IMPROVEMENT OF THE FORECAST OF ECONOMIC PROCESSES PARAMETERS

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ABSTRACT
In work the problem of synthesis of mathematical model of economic process is examined in deterministic statement. It is supposed that the amount of measurements by each variable minimally and coincides with number of variable in model. Such problem can also be named as fast identification of parameters of mathematical model of economic processes. The some possible variants of statement of such problem are considered. The retrospective test calculation on real measurements were executed for comparison with known methods.

Keywords: economic processes, identification parameters, different statements, regularization method

1. INTRODUCTION

The classical problem of parameters identification of linear stationary many-dimensional model consists in synthesis of linear connection between the chosen characteristics of process $q_1, q_2, , , , q_n$ (Grop 1979, Seidg and Melsa 1974). For simplicity we shall consider only problem of construction of linear model:

$$q_1 = z_1 q_2 + z_2 q_3 + z_{n-1} q_n + z_n , \qquad (1)$$

where $z_1, z_2, , , , z_n$ – unknown coefficients of stationary mathematical model.

Let's denote $z^T = (z_1, z_2, , , , z_n)$, $(.)^T$ – the mark of transposition. It is supposed that for everyone variable q_i $(i = 1,2,,,,n)$ we have m of measurements q_{ik} $(k = 1,2,,,,m)$, $n < m$. Let's designate $q_i^T = (q_{i1}, q_{i2}, ,,, q_{im})$.

This problem can be reduced to the solution of the redefined linear non-uniform system of the algebraic equations which are being executed, as a rule, by method of least squares (Grop 1979, Seidg and Melsa 1974). It is supposed that statistical characteristics for all variables are given. However, the similar information can be received only on the basis of numerous experiments during long time (infinite time). For a big interval of time the hypothesis about a constancy of properties of the considered process (for example, constancy of coefficients of linear model) can not be fair. So it makes the process of parameters identification of linear mathematical model groundless.

Last years some new approaches to the solution of a problem of parameters identification of mathematical model have been offered in which the statistical characteristics of variables are not used (Kunchevich, Lichak and Nikitenko 1988, Polajk and Nazin 2006, Gubarev and Tigunov 2006). Concerning an errors of measurements the hypothesis is accepted that they are limited on absolute size to some given constant. Such initial preconditions in the greater degree correspond to a real situation. In work (Kunchevich, Lichak and Nikitenko 1988), for example, the problem of solution of system of the linear algebraic equations is considered at which the right part and matrix of system are approximately given. Naturally the result of the solution of a problem in such statement will be the some set of the possible solutions. It is shown that if all matrixes of system are non-degenerate the set is limited and correspond to the convex polyhedrons.

In article (Polajk and Nazin 2006) the problem of obtaining of the guaranteed multiplicity characteristics for researched variable process is formulated. If in family of the approached matrixes of linear system have not singular matrixes the effective algorithm of the solution of the put problem is offered.

In the given work the problem of synthesis of parameters of multivariate regress is considered in which the information about statistical properties of measurements are not used also (Kunchevich, Lichak and Nikitenko 1988, Polajk and Nazin 2006, Gubarev and Tigunov 2006). However in this case it is supposed that the singular matrixes and close to them belong to possible matrixes of linear system with guarantee (Menshikov 2004). The number of measurements is minimal and equals to number of variables of researched process. It allows attribute such algorithm to fast algorithms of identification of parameters.

We shall present the problem of synthesis of linear mathematical model with n variables relatively q_1 for number of measurements $m = n$, as a problem of the solution of system (Menshikov 2004)

$$A_p(q_2, q_3, \ , \ , \ q_n)\, z = q_1 , \qquad (2)$$

where the operator $A_p(q_2, q_3, \ , \ , \ q_n)\, z$ is determined as follows

$$A_p(q_2, q_3, \ , \ , \ q_n)\, z = z_1 q_2 + z_2 q_3 + z_{n-1} q_n + z_n e\,,$$

e is the unit vector of dimension n .
As the measurements of variables are received experimentally it is assumed that each measurement q_{ij}, $1 \le i, j \le n$ has some error the maximal size of which is known:

$$\left| q_{ij} - q_{ij}^{ex} \right| \le \delta_i\,,\ 1 \le j \le n, i = 1,2,\ ,\ ,\ ,\ n\,, \tag{3}$$

where q_{ij}^{ex} is exact measurements of variable q_i .
The similar information of measurement errors, as a rule, is known a priori. The statistical characteristics of errors of measurements are unknown.
Let us denoted vector p as vector from space $R^n \oplus R^n \oplus R^n \oplus \ldots . \oplus R^n = R^{n(n-1)}$:

$$p^T = (q_{21},\ ,\ ,\ ,\ q_{2n}, q_{31},\ ,\ ,\ q_{3n},\ ,\ ,\ ,\ q_{n1},\ ,\ ,\ , q_{nn})\,,$$

where R^n is Euclidean vector space.
Each vector q_i can accept meanings in some closed area $D_i \subset R^n$ by virtue of inequalities (3). Vectors p can accept meanings in some closed area $D = D_2 \oplus D_3 \oplus D_4 \oplus \ldots . \oplus D_n \subset R^{n(n-1)}$. The certain operator A_p associates with each vector p from area D. The class of operators $\{A_p\} = K_A$ will correspond to the set $D \subset R^{n(n-1)}$.
Shall we rewrite (2) as

$$A_p z = u_{\delta_1}\,, \tag{4}$$

where
$u_{\delta_1} = q_1;\ u_{\delta_1} \in U = R^n;\ z \in Z = R^n;\ \left\| u_{\delta_1} - u_1^{ex} \right\| \le \delta_1,\ u_1^{ex}$ – exact right part of (4) ; $\left\| A^{ex} - A_p \right\|_{Z \to U} \le h,\ A^{ex}$ – exact operator in (4); $\|.\|$ is the norm of a vector in Euclidean space R^n .
Let us consider now the set of the solutions of the equation (4) with the fixed operator $A_p \in K_A$:

$$Q_{\delta_1, p} = \{ z :\ \left\| A_p z - u_{\delta_1} \right\| \le \delta_1 \}\,.$$

The set $Q_{\delta_1, p}$ is limited if $\Delta = \det A_p \ne 0$ and unlimited if $\Delta = \det A_p = 0$.

2. PROBLEM STATEMENTS

Any vector z from set $Q_{\delta_1, p}$ is the good mathematical model of process so this vector after action of the operator A_p vector $A_p z$ coincides with the given vector q_1 with accuracy of measurement δ_1 . For choice of particular model from set $Q_{\delta_1, p}$ it is necessary to use additional conditions. If such conditions are absent then it is possible to accept as the solution (4) the element $z_p = Q_{\delta_1, p}$ for which the equality is carried out (Menshikov and Nakonecnhij 2003)

$$\left\| z_p \right\|^2 = \inf_{z \in Q_{\delta_1, p}} \| z \|^2\,. \tag{5}$$

The vector $z_p = Q_{\delta_1, p}$ is possible to interpret as a maximum steady element to the change of the factors not taken into account (most stable part), as the influence of these factors will increase norm of a vector z_p (Menshikov and Nakonecnhij 2003). Such a property of the solution z_p is especially important if one takes into account that the vector z_p further will be used for forecasting real processes (parameter q_1).
Consider now the set $Q^* = \bigcup_{p \in D} Q_{\delta_1, p}$ (Menshikov 2004).
Let us consider an extreme problem

$$\left\| z^* \right\|^2 = \inf_{p \in D} \inf_{z \in Q_{\delta_1, p}} \| z \|^2\,. \tag{6}$$

The vector $z^* = Q^*$ is an estimation from below of possible solutions of the equation (4). The similar problem in classical identification is not examined.
The statement of the following extreme problem is possible also:

$$\left\| z_{\sup}^* \right\|^2 = \sup_{p \in D} \inf_{z \in Q_{\delta_1, p}} \| z \|^2\,. \tag{7}$$

The vector $z_{\sup}^* \in Q^*$ has the greatest norm among the solutions of a problem of synthesis on sets $Q_{\delta_1, p}$. The similar problem in the literature is not considered either.
Models z^*, $z_{\sup}^*$ can be used for short-term forecasting of change of variable q_1 as on the one hand models z^*, $z_{\sup}^*$ are received by an rapid way and on the other hand these models are steadiest to the change of the factors not taken into account.

Except (6), (7) it is possible to examine the following statements of problems:

$$\left\| z_{0,0,...,1} \right\|^2 = \inf_{q_2 \in D_2} \inf_{q_3 \in D_3} ... \inf_{q_{n-1} \in D_{n-1}} \sup_{q_n \in D_n} \inf_{z \in Q_{\delta_1, p}} \| z \|^2 \quad (8)$$

$$\left\| z_{0,0,...,1,1} \right\|^2 = \inf_{q_2 \in D_2} \inf_{q_3 \in D_3} ... \sup_{q_{n-1} \in D_{n-1}} \sup_{q_n \in D_n} \inf_{z \in Q_{\delta_1, p}} \| z \|^2 \quad (9)$$

. . . .

$$\left\| z_{0,1,...,1,1} \right\|^2 = \inf_{q_2 \in D_2} \sup_{q_3 \in D_3} ... \sup_{q_{n-1} \in D_{n-1}} \sup_{q_n \in D_n} \inf_{z \in Q_{\delta_1, p}} \| z \|^2 \quad (10)$$

In some cases it is expedient to consider the following problems of identification of parameters:

$$\left\| z^{0,0,...,0} \right\|^2 = \inf_{z \in Q_{\delta_1, p^{0,0,...,0}}} \| z \|^2 , \quad (11)$$

$$\left\| z^{0,0,...,1} \right\|^2 = \inf_{z \in Q_{\delta_1, p^{0,0,...,1}}} \| z \|^2 , \quad (12)$$

. . . .

$$\left\| z^{1,1,...,1} \right\|^2 = \inf_{z \in Q_{\delta_1, p^{1,1,...,1}}} \| z \|^2 , \quad (13)$$

where vector $p^{0,0,...,0}$ has the minimal possible size of all components of vector *p*, $p^{0,0,...,1}$ has the minimal possible size of components $q_1, q_2, ..., q_{n-1}$ and has the maximal size of q_n ; . . . ; vector $p^{1,1,...,1}$ has the maximal possible size of all components of vector *p*.

It is possible to consider the following extreme problem

$$\left\| A_{p^{opt}} z_{\delta_1}^{pl} - u_{\delta_1} \right\|^2 = \inf_{z_a \in Q^*} \sup_{A_p \in K_A} \left\| A_p z_a - u_{\delta_1} \right\|^2 , \quad (14)$$

where z_a is the solution of extreme problem

$$\| z_a \|^2 = \inf_{z \in Q_{\delta_1, a}} \| z \|^2 . \quad (15)$$

Let's called solution $z_{\delta_1}^{pl}$ as ***more plausible mathematical model***.

Use of such model with the purpose of the forecast allows to receive the characteristic q_1 with the least maximal deviation from experiment with possible variations of variables $q_2, q_3, \; , \; , \; , \; q_n$ within the given errors.

Theorem. Solution $z_{\delta_1}^{pl}$ of extreme problem (14) exists, uniquely and is steady to small changes of the initial data if the vector $z_{\delta_1}^{pl}$ is defined uniquely from condition (14) (Menshikov 2004).

3. METHODS OF SOLUTION

One of possible ways of the solution of extreme problems (6) - (10) is use in accounts of special mathematical models of researched processes (Menshikov 2005, Menshikov 2006). If the special mathematical models exist then the solution of extreme problems (6) - (10) can are replaced with more simple extreme problems with the precisely given operator such as problems (11) - (13). Further solution is carried out by regularization method of Tikhonov with a choice of regularization parameter by discrepancy method (Tikhonov and Arsenin 1979).

For approximate solution of extreme problem (14) the interval of change of each components of vector *p* is divided by a uniform grid p_m. The number of the operators in set will be final $K_A = \{A_1, A_2, \; , \; , A_N\} = \{A_i\}$. For each operator A_i the solution z_i is defined. Further the approached solution of an extreme problem (14) is being defined by simple sorting.

$$\left\| A_{p^{opt}} z_{\delta_1}^{pl} - u_{\delta_1} \right\|^2 = \inf_{z_a \in Q^*} \sup_{A_p \in K_A} \left\| A_p z_a - u_{\delta_1} \right\|^2 =$$

$$= \min_j \max_i \{ \left\| A_i z_j - u_{\delta_1} \right\|^2 \}, \; 1 \le i \le m, 1 \le j \le m \; .$$

where z_j satisfies the condition

$$\left\| A_j z_j - u_{\delta_1} \right\|^2 = \delta^2 .$$

4. TEST CALCULATION

The problem of construction econometric model was chosen for test calculation on the data which are given in work (Malenvo 1975, tabl.1). In the beginning for first four years was constructed econometric model by a method of the least squares in the assumption that the errors of measurements satisfies to the normal law of distribution. The minimization was carried out on first variable q_1 (on a vertical). As a result of accounts the following mathematical model was received:

$$q_1 = 0.018 q_2 - 2.35 q_3 + 0.0076 q_4 + 22.27 . \quad (16)$$

Table 1 Import, manufacture, change of stocks and consumption in France (millard. franks, in the prices 1959).

Number (Years)	Import q_{1i}	Manu-facture q_{2i} $_{2t}$	change of stocks q_{3i} $_{3it}$	Consumption q_{4i} $_{4t}$
1 (1949)	15.9	149.3	4.2	108.1
2 (1950)	16.4	161.2	4.1	114.8
3 (1951)	19.0	171.5	3.1	123.2
4 (1952)	19.1	175.5	3.1	126.9
5 (1953)	18.8	180.8	1.1	132.1
6 (1954)	20.4	190.7	2.2	137.7
7 (1955)	22.7	202.1	2.1	146.0
8 (1956)	26.5	212.4	5.6	154.1
9 (1957)	28.1	226.1	5.0	162.3
10 (1958)	27.6	231.9	5.1	164.3

Further in the assumption, that the errors of measurements do not surpass 5 %, the mathematical models $z_{17}, z_{18}, z_{19}, z_{20}$ as result of the solution of the following extreme problems were constructed:

$$\|z_{17}\|^2 = \inf_{z \in Q_{\delta_1, p^{0,0,0}}} \|z\|^2, \tag{17}$$

$$\|z_{18}\|^2 = \inf_{z \in Q_{\delta_1, p^{0,0,1}}} \|z\|^2, \tag{18}$$

$$\|z_{19}\|^2 = \inf_{z \in Q_{\delta_1, p^{0,1,1}}} \|z\|^2, \tag{19}$$

$$\|z_{20}\|^2 = \inf_{z \in Q_{\delta_1, p^{1,1,1}}} \|z\|^2, \tag{20}$$

where

$p^{0,0,0} = (q_2^0, q_3^0, q_4^0)^T$, $p^{0,0,1} = (q_2^0, q_3^0, q_4^1)^T$,
$p^{0,1,1} = (q_2^0, q_3^1, q_4^1)^T$, $p^{1,1,1} = (q_2^1, q_3^1, q_4^1)^T$,
q_k^0 is minimal possible size of q_k, q_k^1 is maximal possible size of q_k. The following econometric models correspond to solutions $z_{17}, z_{18}, z_{19}, z_{20}$:

$$q_1 = -0.099 q_2 - 1.137 q_3 + 0.24 q_4 + 7.03, \tag{17a}$$

$$q_1 = -0.09 q_2 - 1.14 q_3 + 0.27 q_4 + 7.03, \tag{18a}$$

$$q_1 = -0.1 q_2 - 1.14 q_3 + 0.27 q_4 + 7.03, \tag{19a}$$

$$q_1 = -0.09 q_2 - 1.03 q_3 + 0.24 q_4 + 7.03. \tag{20a}$$

The forecast of size q_1 on these models for 1953, 1954 is given in the table 2. For comparison with real measurements (table 1, the second column) is possible to make a conclusion that best short-term forecast gives model (17a) among models (16), (17a) - (20a).

Table 2 Forecast of economic parameters for 1953; 1954.

N	Tab.1	mod. (16)	mod. (17a)	mod (18a)	Mod. (19a)
5	18.8	23,9	19,7	24,84	23,12
6	20.4	21,6	18,8	24,17	22,38

The results of calculations on real measurements by regularization method (Tikhonov and Arsenin 1979) have shown that mathematical models $z_{17}, z_{18}, z_{19}, z_{20}$ describe the real situation more exactly than classical econometrical models z_{16}.

It is necessary to note that the quality of the forecast for the long period is worse, than for the short period. It is expected effect as the offered algorithms were designed for short-term forecast.

Choice in practical problems the certain mathematical model is being determined of the specificity of a concrete problem and final goal of use of mathematical model. However the best model for the forecast can not be determined a priori.

CONCLUSION

The offered approach to a problem of identification of parameters of static linear mathematical model allows to expand a class of the possible solutions (mathematical models) up to maximal possible. Some variants of statement of such problems of parameters identification are considered. There are no basic restrictions for distribution of such approach to nonlinear problems of identification, in author opinion.

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