A LOT-SIZE SIMULATION MODEL WITH BATCH DEMAND WITH SPECIAL ATTENTION TOWARDS THE HOLDING COSTS

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ABSTRACT
The problem deals with the optimization of a multi-echelon supply chain with, at the downstream end, the final customer with random demand but with a pre-determined service level. In such a chain with several levels including production and distribution, safety levels appear for various types of products. Decisions on safety stock are made based on various costs, including the holding cost. It is shown how this holding cost could be calculated and whether it should be based on purchase prices or selling price. In a stochastic scenario it is not so clear what the consequences are of using the wrong type of price. A simulation in Arena has been constructed to show an example of such a supply chain, under various levels of uncertainty and various types of demand distributions.

Keywords: buyer-vendor system, multi-echelon supply chain, uncertain demand

1. BACKGROUND AND LITERATURE

The problem under study deals with the optimization of a multi-echelon supply chain. Such a supply chain consists of at least two levels, of which at the downstream end we have the final customer for which a service level has to be fulfilled. As demand from this final customer is random, safety stock needs to be provided, but as there are several levels with production and distribution, these safety levels appear for raw materials, intermediate products and finished products. The decision about service levels in between and the levels of safety stock is a decision matter of the supply chain, and is not seen by the final customer. He just wants goods delivered according to a pre-specified service level.

While the cost of holding inventory includes the opportunity cost of the money invested, expenses for running the warehouse, handling, insurance, losses for deterioration and damage, it is generally accepted that the largest portion of the holding cost is made up of the opportunity cost of capital (Silver et al. 1998). Thus in many traditional models the following convention is adopted for the holding cost per year:

$$\alpha v E(I) \tag{1}$$

where v is the unit variable cost to be invested for every unit placed in inventory, *E(I)* is the average inventory in unit, and α is defined as the return on investment that could be earned on the next best alternative for the company.

For practical purposes, the question arises which cost elements and how to calculate from these the correct value for v. Furthermore it has to be looked at how to handle other variable out-of-pocket costs, in case they are considered important, like the cost of insurance or the rent of warehouse space. Starting from the important contribution by Grubbström (1980), and further development by Van der Laan and Teuntner (2002), this research develops some further analysis.

In Net Present Value (NPV) analysis, all cash flows, which are related to an activity, are valued by their time of occurrence using one common discount factor α. When applied to our practice, the NPV framework provides annuity stream (AS) profit functions for an inventory system. The NPV approach is powerful in deriving optimal inventory decisions in cases where the moments in time that cash flows occur are not based on the movements of product in the chain.

For certain classes of production and inventory problems, the difference between the classical approach and the NPV framework seems to be large, as shown in Grubbström and Thorstenson (1986) and in Teuntner and Van der Laan (2002). Why this difference appears is still a major issue to explain and to understand. Starting from Grubbström (1980 and further work by Haneveld and Teuntner (1998), it is shown that linearisations of the AS functions can be directly compared with the functions derived in the classical approach.

The linearization of the AS functions constructs a link between the NPV analysis and classical frameworks but sometimes produces counterintuitive results. One example is the batch sales economic order quantity model, which this paper studies in further detail. The optimal lot size in this model is the basic EOQ result but the inventory is to be valued at sales price rather than at invested costs, as the classical

This contradiction is well-known, but it is shown in Beullens and Janssens (2011) that this outcome is not the only valid outcome. Their model introduces the concept of the anchor point, which allows to construct NPV models under either push and pull conditions. When the first activity has to start at some fixed, but arbitrary point of time in the future, the anchor point coincides with this fixed time in the future. In many classical production-inventory models optimal current decisions are restricted by the past.

The case which is studied in this paper is the elementary lot-size model with batch demand from Grubbström (1980) and Kim et al. (1984). A producer fulfills deterministic demand that occurs at a constant rate of y product units per year in batches of size Q. Stock-outs are not allowed and the producer has matched his production rate to the demand rate. The following additional information is required: the sales price (w), the discount rate (α), and the variable cost per product for producing at annual volume y ($c(y)$). The cash flows involved are: (1) a set-up cost s at the start of every cycle $T=Q/y$; (2) production cost equal to $c(y)y$ which is a continuous stream during the year; (3) income equal to wyT arises upon delivery of Q units at the end of every cycle. The annuity stream s can be calculated, from which the optimal order quantity and the holding cost can be obtained. .

The question seems to be whether the opportunity cost of capital should be made or at the rate what can be generated through sales of this inventory and that, by this, classical inventory theory is wrong. In Beullens and Janssens (2011) it is shown that the difference between results by the NPV approach and classical inventory theory depends on the choice of the location of the anchor point. Literature always has assumed that the start of the most upstream process in the supply chain is fixed. Larger production volumes delay the downstream activities and the final sales.

2. BASIS OF THE SIMULATION MODEL

In order to study the effect of using the wrong type of holding cost a simulation model is built making use of the Arena simulation software. A two-level supply chain is simulated in which customers have a demand to a retailer. This type of chain is called a buyer-vendor system (Goyal and Gupta 1989). It is a type of vertical integration in which the buyer and the vendor co-operate by synchronizing production with demand. The objective of this type of co-operation is joint profit maximisation. The dynamic of the inventory levels in this system is shown in Figure 1. The buyer puts orders in batches of size Q and faces a constant demand (y). The vendor works at a finite production rate $R \geq y$. In Figure 1, it is assumed that delivery time equals zero and no shortages exist. However, the buyer-vendor system in our simulation makes use of a stochastic demand and delivery time, in which shortages may appear.

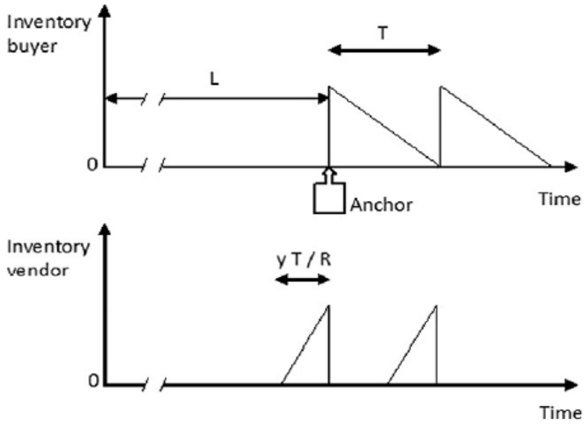

Figure 1: Buyer-vendor system with finite production rhythm

The simulation model makes use of the following data for the buyer-vendor system. The yearly demand y equals 3960 units, which means that demand during a simulation run of one year of 360 days equal 11 units per day. The vendor faces a fixed set-up cost per production (s_p) equal to 600 and a fixed shipment cost per shipment (s_s) equal to 25. The variable cost per product (c) equals 10. The buyer (retailer) faces a fixed cost per order (s_b) equal to 300 and pays a fixed price per unit (w) equal to 20. The sales price at the retailer's site (p) equals 40. The fixed ordering cost includes both the transportation cost and the administrative cost. The holding cost equals α = 20% of the purchase value of the product. The backorder cost is independent of time and is equal to B_2 = 10.

The retailer makes use of an (s, Q) inventory policy, in which s represents the re-order point and Q the fixed order quantity for placing his orders to the production site. The vendor aims to synchronise his production with the buyer's demand, i.e. R = 11 units per day. If delivery time is strictly positive, production should start before the start of the demand as the vendor wants to have the required order size ready for delivery.

In most buyer-vendor systems the vertical co-operation is realised by determining a fixed order quantity by means of the following formula:

$$Q^* = \sqrt{\frac{2*(s_b+s_p+s_s)*y}{\alpha(2p-c)\frac{y}{R}+\alpha c}} \tag{2}$$

With the data mentioned before, formula (2) leads to an order size of 676.66 units. In the simulation we will use the closest multiple of daily demand which is 61 * 11 = 671 units. The total relevant cost (TRC) to be used in the simulation model is the cost function as defined by Banerjee (1986) and by Goyal (1988):

$$TRC = (s_p + s_b + s_s) * O + \alpha c * avg(invr) + \alpha w * avg(invm) + B_2 * ES \quad (3)$$

where

- *O:* number of orders/production runs per year
- *avg(invr)*: yearly average inventory at retailer's site
- *avg(invm)*: yearly average inventory at manufacturer's site.

3. EXPERIMENTS AND RESULTS

Table 1: Sensitivity analysis: scenario 1 – Normal distribution

Normal Distribution		**Variation Coefficient (σ_L/μ_L)**		
μ_L ↓	Element	**0,2**	**0,6**	**1**
2	σ_L	0.4	1.2	2
	Inventory Manuf.	334	333	333
	Inventory Retail	333	333	332
	# Shortages	18	32	49
	Orders Not satisfy	3	3	3,02
	Orders satisfy	2,52	2,56	2,54
	TRC	**€ 7.726**	**€ 7.866**	**€ 8.030**
	P_2	99,55%	99,19%	98,76%
6	σ_L	1,2	3,6	6
	Inventory Manuf.	337	337	336
	Inventory Retail	334	334	339
	# Shortages	33	88	115
	Orders Not satisfy	3,12	2,98	3,2
	Orders satisfy	2,6	2,7	2,5
	TRC	**€ 7.890**	**€ 8.440**	**€ 8.728**
	P_2	99,15%	97,79%	97,09%
10	σ_L	2	6	10
	Inventory Manuf.	339	339	339
	Inventory Retail	334	335	333
	# Shortages	48	139	223
	Order Not Satisfy	3,1	3,18	3,18
	Orders Satisfy	2,76	2,66	2,66
	TRC	**€ 8.044**	**€ 8.958**	**€ 9.790**
	P_2	98,77%	96,47%	94,35%

The simulation model is run for a scenario without safety stock and a scenario with safety stock.

The case of no safety stock is simulated under the following conditions: the demand follows a Poisson distribution and the lead time *L* follows a Normal distribution, with various parameter values of the mean value μ_L and its standard deviation σ_L. Table 1 show some simulation results for various parameter values. The table shows the average yearly inventory at the manufacturer's site (Inventory Manuf.), the average yearly inventory at the retailer's site (Inventory Retail), the number of shortages on a yearly basis (# Shortages), the number of times per year that a manufacturer cannot deliver the full order quantity (Orders Not satisfy), the number of times per year that a manufacturer can deliver the full order quantity (Orders satisfy), total relevant cost (TRC), the P_2-service level (also called fill rate) (P_2).

Different combinations of the parameter values on the lead time distribution do not lead to big changes into average inventory levels both at the retailer's and at the manufacturer's site.

It can be expected that the number of units short (# shortages) increases with an increasing level of lead time variability (as there is no safety stock). This increase in the numer of units short leads to a lower service level (P_2) and to an increase of the shortage cost part in the total relevant cost (TRC). The decrease in service level is more explicit when the lead time becomes bigger.

In a second scenario it is assumed that th retailer hold a level of safety stock. The level depends on the parameters of the distribution of demand during lead time. Via the *Input Analyzer* (Arena software) the distribution during lead time is determined in an empirical way. The best fitting distribution is used to determine the re-order corresponding to a pre-specified service level. Figure 2 shows such an empirical distribution and also the best fitting distribution (based on the Mean Square Error criterion), which in this case is the Normal distribution (for μ_L = 22.2 and σ_T = 7.99).

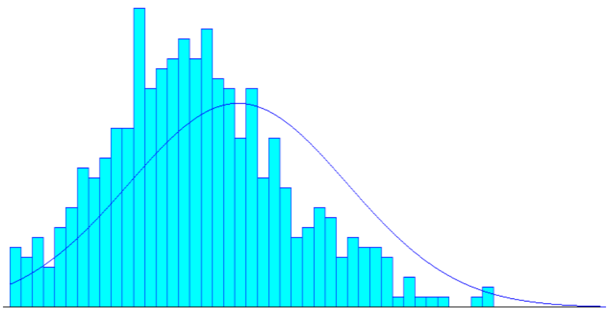

Figure 2: Distribution of demand during lead time (μ_L = 22.2 and σ_T = 7.99).

Table 2: Sensitivity analysis: scenario 2 – Normal distribution

Normal Distribution		Variation Coefficient (σ_L/μ_L)		
μL ↓	Element	**0,2**	**0,6**	**1**
2	σ_L	0.4	1.2	2
	Inventory Manuf.	332	335	334
	Inventory Retail	344	351	359
	# Shortages	2	3	6
	Orders Not satisfy	3,38	3,32	3,4
	Orders Satisfy	2,26	2,26	2,22
	TRC	**€ 7.610**	**€ 7.654**	**€ 7.714**
	P_2	99,95%	90,92%	99,85%
6	σ_L	1,2	3,6	6
	Inventory Manuf.	337	337	339
	Inventory Retail	353	401	415
	# Shortages	4	8	12
	Orders Not satisfy	3,44	3,48	3,4
	Orders Satisfy	2,32	2,5	2,56
	TRC	**€ 7.676**	**€ 7.908**	**€ 8.008**
	P_2	99,90%	99,82%	99,65%
10	σ_L	2	6	10
	Inventory Manuf.	337	337	339
	Inventory Retail	365	423	440
	# Shortages	5	20	36
	Orders Not satisfy	3,32	3,42	3,44
	Orders satisfy	2,64	2,62	2,58
	TRC	**€ 7.734**	**€ 8.116**	**€ 8.348**
	P_2	99,87%	99,54%	99,10%

From Table 2 it can be learned that the total relevant cost (TRC) show a smaller increase in case a safety stock is used. The service level is always above the 99% level. As could be expected, the level of safety stock is higher with a higher level of lead time variability. The increase in the total relevant cost (TRC) is mainly due to the increase in inventory cost. When the lead time is small (μ_L= 2), the increase in inventory level is 4.36%, but when it comes to an intermediate level of lead time (μ_L= 6), the increase is more than 17%. For a high level of lead time it is even more than 20%.

4. CONCLUSIONS

A simulation model has proven to be of high value in order to investigate the effect of lead time variability in a buyer-vendor co-operation. While both partners are seeking to maximise their joint profit, they should also worry about variability in delivery as it lay destroy their joint positive ideas. They should use the arguments brought forward by the simulation to negotiate with their third party logistics providers.

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