A MULTI-ITEM MULTI-RACK APPROACH FOR DESIGNING LIFO STORAGE SYSTEMS: A CASE STUDY FROM THE FOOD INDUSTRY

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ABSTRACT
In a variety of industrial sectors rack storage is adopted for holding stock-keeping units (SKUs) between production (or purchasing from external supplier) and delivery. It is well known that, among the different types of rack storage, racks accessed in a Last-In-Fist-Out manner are the most economically convenient solutions. Nevertheless, especially when product shelf-life is critical as occurs in the food industry, the design of LIFO storage systems is not trivial. Thus, this paper presents an approach able to take into consideration two different measures for assessing the performance of a storage system, with the aim of assigning each item type in inventory to the optimal type of lane racks. Lastly, a significant case study from the food industry is discussed.

Keywords: LIFO storage, efficiency, optimal design, food industry

1. INTRODUCTION

The design of rack storage systems into a multi-item environment presents several challenges and opportunities. A first distinction that needs to be made is between racks accessed according to a First-In-Fist-Out (FIFO) policy and a Last-In-Fist-Out (LIFO) policy. The reader may refer to Bartholdi and Hackman (2006) for a review of the main advantages and disadvantages of the different types of rack storage.

In particular, LIFO racks are the most space-saving (i.e. economically convenient) solution but require periodic replenishing/emptying cycles in order to allow all the stock-keeping units (SKUs), which are not independently accessible, to be retrieved within a reasonable period of time. As a consequence, especially in industries such as food products, where items in inventory typically have critical shelf lives, the adoption of LIFO solutions requires a thorough analysis taking into account more than a single performance measure.

One of the most commonly used measures for assessing the performance of storage systems is *space utilization* (see, e.g., surveys such as Van Den Berg, 1999, and Gua et al., 2010). Briefly, space utilization refers to the amount of aisle space per pallet location. The higher the capacity of a lane is, the higher the value of space utilization is (the same aisle space is spread over a larger number of pallet locations).

Recently, Ferrara et al. (2011) have stressed the importance of integrating such a traditional performance measure with a new indicator, called *storage efficiency*. Storage efficiency (see Ferrara et al., 2011) is a performance measure related to the number of pallet locations that are actually occupied by SKUs. Note that, once the first SKU of a certain item type has been placed into a LIFO lane, that lane is devoted to that item type until it becomes empty again. In other words, the other pallet locations, even if empty, are “constrained” (i.e. they can accept that specific item type only). As a consequence, storage efficiency for LIFO racks is typically less than 1. Moreover, as the number of “constrained” pallet locations arises, the adoption of such a solution becomes more critical. Nevertheless, works such as Ferrara et al. (2011) and Rimini et al. (2011) show that it is possible to maintain satisfactory values of the storage efficiency also in LIFO storage systems, if they are properly designed and operated.

Hence, the main contribution of this work paper is to integrate the two performance measures, i.e. space utilization and storage efficiency, into an innovative approach for designing optimal configurations of LIFO rack systems.

The remainder of the paper is organized as follows. Section 2 describes the problem under analysis. Section 3 presents the solution approach for assigning each item type in inventory to the optimal lane type. In Section 4 the solution approach is applied to a real case study from the food industry. Finally, Section 5 draws some conclusions.

2. PROBLEM DESCRIPTION

2.1. Objective

The focus of this paper is on storage systems where a set I of different item types (i.e. types of products presenting different input/output flows and physical features) is held as inventory in LIFO lane racks. Specifically, a set J of different lane types is assumed to be available, where lane types differ from each other in their capacity (expressed in number of available pallet locations).

In this paper a case study from the food industry is presented. Since the focus is on perishable products, the capacity of the LIFO lane should not increase too much in order to avoid that any SKU spends an excessive time in inventory. Specifically, the height of all the lanes is supposed to be one pallet (pallets cannot be stacked one on top of each other). Nevertheless, for an efficient use of space, lane racks may have more than a single level. Let us denote with H the number of levels of independent LIFO lanes of the same type. As an example, Figure 1 shows an agglomeration of H=2 levels of lanes with a capacity of 3 pallet locations each.

The objective of the study is to develop a solution approach for assigning each item type to the lane type that best suits the inventory requirements by taking into account both space utilization and storage efficiency.

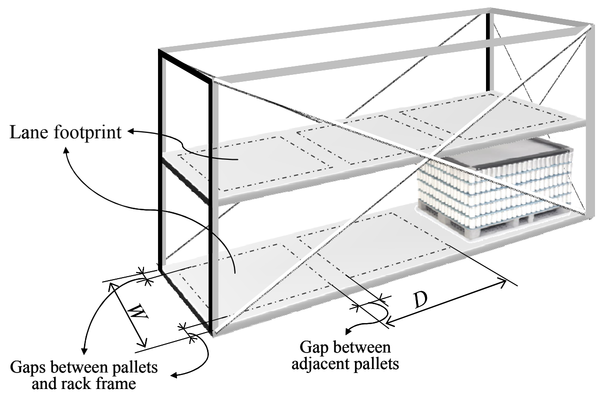

Figure 1: 2-level aggregation of 3-pallet deep lanes

2.2. Problem Data

Once the set I of item types and the set J of lane types has been identified, problem data must be collected or simulated when not available.

Typically, any Enterprise Resource Planning (ERP) system provides the firm with information about the input/output flows of each item type. Thus, even if actual data on inventory levels are not available, they can be easily simulated. We assume to consider a period of time composed of T time units (t.u.). Let us denote with q_{i0} the inventory level of item type i at the beginning of the first time unit t=1. Then, the inventory level of item type i at the end of t.u. t is as follows:

$$q_{it} = q_{it-1} + p_{it} - o_{it}\,, \quad \forall i,t\,. \tag{1}$$

where p_i is the input flow of i during t (coming from the production area or from external suppliers) and o_i is the output flow of i during t (going toward the shipping area).

As regards lane racks, let us denote with C_j the capacity of lane type j, where $C_{j'} \neq C_{j''}\,, \ \forall j', j'' \in J$.

Note that, given a certain item type i and lane type j (i.e. of capacity C_j), the average number of lanes of that type necessary for storing i is:

$$N_{ij} = \left\lceil \frac{\sum_t q_{it}}{C_j\, T} \right\rceil, \quad \forall i,j\,, \tag{2}$$

We assume that pallet dimensions are fixed, e.g. the dimensions of a standard pallet. In Figure 1, two 3-pallet deep lanes (i.e. C_j=3) are shown, stacked on top of each other in a 2-level agglomeration. For each lane we denote as W the total pallet location width (pallet width + gaps) and as D the total pallet location depth (pallet depth + gaps). Thus, the width of any lane is W and the total depth of a lane type j, i.e. with a capacity C_j, is $D\,C_j$. As a result, the footprint F_j of a lane type j is as follows:

$$F_j = WD\, C_j, \tag{3}$$

The lane footprints are indicated by light-grey areas in Figure 1.

Since lanes are assumed to be stacked in agglomerations of H levels each (e.g. H=2 in Figure 1), lanes of the same agglomeration are accessed from the same aisle. Hence, in order to compute the total amount of horizontal space required in the system, one-half the aisle width (denoted as A/2) must be considered for each agglomeration of H lanes. Note that the aisle width A depends on the type of forklift trucks or Automated Guided Vehicles (AGVs) used in the storage system for pallet handling.

Given the input data set discussed in this section, the solution approach for allocating each item type to the optimal lane type is discussed in the sequel.

3. SOLUTION APPROACH

3.1. Storage Efficiency and Inefficiency Weight

According to the notation and conventions defined in the previous section, the storage efficiency related to lanes of capacity C_j housing q_{it} SKUs of item type i during time unit t (see Ferrara et al., 2011) is:

$$e_{ijt} = \frac{q_{it}}{C_j \left\lceil \frac{q_{it}}{C_j} \right\rceil}, \quad \forall i,j,t\,. \tag{4}$$

Thus, the average storage efficiency for item type i and lane type j during the whole observation period T can be expressed as follows:

$$e_{ij} = \frac{\sum_t e_{ijt}}{T}, \quad \forall i,j\,. \tag{5}$$

As explained in Ferrara et al. (2011), a threshold level e_j^T can be identified for the storage efficiency of LIFO storage racks of capacity C_j. The threshold level e_j^T is the minimum storage efficiency required to store any item type into lanes of type j. It can be computed simply as the ratio between the space utilization of selective racks and the space utilization achievable with LIFO racks of capacity C_j. The comparison with selective racks is motivated by the trade-off between space utilization and storage efficiency. If C_j=1, i.e. in case of selective racks, the storage efficiency is always 1 for any item type, while the amount of aisle space required per pallet location arises. Thus, if both the performance measures are jointly considered, the efficiency threshold allows us to estimate the convenience of adopting lane racks instead of selective racks. Thus, given the storage efficiency computed according to Eq. (5) , if $e_{ij} \geq e_j^T$, it is more convenient to stock item type i into lanes of capacity C_j>1. Otherwise, the adoption of selective racks is suggested.

For the sake of convenience, we now define the inefficiency weight w_{ij} as the complement of the storage efficiency. So,

$$w_{ij} = 1 - e_{ij}, \quad \forall i, j\,. \tag{6}$$

This parameter is used in the mathematical formulation of the linear-programming model discussed in the next section.

3.2. Optimization Model

In this section a linear allocation model is presented in order to assign each item type to the optimal lane type. The objective is to minimize the total lane and floor space (including gaps between lanes and aisle space) required on average to store the SKUs of all the item types under analysis. Moreover, solutions presenting low values of the inefficiency weight, i.e. high values of the storage efficiency, are preferred.

Prior to the model formulation, the following additional assumptions must be made:

- The storage system is assumed to be uncapacitated (a large number of lanes of any type is available);
- Each item type can be assigned to a single lane type;
- Given a certain item type i, there exist at least one lane type j so that $e_{ij} \geq e_j^T$;
- All the agglomerations of lanes have the same number of levels.

The integer linear-programming model is as follows:

$$\min \sum_{i \in I} \sum_{j \in J} F_j N_{ij}\, w_{ij} x_{ij} + W \frac{A}{2} \sum_{j \in J} z_j \tag{7}$$

subject to

$$\sum_{j \in J} x_{ij} = 1, \quad \forall i\,, \tag{7.1}$$

$$e_j^T x_{ij} \leq \left(1 - w_{ij}\right), \quad \forall i, j\,, \tag{7.2}$$

$$\frac{1}{H} \sum_{i \in I} N_{ij} x_{ij} \leq z_j\,, \quad \forall j\,, \tag{7.3}$$

$$z_j \text{ integer}\,, \tag{7.4}$$

$$x_{ij} \in \{0,1\}, \tag{7.5}$$

where the input data are as follows:

- $i \in I$ item type;
- $j \in J$ lane type;
- F_j footprint of lane type j according to Eq. (3);
- e_j^T threshold level for the storage efficiency of lane type j (refer to Section 2);
- N_{ij} number of lanes type j necessary on average for storing item type i according to Eq. (2);
- w_{ij} inefficiency weight for the assignment item type i into lane type j (refer to Section 2);
- A aisle width;
- W lane width;
- H number of levels of a lane agglomeration;

The decision variable is x_{ij}. It is a binary variable that in the optimal solution is set to 1 if item type i is assigned to lane type j, 0 otherwise. Variable z_j allows the objective function to be linear: according to constraints (7.3) and (7.4), z_j assumes integer values only and represents the minimum number of agglomerations of H lanes of type j.

The objective function is composed of two contributions:

- The first term represents the total lane footprint for storing all the item types under analysis, weighted by the inefficiency weights w_{ij} . Hence, since the objective function defines the optimization problem as a minimization task, solutions with low values of the inefficiency weight are preferred;
- The second term represents the aisle space by considering a value equal to $\left(W \frac{A}{2}\right)$ for each agglomeration of H lanes (recall the above definition of z_j and the correspondent constraints).

Constraint (7.1) states that each item type must be assigned to a single lane type. Constraint (7.2) guarantees that each item type i can be assigned to lane type j only if the corresponding storage efficiency, equal to the complement of w_{ij} according to Eq. (6), is higher than the threshold level as defined in Section 2. Constraints (7.3) and (7.4) regard the values assumed by z_j, as described above. Constraints (7.5) is the classical integrity constraint.

In the optimal solution, each item type is assigned to the optimal lane type. Thus, the number of lanes of each type that should be installed in the storage area is:

$$N_j^* = \sum_i N_{ij} x_{ij}^*, \quad \forall j, \tag{8}$$

where x_{ij}^* represent the values assumed by the decision variable in the optimal solution.

Similarly, the total storage efficiency for the whole storage system in the optimal solution can be computed as follows:

$$e^* = \frac{\sum_j \sum_i e_{ij} N_{ij} x_{ij}^*}{\sum_j \sum_i N_{ij} x_{ij}^*}. \tag{9}$$

4. CASE STUDY

This case study has been done in collaboration with an Italian company producing glass and plastic containers for food products such as pasta sauces, pickled vegetables and marmalades.

4.1. Input Data

Item types differ from each other in either the type of container or the type of product or both. Thus, the same sauce in a 314 ml container or in a 720 ml container corresponds to two different item type. Specifically, the set I under analysis includes 1 597 item types.

The ERP system provides the analyst with daily information about the input/output flows of all item types. Thus, a past period of 228 days has been considered and the inventory level for each item type at the end of each day within the observation period has been evaluated according to Eq. (1). As an example, Table 1 shows the inventory levels of 4 different item types over 5 days.

Table 1: Inventory levels for 3 item types over 5 days

Days	Item Types i			
	1	*2*	*3*	*4*
Day_1	18	18	18	13
Day_2	23	15	15	4
Day_3	32	32	16	12
Day_4	5	5	5	2
Day_5	3	3	0	0

The set J of lane types comprises 13 typologies of lane racks that can potentially be installed in the storage area. Each lane type j has a certain capacity C_j. Let us consider standard pallets, i.e. width of 1.2 m and depth of 0.8 m. Since the dimensions of a pallet location should include the gaps between SKUs and the rack frame (see Figure 1), the width of a pallet location W can be set to about 1.5 m and the depth D to 0.85 m. Thus, the footprint of any lane type j can be computed according to Eq. (2). Table 2 lists the available lane types along with their capacities (expressed in terms of pallet locations) and footprints (expressed in square meters).

Table 2: Set of lane types

Lane Types j	C_j [# of pallet locations]	F_j [m^2]
1	3	3 825
2	4	5 100
3	5	6 375
4	6	7 650
5	7	8 925
6	8	10 200
7	9	11 475
8	10	12 750
9	11	14 025
10	12	15 300
11	13	16 575
12	14	17 850
13	15	19 125

Lanes of the same types are stacked in agglomerates. For each agglomerate the number of levels (i.e. number of lanes) is H=6. In this case study an AGVs system is adopted for pallet handling. So, the aisle width A can be set to 3.6 m.

4.2. Model Implementation and Results

Given the data set discussed above, the solution approach described in Section 3 can be applied to the case study.

Firstly, the threshold levels of the storage efficiency have been computed for each lane type according to Ferrara et al. (2011). The threshold levels are reported in the second column of Table 3.

Table 4: Storage Efficiency and Threshold levels

Lane Types j	Thres-hold e_j^T	Storage Efficiency e_{ij}			
		Item Types i			
		1	*2*	*3*	*4*
1	**99%**	99%	99%	91%	94%
2	**89%**	76%	92%	89%	97%
3	**83%**	87%	84%	76%	98%
4	**78%**	75%	94%	86%	85%
5	**75%**	64%	93%	81%	91%
6	**73%**	56%	81%	71%	83%
7	**72%**	93%	77%	63%	74%
8	**70%**	84%	69%	57%	98%
9	**69%**	80%	79%	87%	89%
10	**68%**	74%	90%	81%	82%
11	**67%**	68%	83%	87%	75%
12	**67%**	63%	77%	81%	70%
13	**66%**	59%	72%	76%	71%

Then, the storage efficiency e_{ij} of each item type i into lane type j can be obtained according to Eq. (5). For the sake of clarity, Table 4 shows the values of the storage efficiency for 4 item types if assigned to the lane types under analysis. As described in Section 3.1, only if $e_{ij} \geq e_j^T$ it could be convenient to assign item type i to lane type j. As an example, if item type i=1 is assigned to lane type j=3 (i.e. 5-pallet deep lanes according to Table) the corresponding storage efficiency is 87%. Since it is higher than the threshold level (83%), such an assignment is allowed. On the other hand, if the same item is assigned to lane type j=2 (i.e. 4-pallet deep lanes) the storage efficiency is lower than the threshold level. As a consequence, if lane type j=2 were the only one available in the system, it would be more convenient to store i into selective racks.

Since the liner-programming model described in Section 3.2 makes use of the parameter w_{ij} as defined by Eq. (6), in Table 5 the values of the storage efficiency are converted into inefficiency weights. If a certain assignment is not allowed (i.e. $e_{ij} < e_j^T$) a dash (-) is entered in the corresponding cell. Recall that in our solution approach any assignment that does not satisfy $e_{ij} \geq e_j^T$ is prevented by constraint (7.2).

Table 5: Inefficiency Weights

Lane Types	Inefficiency Weights, w_{ij}			
	Item Types i			
j	*1*	*2*	*3*	*4*
1	0.01	0.01	-	-
2	-	0.08	0.11	0.03
3	0.13	0.16	-	0.02
4	-	0.06	0.14	0.15
5	-	0.07	0.19	0.09
6	-	0.19	-	0.17
7	0.07	0.23	-	0.26
8	0.16	-	-	0.02
9	0.20	0.21	0.13	0.11
10	0.26	0.10	0.19	0.18
11	0.32	0.17	0.13	0.25
12	-	0.23	0.19	0.30
13	-	0.28	0.24	0.29

The linear-programming model was solved by using ILOG CPLEX 10.1 on a Pentium IV-3.2 GHz PC.

Once the optimal solution has been found, the number of necessary lanes of each type can be determined according to Eq. (8) as reported in Table 6.

The total amount of space required on average to store the SKUs and support the lane racks is 82 485 m^2 (78 894 m^2 of lane space + 3 591 m^2 of aisle space). Finally, according to Eq. (9) the total storage efficiency is 90,5%.

Since both the space requirements and the total storage efficiency are satisfactory and consistent with the company needs, this design solution can be adopted in practice.

Table 6: Number of lanes in the optimal solution

Lane Types j	# of necessary lanes
1	78
2	1 260
3	1 254
4	960
5	948
6	660
7	630
8	438
9	354
10	318
11	558
12	186
13	336

5. CONCLUSIONS

The paper addresses the design of LIFO storage systems. Particularly, a new solution approach able to assign each item type to the optimal lane type has been developed. The application to a significant case study from the food industry has been described and the ability of the method to produce a satisfactory solution has been proved.

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