ADDITIVE FAULT TOLERANT CONTROL FOR DELAYED SYSTEM

Nouceyba Abdelkrim $^{(1,3)}$, *Adel Tellili* $^{(1,2)}$ *and Mohamed Naceur Abdelkrim* $^{(1,3)}$

[1]Université de Gabès, Unité de recherche Modélisation, Analyse et Commande des Systèmes (MACS)
[2]Institut Supérieur des Systèmes Industriels de Gabès (ISSIG)
[3] Ecole Nationale d'Ingnieurs de Gabs (ENIG)
e-mail: nouceyba.naceur@laposte.net
e-mail: Adel_Tellili@Lycos.com
e-mail: naceur.abdelkrim@enig.rnu.tn

ABSTRACT

The additive fault tolerant control (FTC) for delayed system is proposed in this paper. To design the additive control, two steps are necessary, the first one is the estimation of the sensor fault amplitude which is realized by using the Luenberger observer and the second one is the addition of the additive fault tolerant control law to the delayed system nominal control. The nominal control law of delayed system is designed by using the Lambert W method.

Index Terms— Additive FTC, delayed system, Lambert W method, Luenberger observer, sensor faults.

1. INTRODUCTION

Time delay phenomena appear naturally in the modeling of many systems. Actually, we remark that the majority of industrial system controls are implemented via a numerical calculator, so if a system haven't an intrinsic time delay, often a time delay appear via the control loop[1], [2]. Time delay systems can be represented by delay differential equations (DDEs), which belong to the class of functional differential equations (FDEs), and have been extensively studied in [3], [4], [5] and [6]. During recent decades, the Lambert W function has been used to develop an approach for the solution of linear time-invariant (LTI) systems of DDEs with a single delay [7], [8]. The Lambert W approach is used to resolve many problem such that stability [9], [10], design by state feedback [11].
Moreover to the presence of time delay, many type of fault can affect the delayed system such as sensor fault, actuator fault and system fault. So it's necessary to design a control able to tolerating potential faults in these systems in order to improve the reliability and availability while providing a desirable performance [12]. This field control is known as fault-tolerant control systems ; it can maintain overall system stability and desired performance in the event of such failures [13-15].
This work deals with the design of an additive FTC developed in [13, 14] as extension to delayed system where its nominal control is designed by using the Lambert W approach.

2. DESIGN OF THE NOMINAL CONTROL OF DELAYED SYSTEM BY USING THE MLW METHOD

In this section we construct the nominal control of delayed system by using the Lambert W method.

2.1. Definition of Lambert W function

Let's $x \in \mathbb{C}$ be a solution, to determinate, of the equation $xe^x = y$ for $y \in \mathbb{C}$, This type of equation can be resolve by using the Lambert W function W_k such that [11,16,17] $x = W_k(y)$.
With:
k branches of Lambert W function and $k \in]-\infty, +\infty[$, $k = 0$ principal branch [11,16,17].
Consider the delayed system:

$$\dot{x}(t) = A\,x(t) + A_d\,x(t-d) \tag{1}$$

where A and A_d are n×n matrices, $x(t)$ is an n×1 state vector and d is a constant time delay. this can be represented by the delayed differential equation (DDE):

$$\dot{x}(t) - A\,x(t) - A_d\,x(t-d) = 0 \tag{2}$$

Let's $x(t) = e^{St}x_0$ be a solution of (2) with S: matrix, with appropriate dimension, to determinate. Replacing the expression of $x(t)$ in (2) we find:

$$(S - A - A_d e^{S(-d)})e^{St}x_0 = 0 \tag{3}$$

by consequence we find:

$$(S - A - A_d e^{S(-d)}) = 0 \tag{4}$$

then:

$$S - A = A_d e^{-dS} \tag{5}$$

multiplying the equation (5) by $(d\,e^{d\,S}e^{-d\,A})$ we find:

$$d(S-A)(e^{(S-A)d}) = A_d\,d\,e^{-A\,d} \tag{6}$$

In this step we can use Lambert W function with $x = d\,(S-A)$ and $y = A_d\,d\,e^{-A\,d}$, by consequence the expression of S is:

$$S = \frac{1}{d}\,W(A_d\,d\,e^{-A\,d}) + A \tag{7}$$

equation (7) represent the characteristic equation for the general time-delay system [9]. The roots of equation (2) are the eigenvalues of the matrix S and can be used to describe the stability of the DDE (2), which represent the stability of general system.

2.2. Determination of the state feedback gain of delayed system

For a scalar DDE with state feedback as shown Eq.(8)

$$\begin{cases} \dot{x}(t) = a\,x(t) + a_d\,x(t-d) + bu(t) \\ u(t) = k\,x(t) \end{cases} \tag{8}$$

With k is the feedback gain determinate by using the Lambert W method The Lambert-W function can assign the real part of the rightmost pole exactly, by using this relation:

$$\mathrm{Re}(S_0 = \frac{1}{d} W_0(a_d d\, e^{-(a+bk)d}) + (a + b\,k)) = \lambda_{desired} \tag{9}$$

For example in [11], for the system (8) with $a = 1$, $a_d = -1$, $b = 1$ and $d = 1$,

$$\mathrm{Re}(S_0 = \frac{1}{d} W_0(a_d d\, e^{-(a+b\,k)d}) + a + b\,k) = -1 \tag{10}$$

Then, the resulting value of k is -3.5978.
After the design of the nominal control of delayed system we will determinate the additive control which can compensate the sensor fault.

3. DESIGN OF THE ADDITIVE CONTROL

The fault accommodation is based on the addition of an additive control u_{ad} to the nominal control law u. To design the additive control, two steps are necessary, the first one is the estimation of the sensor fault amplitude and the second one is the addition of the additive fault tolerant control law to the nominal control of delayed system.

3.1. Sensor fault amplitude estimation

When a sensor fault affects the closed loop system the tracking error between the reference input and the measurement will no longer be equal to zero. In this case, the nominal control law tries to bring the steady-state error back to zero. Hence, in the presence of sensor fault, the control law must be prevented form reacting. This can be achieved by cancelling the fault effect on the control input [13, 14].
A Luenberger observer [1], [18] will be used to estimate the sensor fault amplitude which affects the delayed system. Consider the linear time invariant LTI delayed system:

$$\begin{cases} \dot{x}\,(t) = A\,x\,(t) + A_d\,x\,(t-d) + B\,u\,(t) \\ y\,(t) = C\,x\,(t) \end{cases} \tag{11}$$

If the number of outputs is greater than the number of control inputs, the designer of the control law selects the outputs that must be tracked and breaks down the output vector $y(t)$ as follow:

$$y\,(t) = Cx\,(t) = \begin{pmatrix} C_1 \\ C_2 \end{pmatrix} x\,(t) = \begin{pmatrix} y_1\,(t) \\ y_2\,(t) \end{pmatrix} \tag{12}$$

The feedback controller is required to cause the output vector $y_1\,(t) \in \Re^p (p \leq r)$ to track the reference input vector y_r such that in steady-state:

$$y_r\,(t) - y_1\,(t) = 0 \tag{13}$$

To achieve this objective, an integrator vector $\dot{\varepsilon}\,(t)$ is added which satisfy the following relation:

$$\dot{\varepsilon}\,(t) = y_r\,(t) - y_1\,(t) \tag{14}$$

With: $\varepsilon\,(t) = \int (y_r\,(t) - y_1\,(t))\,dt$
figure (1) represent the adding of integrator and the diagram of estimation and compensation of sensor fault.

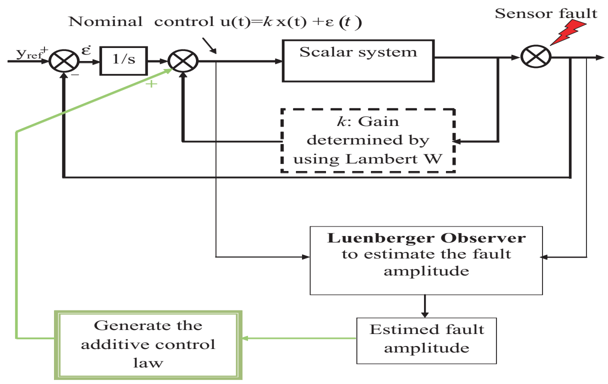

Figure 1. Diagram of additive fault tolerant control.

So the nominal control represented by (8) becomes:

$$u\,(t) = k\,x\,(t) + \varepsilon\,(t) \tag{15}$$

The closed loop system becomes:

$$\begin{cases} \dot{X}\,(t) = A_f\,X\,(t) + A_{df}\,x\,(t-d) + G\,y_r\,(t) \\ y\,(t) = C_f\,X\,(t) \end{cases} \tag{16}$$

with:

$$\begin{cases} X\,(t) = \begin{bmatrix} x\,(t) \\ \varepsilon\,(t) \end{bmatrix}, A_f = \begin{bmatrix} (A + B\,k) & B \\ -C & 0 \end{bmatrix}, \\ A_{df} = \begin{bmatrix} A_d \\ 0 \end{bmatrix} = \begin{bmatrix} A_{df1} \\ A_{df2} \end{bmatrix}, \\ C_f = \begin{bmatrix} C & 0 \end{bmatrix}, G = \begin{bmatrix} 0 \\ 1 \end{bmatrix}. \end{cases}$$

For sensor faults, the output equation given in (11) is broken down according to (12), and can be written as:

$$y\,(t) = C\,x\,(t) + F\,f\,(t) = \begin{pmatrix} y_1\,(t) \\ y_2\,(t) \end{pmatrix} \tag{17}$$

If the delayed system is observable the Luenberger observer can be written as follow:

$$\begin{aligned} \dot{\hat{X}}(t) &= A_f\,\hat{X}(t) + A_{df}\,\hat{x}(t-d) + G\,y_r(t) \\ &+ L\,(y(t) - \hat{y}(t)) \\ \hat{y}(t) &= C_f\,\hat{X}(t) \end{aligned} \tag{18}$$

Where: L: observer gain, $\hat{y}(t)$, $\hat{x}(t-d)$ and $\hat{X}(t)$ are respectively estimated output, delayed estimated state vector and augmented estimated state vector. Then we define the observation error $e(t) = X(t) - \hat{X}(t)$, so the error dynamic of this relation is governed by this equation:

$$\begin{aligned} \dot{e}(t) &= \dot{X}(t) - \dot{\hat{X}}(t) \\ &= A_f\,X(t) + A_{df}\,x(t-d) + G\,y_r(t) \\ &- \Big[A_f\,\hat{X}(t) + A_{df}\,\hat{x}(t-d) + G\,y_r(t) \\ &+ L\,(y(t) - \hat{y}(t))\Big] \\ \dot{e}(t) &= M\,e(t) + A_{df}\,\tilde{x}(t-d) - L\,F\,f(t) \end{aligned} \tag{19}$$

with:

$$\begin{cases} M = A_f - LC_f = \begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix} \\ \tilde{x}(t-d) = x(t-d) - \hat{x}(t-d) \end{cases}$$

Knowing that in steady state where $\dot{e}(t) = 0$, then we can write this equation:

$$\dot{e}(t) = 0 = M\,e(t) + A_{df}\,\tilde{x}(t-d) - L\,F\,f(t) \tag{20}$$

And by consequence:

$$L\,F\,f(t) = M\,e(t) + A_{df}\,\tilde{x}(t-d) \tag{21}$$

In our case F is scalar and equal to 1, the gain of observer $L = \begin{bmatrix} L_1 & L_2 \end{bmatrix}^T$ is designed by using the augmented system, then LF is not invertible, for this reason we make the following decomposition:

$$L_1\,F\,\hat{f}(t) = M_{11}\,\tilde{x}(t) + M_{12}\,\tilde{\varepsilon}(t) + A_{df1}\,\tilde{x}(t-d) \tag{22}$$

Then the expression of the sensor fault amplitude is given by this equation:

$$\begin{aligned} \hat{f}(t) = (L_1\,F)^{-1}\,&(M_{11}\,\tilde{x}(t) + M_{12}\,\tilde{\varepsilon}(t) \\ &+ A_{df1}\,\tilde{x}(t-d)) \end{aligned} \tag{23}$$

3.2. Compensation of sensor fault

The compensation for sensor fault effect on the closed-loop system can be achieved by adding a new control law to the nominal one [13, 14]:

$$u(t) = k\,x(t) + \varepsilon(t) + u_{ad}(t) \tag{24}$$

The output and the integrator are affected such that:

$$\begin{cases} y(t) = C\,x(t) + F\,f(t) \\ \varepsilon(t) = \varepsilon(t) + f_\varepsilon(t) \\ f_\varepsilon(t) = \int (-F\,f(t))\,dt \end{cases} \tag{25}$$

If $C = 1$, by using (24) and (25) we can write the control law as follow:

$$u(t) = k\,x(t) + k\,F\,\hat{f}(t) + \varepsilon(t) + f_\varepsilon(t) + u_{ad}(t) \tag{26}$$

The effect of sensor fault, in control and by consequence in system, can be compensated by using the fault amplitude estimation and by calculating the additive control as follow:

$$u_{ad}(t) = -\,k\,F\,\hat{f}(t) - \ f_\varepsilon(t) \tag{27}$$

4. SIMULATION EXAMPLE

4.1. Estimation and compensation of sensor fault

Consider the linear time invariant delayed system:

$$\begin{cases} \dot{x}(t) = -x(t) - x(t-d) + u(t) \\ y(t) = x(t) \end{cases} \tag{28}$$

With: $d = 1s$.
The state feedback control determined by using the relation (9) leads to the numerical value of gain $k = -21.86$, this gain is obtained by placing -3 as desired eigenvalue of S.
The closed loop-system augmented by integrator becomes:

$$\begin{cases} \dot{X}(t) = \begin{bmatrix} -22.86 & 1 \\ -1 & 0 \end{bmatrix} X(t) + \begin{bmatrix} -1 \\ 0 \end{bmatrix} x(t-d) \\ \qquad + \begin{bmatrix} 0 \\ 1 \end{bmatrix} y_r(t) \\ y(t) = \begin{bmatrix} 1 & 0 \end{bmatrix} X(t) \end{cases} \tag{29}$$

Then the nominal control law take this form:

$$u(t) = -21\,.\,86\;x(t)\ +\ \varepsilon(t) \tag{30}$$

To estimate the sensor fault amplitude we use the Luenberger observer represented by (18). The observer gain L is determined such that the poles of $(A_f - LC_f)$ equal 20 times of the ones of (A_f). So the numerical value of L is: $L = \begin{bmatrix} 838 & 398 \end{bmatrix}^T$ It's clear that the matrix F is scalar and the gain L is a vector, so (LF) is not invertible, for this reason we use the decomposition represented by equation (22). Let us now examine the influence of sensor fault on the delayed system and the way to compensate for their effect. We will consider the type of sensor fault as a bias. The value of sensor fault is initialized to zero and it's switched to 0.1 at moment $t = 400s$, then the output equation of system (28) becomes:

$$y_f(t) = y(t) + 0.1 \tag{31}$$

The input reference is equal to: $y_r(t) = 0.5$.

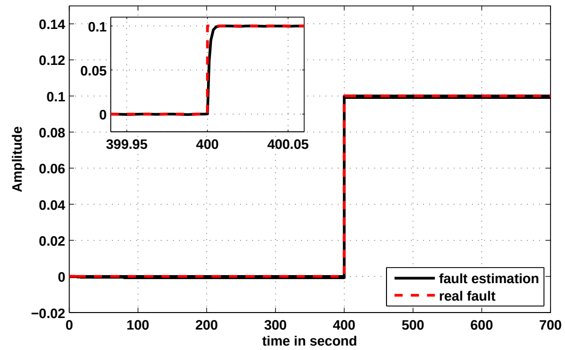

Figure 2. Sensor fault estimation.

Figure 2 shows that the observer can estimate the amplitude of sensor fault.

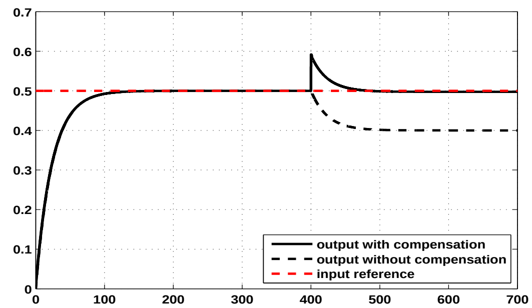

Figure 3. Time delay system output with and without fault compensation.

Figure 3 shows the time evolution of the output of delayed system in the occurrence of sensor fault. It's clear, in the curve of the output with compensation, that the additive control added to the nominal control can compensate the effect of sensor fault.

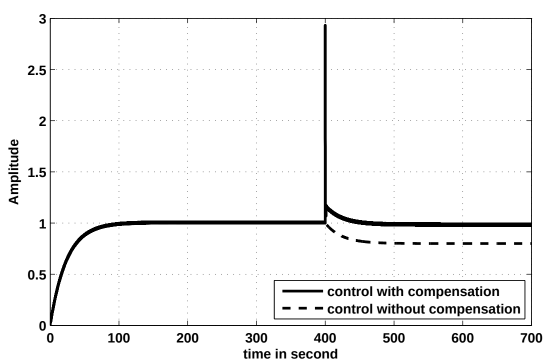

Figure 4. Time delay system control with and without fault compensation.

Figure 4 shows that the control law takes its steady value after compensation of sensor fault, but at a fault occurrence time the control law effect an important deviation. This deviation due to the important value of the observer gain and it is relative to the dynamic of same delayed system. The deviation can be accepted if we haven't constraints in the control law.

4.2. Effect of observer gain value to the compensation

In this section we present the effect of the observer gain value of the fault compensation error.

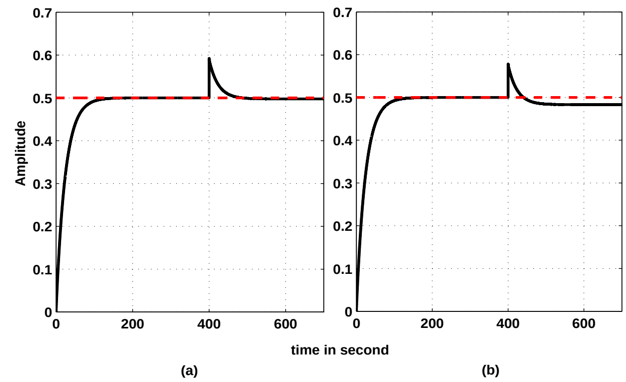

Figure 5. Time delay system output with fault compensation for different value of observer gain.

a: Case of L determinate such that the poles value of $(A_f - LC_f)$ equal 20 times of the ones of (A_f).
b: Case of L determinate such that the poles value of $(A_f - LC_f)$ equal 3 times of the ones of (A_f).

It's clear that if the value of L is more important, the error of compensation tend to zero.

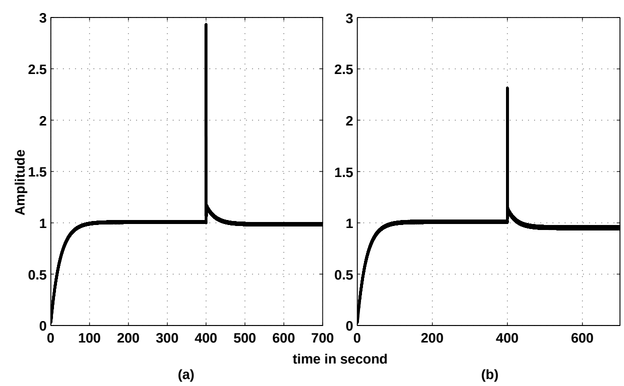

Figure 6. Time delay system control with fault compensation for different value of observer gain.

a: Case of L determinate such that the poles value of $(A_f - LC_f)$ equal 20 times of the ones of (A_f).
b: Case of L determinate such that the poles value of $(A_f - LC_f)$ equal 3 times of the ones of (A_f).

In other part, if the value of L is more important the deviation of control in the occurrence of fault is more important. The optimal case is choose according to the desired performance, for example, if we haven't constraints in the control law we can choose the case (a), but if the control law had a saturation we must reduce the value of L by accepting an error of fault compensation, this case we encouraged to study the fault tolerant control with constraints.

5. CONCLUSION

In this paper we have treated the fault tolerant control of time-delayed system by using the Luenberger observer to estimate the fault amplitude. An additive control is added to the nominal control law determinate by using Lambert W approach to compensate the effect of fault.
We note that the use of observer provoke a deviation in the control evaluation at time of fault occurrence this we encourage to study the fault tolerant control with constraints.

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