SOLVING FULLY FUZZY LINEAR PROGRAMMING PROBLEM WITH EQUALITY AND INEQUALITY CONSTRAINTS

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ABSTRACT

Fuzzy linear programming problems are models in which all parameters as well as the variables are represented by fuzzy numbers and are known as FFLP problems. In this paper a new method is proposed to find the fuzzy optimal solution of FFLP problems including both equality and inequality constraints. Numerical examples are solved to show capability of the proposed method in real life situations that the inherent fuzziness of a decision problem demands a fuzzy decision to be taken. The main advantage of the proposed method is simplicity of computations.

Keywords: Fully Fuzzy Linear Programming Problems, Triangular Fuzzy numbers, fuzzy ranking.

1. INTRODUCTION

The first application of FST to decision-making processes was presented by Bellman and Zadeh (1965) and the concept of fuzzy mathematical programming on general level was first proposed by Tanaka et al. (1973). The first formulation of fuzzy linear programming (FLP) is proposed by Zimmermann (1978).Various fuzzy linear programming techniques are surveyed in literature which are classified into two main classes: fuzzy linear programming (Verdegey 1982, Warners 1987a, Warners 1987b, Zimmerman 1976, Chanas 1983, Verdegey 1984a, Verdegey 1984b) and possibilistic linear programming (Carlsson and Corhonen 1986, Ramik and Rimanek 1985, Tanaka Ichihashi and Asai 1984, Lai and Hwang 1992, Rommelfanger Hanuscheck and Wolf 1989, Buckley 1988). For the sake of simplicity, usually these techniques consider only crisp solutions of the fuzzy problems (Stanciulescu Fortemps Install and Wertz 2003).

Generally speaking, in fuzzy linear programming problems, the coefficients of decision variables are fuzzy numbers while decision variables are crisp ones. This means that in an uncertain environment, a crisp decision is made to meet some decision criteria (Tanaka Guo and Zimmermann 2000), thus the decision-making process is constrained to crisp decisions that hide the fuzzy aspect of the problem. Buckley and Feuring (2000), have developed a kind of fuzzy linear programming problems in which all the parameters as well as the variables are represented by fuzzy numbers which is known as FFLP problems. Then some other authors have proposed different methods to solve FFLP problems under two categories: FFLP problems with inequality constraints (Maleki Tata and Mashinchi 2000, Buckley and Feuring 2000, Hashemi Modarres and Nasrabadi 2006, Allahviranloo Lotfi Kiasary Kiani and Alizadeh 2008) and FFLP problems with equality constraints (Dehghan Hashemi and Ghatee 2006, Lotfi Allahviranloo Jondabeha and Alizadeh 2009, Kumar Kaur and Singh 2011). As pointed out by Buckley and Feuring (2000), searching for the optimal solutions of FFLP problems is very difficult. They used evolutionary algorithm to find the solution.

The paper is organized as follows: in section 2, some basic definitions of fuzzy set theory and arithmetic between two triangular fuzzy numbers are reviewed; In Section 3 formulation of FFLP problems are discussed; In Section 4 a new method is proposed for solving FFLP presented in section3; To illustrate the proposed method, numerical examples are solved in Section 5; Conclusion is drawn in Section 6

2. PRELIMINARIES

Definition 2.1.

A fuzzy number $\tilde{A} = (m_A, w_A, \acute{w}_A)$ *is a LR type if and only if:*

$$A(x) = \begin{cases} L\left(\frac{m_A - x}{w_A}\right) & -\infty < x \le m_A \\ R\left(\frac{x - m_A}{\acute{w}_A}\right) & m_A \le x < +\infty \end{cases} \quad w_A, \acute{w}_A \ge 0$$

Where m_A is the center (core) and w_A and $\acute{w}_A$ are the left and right bandwidths of $\tilde{A}$ respectively. This is a parametric form of fuzzy number $\tilde{A}$, so we can show it as a triangular shape as follow:

$\tilde{A} \equiv (m, w_A, \acute{w}_A)_{LR}$

Definition 2.2.
A LR type Fuzzy number is said to be a triangular fuzzy number if and only is its membership function is as follow:

$$A(x) = \begin{cases} 1 - \frac{m_A - x}{w_A} & m_A - w_A < x \le m_A \\ 1 - \frac{x - m_A}{\acute{w}_A} & m_A \le x < m_A + \acute{w}_A \\ 0 & otherwise \end{cases}$$

The graphical display of a triangular fuzzy number A is shown in figure (1).

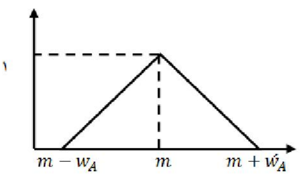

Figure 1: Graphical display of a triangular fuzzy number

Definition 2.3.
Given two triangular fuzzy numbers $\tilde{A} = (a, w_A, \acute{w}_A)_{LR}$ and $\tilde{B} = (b, w_B, \acute{w}_B)_{LR}$, $\tilde{B} < \tilde{A}$ if and only if:

b < a and then $w_B + \acute{w}_B \ge w_A + \acute{w}_A$

And $\tilde{B} = \tilde{A}$ if and only if

b = a and $w_B + \acute{w}_B = w_A + \acute{w}_A$

Definition 2.4.
A triangular fuzzy number $\tilde{A} = (a, w_A, \acute{w}_A)_{LR}$ is said to be non-negative fuzzy number if and only if $a - w_A \ge 0$

Definition 2.5.
Given two fuzzy numbers $\tilde{A} = (a, w_A, \acute{w}_A)_{LR}$ and $\tilde{B} = (b, w_B, \acute{w}_B)_{LR}$ with continuous nondecreasing function over $[0, \infty)$, fuzzy Arithmetic operations are defined as follows:

$\tilde{A} + \tilde{B} = (a + b, w_A + w_B, w_A \acute{+} \acute{w}_B)_{LR}$

$\tilde{A} - \tilde{B} = (a - b, w_A + \acute{w}_B, w_A \acute{+} w_B)_{LR}$

$(a, w_A, \acute{w}_A)_{LR}.(b, w_B, \acute{w}_B)_{LR} \approx (a.b, aw_B + bw_A, a\acute{w}_B + b\acute{w}_A)_{LR}$ *for $\tilde{A} > 0$ and $\tilde{B} > 0$*

$(a, w_A, \acute{w}_A)_{LR}.(b, w_B, \acute{w}_B)_{LR} \approx (a.b, -a\acute{w}_B + bw_A, -aw_B + b\acute{w}_A)_{LR}$ *for $\tilde{A} < 0$ and $\tilde{B} > 0$*

3. FULL FUZZY NUMBER LINEAR PROGRAMMING PROBLEMS (FFLP)

A Full fuzzy number linear programming problem type (FFLP) is defined as:

$Max\ (or\ Min)\ \tilde{Z} = (\tilde{C}^T \otimes \tilde{x})$

$$Subject\ to{:}\ \tilde{A} \otimes \tilde{x} \overset{\le}{\underset{\ge}{(=)}} \tilde{b} \qquad (3.1)$$

$\tilde{x} \ge 0$

Where $\tilde{x} = (x_m, w, \acute{w}), \tilde{C} = (c, w_c, \acute{w}_c), \tilde{A} = (A, w_A, \acute{w}_A), b = (b, b_w, b_{\acute{w}}), x_m = \text{core}\ (\tilde{x}), c = \text{core}(\tilde{C}), A = \text{core}(\tilde{A}), b = \text{core}\ (\tilde{b})$ and $w, \acute{w}, w_c, \acute{w}_c, w_A, \acute{w}_A, b_w$ and $b_{\acute{w}}$ are the left and right bandwidths of $\tilde{x}$, $\tilde{C}$, $\tilde{A}$ and $\tilde{b}$ respectively. $\tilde{x} \epsilon F^n, \tilde{b} \epsilon F^M, \tilde{A} = [a_{ij}]^{M \times n} \epsilon F^{M \times n}, \tilde{C} \epsilon F^n$.

Therefore the above FFLP problem can be written as:

$Max\ (or\ Min)\ \tilde{Z} = \sum_{j=1}^{n} (c_j, w_{c_j}, \acute{w}_{c_j}) \otimes (x_j, w_j, \acute{w}_j)$

Subject to:

$$\sum_{j=1}^{n} \left(a_{ij}, w_{a_{ij}}, \acute{w}_{a_{ij}}\right) \otimes \left(x_j, w_j, \acute{w}_j\right) \begin{pmatrix} \le \\ = \\ \ge \end{pmatrix} \left(b_i, w_{b_i}, \acute{w}_{b_i}\right)$$

$\forall i = 1, 2, \dots, M,\ j = 1, \dots, n$

$(x_j, w_j, \acute{w}_j)\ non-negative\ j = 1, \dots, n \qquad (3.2)$

Definition 3.1.
We say that fuzzy vector $\tilde{x} = (x_{jm}, w_j, \acute{w}_j)$ is a fuzzy feasible solution to the problem (3.2) if $\tilde{x}$ satisfies the constraints of the problem.

Definition 3.2.
A fuzzy feasible solution $\tilde{x}^*$ is a fuzzy optimal solution for (3.2), if for all fuzzy feasible solution $\tilde{x}$ for (3.2), we have $\tilde{c} \otimes \tilde{x}^* \tilde{\ge} (\tilde{\le}) \tilde{c} \otimes \tilde{x}$

4. PROPOSED METHOD TO FIND THE FUZZY OPTIMAL SOLUTION OF THE FFLP PROBLEM.

In this section, a new method is proposed to find the fuzzy optimal solution of the FFLP problems:

First, using the arithmetic operations presented in definition (2.5), the problem (3.2) is written in the form of problem (4.1):

$$Max\ (Min)Z = [\sum_{j=1}^{n} c_j x_j , \sum_{j\epsilon C_p}\left(c_j w_j + x_j w_{c_j}\right) + \sum_{j\epsilon C_{\acute{p}}}\left(-c_j \acute{w}_j + x_j w_{c_j}\right), \sum_{j\epsilon C_p}\left(c_j \acute{w}_j + x_j \acute{w}_{c_j}\right) + \sum_{j\epsilon C_{\acute{p}}}\left(-c_j w_j + x_j \acute{w}_{c_j}\right)]$$

$$s.t. \left[\sum_{i=1}^{M}\sum_{j=1}^{n} a_{ij}\, x_j, \sum_{a_{ij}\epsilon A_p}\left(a_{ij} w_j + x_j w_{a_{ij}}\right) + \sum_{a_{ij}\epsilon A_{\acute{p}}}\left(-a_{ij} \acute{w}_j + x_j w_{a_{ij}}\right), \sum_{a_{ij}\epsilon A_p}\left(a_{ij} \acute{w}_j + x_j \acute{w}_{a_{ij}}\right) + \sum_{a_{ij}\epsilon A_{\acute{p}}}\left(-a_{ij} w_j + x_j \acute{w}_{a_{ij}}\right)\right] \begin{matrix}\le\\(=)\\\ge\end{matrix} (b_i, w_{b_i}, \acute{w}_{b_i})$$

$$\forall i = 1,2,\dots,M\ , j = 1,\dots,n \qquad (4.1)$$

$$(x_j, w_j, \acute{w}_j)\ non-negative\ j = 1,\dots,n$$

Where:

$C_p = \{j|\tilde{c}_j \ \ is\ non\ negative\}$

$C_{\acute{p}} = \{j|\tilde{c}_j \ \ is\ negative\}$

$A_p = \{j|\tilde{a}_{ij} \ \ is\ non\ negative\}$

$A_{\acute{p}} = \{j|\tilde{a}_{ij}\ is\ negative\}$

The objective function denotes a triangular fuzzy number which its center, left side and right side bandwidth are as follows:

$$Z_m = \sum_{j=1}^{n} c_j x_j$$

$$w_Z = \sum_{j\epsilon C_p}\left(c_j w_j + x_j w_{c_j}\right) + \sum_{j\epsilon C_{\acute{p}}}\left(-c_j \acute{w}_j + x_j w_{c_j}\right)$$

$$\acute{w}_Z = \sum_{j\epsilon C_p}\left(c_j \acute{w}_j + x_j \acute{w}_{c_j}\right) + \sum_{j\epsilon C_{\acute{p}}}\left(-c_j w_j + x_j \acute{w}_{c_j}\right)$$

Furthermore the left hand sides of the constraints are triangular fuzzy numbers as well. The components are as follows:

$$(AX)_m = \sum_{i=1}^{M}\sum_{j=1}^{n} a_{ij}\, x_j$$

$$(AX)_w = \sum_{a_{ij}\epsilon A_p}\left(a_{ij} w_j + x_j w_{a_{ij}}\right) + \sum_{a_{ij}\epsilon A_{\acute{p}}}\left(-a_{ij} \acute{w}_j + x_j w_{a_{ij}}\right)$$

$$(AX)_{\acute{w}} = \sum_{a_{ij}\epsilon A_p}\left(a_{ij} \acute{w}_j + x_j \acute{w}_{a_{ij}}\right) + \sum_{a_{ij}\epsilon A_{\acute{p}}}\left(-a_{ij} w_j + x_j \acute{w}_{a_{ij}}\right)$$

As mentioned in the definition (2.3), a triangular fuzzy number will be larger than another triangular fuzzy number, if its center is larger and its bandwidths are smaller than the other one. So, to maximize (minimize) the value of the objective function that is a triangular fuzzy number, we can maximize (minimize) the mean and minimize (maximize) the bandwidths. In addition, constraints show a comparison of two triangular fuzzy number and we can use the definition (2.3) for such a comparison.

So in the second stage the problem (4.1) is converted to the following crisp LP problem using the fuzzy ranking method defined in definition (2.3). :

$$Max\ (Min)Z_m = \sum_{j=1}^{n} c_j x_j$$

$$Min\ (Max)(w_Z + \acute{w}_Z) = \sum_{j\epsilon C_p}\left(c_j w_j + x_j w_{c_j}\right) + (c_j \acute{w}_j + x_j \acute{w}_{c_j}) + \sum_{j\epsilon C_{\acute{p}}}\left(-c_j \acute{w}_j + x_j w_{c_j}\right) + (-c_j w_j + x_j \acute{w}_{c_j})$$

$$s.t.\ \ \sum_{i=1}^{M}\sum_{j=1}^{n} a_{ij}\, x_j \begin{matrix}\le\\(=)\\\ge\end{matrix} b_i$$

$$\sum_{a_{ij}\epsilon A_p}\left(a_{ij} w_j + x_j w_{a_{ij}}\right) + \left(a_{ij} \acute{w}_j + x_j \acute{w}_{a_{ij}}\right) + \sum_{a_{ij}\epsilon A_{\acute{p}}}\left(-a_{ij} \acute{w}_j + x_j w_{a_{ij}}\right) + \left(-a_{ij} w_j + x_j \acute{w}_{a_{ij}}\right) \begin{pmatrix}\ge\\=\\\le\end{pmatrix} w_{b_i} + \acute{w}_{b_i}$$

$$\forall i = 1,2,\dots,M\ , j = 1,\dots,n \qquad (4\text{-}2)$$

Where:

$C_p = \{j|\tilde{c}_j \ \ is\ non\ negative\}$

$C_{\acute{p}} = \{j|\tilde{c}_j \ \ is\ negative\}$

$A_p = \{j|\tilde{a}_{ij} \ \ is\ non\ negative\}$

$A_{\acute{p}} = \{j|\tilde{a}_{ij}\ is\ negative\}$

In the next stage we find the optimal solution x_j, w_j and $\acute{w}_j$ by solving the problem (4.2). Then we put the values x_j, w_j, $\acute{w}_j$ in $\tilde{x} = (x_{jm}, w_j, \acute{w}_j)$ and find the fuzzy optimal solution. Finally, we calculate the fuzzy optimal value of the objective function by putting $\tilde{x}$ in $\sum_{j=1}^{n}(c_j, w_{c_j}, \acute{w}_{c_j}) \otimes (x_j, w_j, \acute{w}_j)$.

5. EXAMPLES

In this Section two FFLP examples including equality and inequality constraints as well as some negative fuzzy coefficients are solved to show the capability of the proposed method in different situations.

Example 5-1:

To illustrate the efficiency of the method we consider an example used by Kumar Kaur and Singh (2011). Choosing this example is intentionally to compare the efficiency of our method with the efficiency of their method. This example includes equality constraints with $\tilde{a}_{ij} \geq 0, i = 1, \dots, \mathrm{M}\ and\ j = 1, \dots, n.$

In the form we have presented our equations and models in our paper, the Kumar example can be rewritten as bellow:

$Max\, Z_1 = (2,1,1) \otimes (x_{1m}, w_1, \acute{w}_1) + (3,1,1) \otimes (x_{2m}, w_2, \acute{w}_2)$

$s.t.\quad (1,1,1) \otimes (x_{1m}, w_1, \acute{w}_1) + (2,1,1) \otimes (x_{2m}, w_2, \acute{w}_2) = (10,8,14)$

$(2,1,1) \otimes (x_{1m}, w_1, \acute{w}_1) + (1,1,1) \otimes (x_{2m}, w_2, \acute{w}_2) = (8,7,13)$

According to the proposed method the above LP is converted to:

$\mathrm{Max}\, 2x_{1m} + 3x_{2m}$

$\mathrm{Min}\, 2x_{1m} + 2x_{2m} + 2w_1 + 2\acute{w}_1 + 3w_2 + 3\acute{w}_2$

Subject to:

$x_{1m} + 2x_{2m} = 10$

$x_{1m} + x_{2m} + 1w_1 + 2w_2 = 8$

$x_{1m} + x_{2m} + 1\acute{w}_1 + 2\acute{w}_2 = 14$

$2x_{1m} + x_{2m} = 8$

$x_{1m} + x_{2m} + 2w_1 + w_2 = 7$

$x_{1m} + x_{2m} + 2\acute{w}_1 + \acute{w}_2 = 13$

$x_{1m} - w_1 \geq 0 \quad , \quad x_{2m} - w_2 \geq 0$

The solution of the problem is:

$(x_{1m}, w_1, \acute{w}_1) = (2,0,0)$

$(x_{2m}, w_2, \acute{w}_2) = (4,1,0)$

$\tilde{z}^* = (16,9,6)$

The optimal solution of the Kumar approach is:

$(x_{1m}, w_1, \acute{w}_1) = (2,1,1)$

$(x_{2m}, w_2, \acute{w}_2) = (4,2,2)$

$\tilde{z}^* = (16,12,17)$

Based on the definition (2.3), our solution finds more precise value for decision variable and better (larger value) for objective function. In a fuzzy environment a more precise decision is more helpful to decision makers.

Example 5-2:

Let us consider the following FFLP and solve it by the proposed method. This example includes inequality constraints in which:

$\exists\ i\ and\ j,\ \tilde{a}_{ij} \leq 0.$

$i = 1, \dots, \mathrm{M},\ j = 1, \dots, n$

The problem is:

$\mathrm{Max}\, Z_1 = (6,5,3) \otimes (x_{1m}, w_1, \acute{w}_1) + (3,1,5) \otimes (x_{2m}, w_2, \acute{w}_2)$

$\mathrm{s.t.}\quad (3,1,1) \otimes (x_{1m}, w_1, \acute{w}_1) + (2,1,1) \otimes (x_{2m}, w_2, \acute{w}_2) \lesssim (16,10,14)$

$(1,2,1) \otimes (x_{1m}, w_1, \acute{w}_1) + (3,2,1) \otimes (x_{2m}, w_2, \acute{w}_2) \lesssim (17,16,13)$

The above LP is converted to:

$\mathrm{Max}\, 6x_{1m} + 3x_{2m}$

$\mathrm{Min}\, 8x_{1m} + 6x_{2m} + 6w_1 + 6\acute{w}_1 + 3w_2 + 3\acute{w}_2$

Subject to:

$3x_{1m} + 2x_{2m} \leq 16$

$2x_{1m} + 2x_{2m} + 3w_1 + 2w_2 + 3\acute{w}_1 + 2\acute{w}_2 \geq 24$

$x_{1m} + 3x_{2m} \leq 17$

$3x_{1m} + 3x_{2m} - w_1 + 3w_2 - \acute{w}_1 + 3\acute{w}_2 \geq 29$

$x_{1m} - w_1 \geq 0 \quad , \quad x_{2m} - w_2 \geq 0$

The solution of the problem is:

$(x_{1m}, w_1, \acute{w}_1) = (5.33,0,1.27)$

$(x_{2m}, w_2, \acute{w}_2) = (0,0,4.76)$

$\tilde{z}^* = (31.98,26.65,37.89)$

As it can be seen, the proposed approach can deal with Fuzzy linear programming problems which contain inequality constraints and some negative fuzzy coefficient as well.

6. CONCLUSION

In this paper a new method is proposed to find the fuzzy optimal solution of FFLP problems; using the proposed method the fuzzy solution of FFLP problems, occurring in real life situation, can be easily obtained .With the aid of this, decision makers are able to obtain fuzzy decisions to reflect the inherent fuzziness of a decision problem, so the need to do sensitivity analysis after obtaining a crisp solution decreases. To illustrate the proposed method FFLP examples including equality and inequality constraints as well as some negative fuzzy coefficients are solved and result shows the capability of the method in different mentioned situations. The main advantage of the proposed method is simplicity of computations.

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