic), z is unknown function, (X, Z, U are afunctional spaces). Let's assume, that the operator  $A_p$ continuously depend on some vector parameters p of mathematical model of process:  $p = (p_1(t), p_2(t), ..., p_N(t))^T$ . The coefficients of matrices  $\tilde{C}, \tilde{D}$  can be chosen as components of such vector parameters p.

It can be shown that in the most practical problems the operator  $A_p$  is completely continuous [20].

Thus, a necessary condition for obtaining the equation (5) concerning required external load  $z_i$  is the

possibility through a number of "sections" to obtain a subsystem of initial system with one unknown external load and one known state variables  $\tilde{x}_j$ ,  $1 \le j \le m$ . It is

easy to demonstrate an example of system such as (1), in which such opportunity is absent.

## 3. SPECIFIC FEATURES OF INTEGRAL EQUATIONS SOLUTIONS

We will consider the integral equation such as (5). From the practical point of view it is convenient to take Z as Banach space of continuous functions C[0,T] or Hilbert space  $W_2^1[0,T]$ , where [0,T] is interval of time on which the functions of external loads are being investigated [20]. As far as the initial experimental data are frequently varying functions, it is convenient to accept U as Hilbert space  $L_2[0,T]$  [20].

Further, we shall suppose that the element  $x_g$  in the equation (5) is exchanged by function  $x_{\delta}$  which approximated given graphic  $x_g$  with a known error:

$$\left\| x_g - x_\delta \right\|_X \leq \delta \,,$$

where  $\delta$  is const,  $\delta > 0$ .

Let's denote by  $Q_{\delta,p}$  the set of the possible solutions of an inverse problem of identification of external load function (6) with the fixed operators  $A_p, B_p$ :

$$Q_{\delta,p} = \{ z : \left\| A_p z - B_p x_{\delta} \right\|_U \le \left\| B_p \right\| \cdot \delta \}.$$

Any function z from set  $Q_{\delta,p}$  may be considered as "good" function of external load as far as the function  $A_p z$  coincides with  $B_p x_g$  with accuracy of approximation.

Thus, the operators  $A_p, B_p$  and any function from the set  $Q_{\delta,p}$  give the triple which will provide adequacy of results of mathematical simulation with accuracy  $\|B_p\| \cdot \delta$ .

We shall name the process of determination of  $z \in Q_{\delta,p}$  as synthesis of function of external load by a method of identification [12,14].

However set of the possible solutions  $Q_{\delta,p}$  at any  $\delta$  has a number of specific features (it is actually incorrect problem) [20]). First, and main of them is that this set is not bounded at any  $\delta$  [20].

Let's consider this feature more in detail due to the fact that it leads to a number of unexpected and unusual consequences.

The set  $Q_{\delta,p}$  contains infinite number of the solutions like any problem with the use of approximate

data. However, the set  $Q_{\delta,p}$  contains functions which can differ one from another on infinite value [12, 20]. It is due to the reason that the operator  $A_p$  in the equation (6), as a rule, is completely continuous.

Thus, the set  $Q_{\delta,p}$  includes the essentially different

functions which are equivalent in sense of the solution of the equation (6) (incorrect problems). Therefore, the basic difficulty will be the selection of the concrete solution from infinite set of the various equivalent solutions. For this purpose it is necessary to involve some additional information [20]. As example, the function which is the more convenient for further use can be selected in this quality [13,14].

More over the inaccuracy of operators  $A_p, B_p$  with

respect to the exact operators can be not taken into account in these synthesis problems [13,14].

### 4. METHODS OF SOLUTION OF IDENTIFICATION EQUATION

For obtaining of the steady solutions of formulated above incorrect problems it is necessary to use the method of Tikhonov's regularization [20].

Let us consider the stabilizing functional  $\Omega[z]$  which has been defined on set  $Z_1$ , where  $Z_1$  is everywhere dense in Z [20]. Consider now the following extreme problem:

$$\Omega[z_{\delta,p}] = \inf_{z \in Q_{\delta,p} \cap Z_1} \Omega[z], \ p \in \mathbb{R}^N$$
(7)

It was shown that under certain conditions the solution of the extreme problem (7) exists, is unique and stable with respect to small change of initial data  $x_g$  [20]. The function  $z_{\delta,p}$  is named *the stable model of external load* after taking into account the only inaccuracy of approximation. The solution of a problem (7) can be non-unique. For the purposes of mathematical simulation any such solution will be acceptable. Such function of external load can be used for mathematical simulation of initial system (1).

Still there are no basis to believe that the function  $z_{\delta,p}$  will be close to real external load  $z_{ex}$ . It is only good and steady function (model) of external load [12,13].

However, such approximate solution can be interpreted in other way. The regularized solution can be treated as steadiest with respect to change of the factors which were not taken into account in mathematical model. These factors may include changes in structure of mathematical model of system, the influence, which were not taken into account, change of conditions of experiment etc. We can prove such interpretation of the approximate solution.

At the synthesis of mathematical model of physical process we will first of all take into account the factors which define a low-frequency part of change of state variables. First, it is due to the fact that this part of a spectrum is well observed during the experiment, as measuring devices do not deform it. Secondly, the highfrequency components of external loads as well as equivalent to them insignificant factors not taken into account quickly die away in process of distribution among inertial elements. Thus, factors of interactions, which were not taken into account and equivalent to them influences, change only high-frequency part of the approximate solutions. If the factors, which are not taken into account, change a low-frequency part of the solutions then it means that mathematical model of process is chosen incorrectly. In work [21] has been shown that the regularized solution  $z_{\delta,p}$  represents result of high-frequency filtration of approximate solution. The greater degree of smoothing of the solution corresponds to greater error of initial data.

Hence, the regularized solution  $z_{\delta,p}$  can be interpreted as the function from set  $Q_{\delta,p}$  which is the steadiest with respect to changes of factors, which are not taken into account. Such quality of the regularized solution is very important when it is used in mathematical simulation of real processes when the results of simulation are steady with respect to small changes factors, which are not taken into account and which are naturally present at any mathematical description of process.

The obtained solution of a synthesis problem of external load function  $z_{\delta,p}$  requires, as a rule, the additional analysis. It is necessary, first of all, to determine transformation of model  $z_{\delta,p}$  with change of operational conditions, for example, change of speed of movement of real dynamical system, change of sizes of various static loads etc. As a result of the analysis, the model  $z_p$  will be obtained which can be used for mathematical modeling of real processes and also at study of new prospective modes of operations.

For the numerical solution of an extreme problem (7) the discrepancy method was used [20]. The problem (7) was replaced by following extreme problem:

$$M^{\alpha}[A_{p}, z_{\delta, p}] = \inf_{z \in Z_{1}} \{ \|A_{p}z - B_{p}x_{g}\|_{U}^{2} + \alpha \Omega[z] \}, (8)$$

where parameter of regularization  $\alpha$  was determined from a condition

$$\left\| A_p z_{\delta p} - B_p x_g \right\|_U = \left\| B_p \right\| \delta .$$
<sup>(9)</sup>

So the obtained function  $z_{\delta,p}$  together with operators  $A_p, B_p$  give the adequate mathematical description of process which is to stable change of initial data (stable adequate mathematical description). In this case the analysis of a problem was reduced to the solution of the Euler's equation for functional (8) if the functional space U is Hilbelt space:

$$A_{p}^{*} A_{p} z_{\delta, p} + \alpha \, \Omega'[z_{\delta, p}] = A_{p}^{*} B_{p} \, x_{g} \,, \tag{10}$$

where  $A_p^*$  is the associate operator to  $A_p$ ;  $\Omega'[z]$  is Frechet's derivative.

The approximate solution of the equation (10) on a uniform discrete grid was carried out by a numerical method.

### 5. EXAMPLE OF PROBLEM SOLUTION OF SYNTHESIS OF ADEQUATE MATHEMATICAL DESCRIPTION

Now we consider in detail the problem of adequate mathematical description synthesis of the main mechanical line of rolling mill [14,22].

The four-mass model with weightless elastic connections is chosen as mathematical model of dynamic system of the main mechanical line of the rolling mill [14,22].

The equations of motion are obtained from the Lagrangian equations of second kind and have the form:

$$\dot{x}(t) = \tilde{C} x(t) + \tilde{D} z(t), \qquad (11)$$

where  $x(t) = (x_1(t), x_2(t), ..., x_6(t))^T$  is vector function of state variables,  $z(t) = (z_1(t), z_2(t))^T$  is vector function of unknown external loads,

$$\widetilde{C} = \begin{pmatrix} 0 & c_{12} & 0 & 0 & 0 & 0 \\ c_{21} & 0 & c_{23} & 0 & c_{25} & 0 \\ 0 & 0 & 0 & c_{34} & 0 & 0 \\ c_{41} & 0 & c_{43} & 0 & c_{45} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{56} \\ c_{61} & 0 & c_{63} & 0 & c_{65} & 0 \end{pmatrix}$$

$$\widetilde{D} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ d_{41} & 0 \\ 0 & 0 \\ 0 & d_{62} \end{pmatrix}.$$

The functions  $x_1(t), x_3(t), x_5(t)$  are obtained by experimental way and are given as graphics  $\tilde{x}_1(t), \tilde{x}_3(t), \tilde{x}_5(t)$  (Fig.1).



Fig.1.The records of functions  $\tilde{x}_1(t), \tilde{x}_3(t), \tilde{x}_5(t)$ .

The problem of synthesis of adequate mathematical description can be formulated as: it is necessary to find such functions of external loads  $z_1(t), z_2(t)$  that the results of simulation  $x_1(t), x_3(t), x_5(t)$  of system (11) with functions of external loads  $z_1(t), z_2(t)$  coincide with experiment  $\tilde{x}_1(t), \tilde{x}_3(t), \tilde{x}_5(t)$  (Fig.1).

Two integral equations for unknown functions  $z_1(t), z_2(t)$  was obtained by method of "sections" [14,22]. The errors of experimental measurements are less than to 7 % in the uniform metrics.

The solutions of identification problem of the stable functions of external loads for experimental data presented in Fig.1 was performed.

The results of calculation are presented in Fig.2.







The mathematical model (11) with the found external loads  $z_1(t), z_2(t)$  give stable adequate mathematical description of real process.

### 6. SYNTHESIS OF ADEQUATE MATHEMATICAL DESCRIPTION FOR CLASS OF OPERATORS

It is necessary to apply various methods of simplification in developing the mathematical model, such as taking into account different forces and their impact on the movement of real system. The various models (with different parameters) of real process or system were obtained by the different authors even in cases, when the structures of the mathematical descriptions (models) are similar. So, it is supposed that the vector parameters p is given inexactly. So vector p can get different values in given closed domain  $D: p \in D \subset \mathbb{R}^N$ .

Some operators  $A_p, B_p$  correspond to each vector from D. The sets of possible operators  $A_p, B_p$  have been denoted as classes of operators  $K_A = \{A_p\}, K_B = \{B_p\}$ . So we have  $A_p \in K_A, B_p \in K_B$ . The deviations of operators  $A_p \in K_A$  between themselves from set  $K_A$  and operators  $B_p \in K_B$  from set  $K_B$  are given:

$$\sup_{p_{\alpha}, p_{\beta} \in D} \left\| A_{p_{\alpha}} - A_{p_{\beta}} \right\| \le h_{1}, \sup_{p_{\gamma}, p_{\lambda} \in D} \left\| B_{p_{\gamma}} - B_{p_{\lambda}} \right\| \le d_{1}$$

Now we transfer to consideration of a more general problem of synthesis of external loads functions in which the inaccuracy of operators  $A_p, B_p$  will be taken into account.

The set of possible solution of equation (6) is necessary to extend to set  $Q_{h_1,d_1,\delta}$  taking into account the inaccuracy of the operators  $A_p, B_p, p \in D$  [20]:

$$Q_{h_1, d_1, \delta} = \{ z : \left\| A_p z - B_p x_{\delta} \right\| \le h_1 \left\| z \right\|_U + B_0 \}$$

where  $B_0 = d_1 \|x_g\| + \|B_p\| \delta$ .

Any function from  $Q_{h_1,d_1,\delta}$  causes the response of mathematical model coinciding with the response of investigated object with an error into which errors of experimental measurements and errors of a possible deviation of parameters of a vector  $p \in D$  are included. A problem of a finding  $z \in Q_{h_1,d_1,\delta}$  will be entitled by analogy to the previous one as a *problem of synthesis for a class of operators* [13,14]. It should be noted that the set of the solutions of a problem of synthesis  $Q_{h_1,d_1,\delta}$  at the fixed operators  $A_p \in K_A, B_p \in K_B$  in  $Q_{h_1,d_1,\delta}$  contain elements with unlimited norm (incorrect problem) therefore the value  $(h_1 ||z||_U + B_0 \cdot \delta)$  can be infinitely large. Formally speaking, such situation is unacceptable as it means that the error of mathematical simulation tends to infinity, if for the simulation of external load is used the arbitrary function from  $Q_{h_1,d_1,\delta}$  as functions of external load.

Hence not all functions from  $Q_{h_1,d_1,\delta}$  will serve as "good" functions of external load.

The function of external loads z(t) in this case can be different. They will depend on final goals of mathematical simulation. For example, we can obtain model  $z_0, z^1$  for simulation of given motion of system as solution of extreme problems [12,13,23]:

$$\Omega[z_0] = \inf_{p \in D} \inf_{z \in Q_{\delta, p} \cap Z_1} \Omega[z], \ p \in \mathbb{R}^N, \quad (12)$$

$$\Omega[z^{1}] = \sup_{p \in D} \inf_{z \in Q_{\delta, p} \cap Z_{1}} \Omega[z], \ p \in \mathbb{R}^{N}.$$
(13)

The function of external loads which is necessary for estimation from below of output of dynamic system (process) can be obtained as solution of the following extreme problem [13,14]:

$$\left\|A_{p_b}z_b\right\| = \inf_{p \in D} \left\|A_p z_{\delta,p}\right\|.$$
(14)

where  $z_{\delta,p}$  is the solution of extreme problem (7).

Another model for estimation from above of output of dynamic system (process) can be obtained as solution the extreme problem [12,13]:

$$\left\|A_{p_c}z_c\right\| = \sup_{p \in D} \left\|A_p z_{\delta,p}\right\|.$$
 (15)

As unitary model  $z_{un}$  we can call the solution of following extreme problem [12,14]:

$$\left\| A_{p_{un}} z_{un} - B_{p_{un}} x_{\delta} \right\| = \inf_{p \in D} \sup_{p \in D} \left\| A_{p} z_{\delta, a} - B_{p} x_{\delta} \right\|_{U}$$
(16)

where  $z_{\delta,a}$  is the solution of extreme problem (7) with  $p = a, a \in D$ .

The triple  $\{A_{p_{un}}, B_{p_{un}}, z_{un}\}$  gives the stable adequate mathematical description for class of operators of process as example of possible one.

The method of special mathematical model selection was suggested to increase of approximate solution exactness of extreme problems [14,15,16].

The real calculations of external loads function  $z_{un}$  on rolling mill were presented as an example in papers [11,14].

For the case which is shown on Fig.1. the function of external load as solution of extreme problem (16) is presented on Fig.3.



Fig. 3. The diagram of model change.

It is necessary to note that results of synthesis do not vary if to change of the initial data within the limits of accuracy of measurements  $\delta$  and to change of the initial dynamic system, so that the inaccuracy of operators  $A_p, B_p$  would not differ from any operators  $A_{p_{\alpha}} \in K_A$ ,  $B_{p_{\alpha}} \in K_B$  on value the less then  $h_1$ ,  $d_1$ accordingly.

The results of calculations show that the estimation from above of accuracy of mathematical modeling with model  $z_{un}$  for all  $A_p \in K_A$  does not exceed 11 % in the uniform metrics with average error of mathematical model parameters of the main mechanical line of rolling mill equal to 10 % and errors of experimental measurements equal to 7 % in the uniform metrics.

Note that taking into account the error of the operators  $A_p \in K_A$ ,  $B_p \in K_B$  leads to more smooth results of identification (see Fig.2. and Fig.3).

The comparative analysis of mathematical modeling with various known models of external loads and experimental data were presented in work [24]. The model of external load  $z_{un}$  better corresponds to new experimental observations.

Now it is possible to give the definition of stable adequate mathematical description of dynamic systems (1).

Definition of stable adequate mathematical description: mathematical description of real process will be considered as stable adequate description with respect to selected variables of state of process model, if under proper limitation on external loads and on the values of variables of state real process with the same additional conditions (initial and boundary) will coincide with experimental measurements of corresponding physical characteristics of real process in given metrics with the accuracy of measurements and exactness of parameters definition of mathematical model. This adequate mathematical description has to be stable with respect to a change of initial data and to a change of experiment's conditions.

# Conclusion

The problems of synthesis of adequate mathematical description of real dynamical system are considered in this paper. One of the possible solutions of abovementioned problems is the choice of function of external loads adapted to dynamical system by the use of identification method. The peculiarities of such approach were investigated. These problems are actually incorrect ones by their nature and that is way for their solution were used the regularization Tikhonov's method. For the case when mathematical model are given approximately different variants of choice of external loads functions which depend on final goals of mathematical simulation are considered. It can be as follows: a copy of given motion of system, different estimation of responses of dynamic system, simulation of best forecast of system motion, the most stable model with respect to small change of initial data, unitary model etc.

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