MONTE CARLO VALUATION OF POWER GENERATING UNITS

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ABSTRACT
Electricity deregulation in North America has enabled owners of small electric generators to become independent power producers (IPP) by selling electricity to the grid. Uncertainty affects both the profit maximizing utilization schedule and the related valuation of the underlying asset, the power generating unit (PGU). In addition to price uncertainty, the small sized power generating units are subject to different constraints and fuel supplies compared to those of large power plants, with higher production costs. Monte Carlo simulation is applied to evaluate the profitability of small PGUs as potential electricity producers and stochastic methods are used to model the prices of electricity and fuel. We extended the mean-reverting stochastic differential equation for the electricity prices with term modeling the price spikes. The spike distribution was done using the 'peak over threshold' approach of the Extreme Value Theory. The profitability measures, net present value and internal rate of return, were calculated by modeling the term structure of the interest rate with extended Nelson-Siegel form.

Keywords: Monte Carlo simulation, price models, Nelson-Siegel method, extreme value theory

1. INTRODUCTION

The electricity industry is concerned with the processes of electricity generation, electricity transmission from location (e.g. power plant) to location (e.g. populated area), and electricity distribution to the customers. Until the beginning of the 1990s, the industry was characterized by a relatively small number of government regulated and vertically integrated monopolies that covered the whole process. The pressure of globalization and the desire for lowering the price of electricity brought about restructuring and deregulation in the electricity markets in many industrial countries. One of the first areas to adopt these changes was the province of Alberta, Canada. The transition to fully deregulated electricity generation in Alberta took several years and established a competitive market, where the spot price of electricity is determined by the forces of supply and demand.

Deregulation of the electricity industry opened the door for owners of small electric generators to become independent power producers and sell electricity to the grid. A typical PGU would be a part of an emergency power system and would be idle most of the time. Most PGUs have a generating capacity of 0.5 – 2MW and are driven by diesel engines, resulting in a high cost for the produced electricity. With uncertainty in the price of fuel and electricity, there is no simple way to evaluate the potential profitability of using such generators to sell electricity to the grid.

One approach is to apply Monte Carlo simulation within the framework of capital budgeting. The first step is to build models for the price processes of the electricity and fuel. For a given realization in these prices the generator would be 'switched on' if the price of electricity exceeds the price of the fuel necessary to produce it. The present value of the generated cash flows are discounted by the appropriate required rate of return to obtain the values of two profitability measures: net present value and the internal rate of return. Generating a large number of sample price paths allows accurate estimates of the profitability measures to be obtained.

The major challenge in this approach is to build a good model for the price of electricity, accommodating its unique features. We modeled these features by specifying a seasonal mean-reverting stochastic model on the basis of standard geometric Brownian motion. To account for the spikes, we first used the 'peak over threshold' method of the Extreme Value Theory to find a high threshold above which the data is well described by the generalized Pareto distribution. We used the threshold excesses to fit an independent 'spike' term that is a product of spike intensity and spike frequency.

An additional source of uncertainty was the discount factor used in the calculation of the net present value and the internal rate of return. We opted for using these two discounted cash flow measures, as opposed to more refined approaches such as real options, because of their intuitive simplicity and ease of communication with decision makers. The simplest approach would have been to specify a required rate of return to use as a discount factor throughout the project lifetime. Because the typical length of the latter is 10-15 years, different heuristic discount factors would yield quite different results. We decided to use instead a benchmark approach with the risk-free rate as a discount factor. We modeled carefully the full term structure of interest rates by fitting the extended Nelson-Siegel form on the available Government of Canada bonds.

2. POWER PLANT VALUATION

The current widely accepted capital budgeting method for assessment of capital investments is the net present value (NPV). It captures the intuitive guiding principle that a project should not be undertaken if the investments in the project outweigh the revenues from the project, or more formally – if the expected rate of return on the investment is less than the opportunity cost of capital. In practice, the calculation of NPV uses the discounted cash flow (DCF) method to obtain the present value (PV) of expected cash flows C_t from the project and compare it with the estimated (initial) capital outlay C_0:

$$NPV = PV\left(\sum_t E\left[C_t\right]\right) - C_0. \tag{1}$$

A project is accepted if its NPV is positive and rejected otherwise. In general, for project that has T+1 number of discrete cash flows that can be both positive (inflows) and negative (outflows) over the project lifetime T, the formula can be written as:

$$NPV = \sum_{t=0}^{T} \frac{E\left[C_t\right]}{(1+r_t)^t}, \tag{2}$$

where the cash flows C_t are explicitly discounted at an appropriate hurdle rate rt (required rate of return) at time, *t*. For the case of power generating units, the cash flows C_t are the net difference between the price at which the generated electrical energy was sold and the price, paid for the fuel that was used to generate that unit of electricity, i.e.

$$C_t = q(P_t^e - hP_t^f) \tag{3}$$

where P_t^e is the spot price of electricity (\$/MWh), P_t^f is the price the fuel was bought at time $\tau < t$ (\$/MMBtu), *h* is the unit heat rate (MMBtu/MWh) and *q* is the unit output level, which, for small PGUs, is usually close to 1. The quantity in the brackets is called the spark spread. Since the generator is only switched on if it is deemed profitable to do so, the spark spread in this case is always positive.

A second popular measure is the internal rate of return (IRR), which is defined as the discount rate corresponding to zero NPV, i.e. it is the rate that solves the equation:

$$\sum_{t=1}^{T} \frac{E\left[C_t\right]}{(1+IRR)^t} - C_0 = 0. \tag{4}$$

For a small number of periodic cash flows, Equation (4) can easily be solved numerically. In the case of frequent cash flows over a long period of time, T, a different approach is needed. We adopt a Future Value (FV) point of view (Ourdev and AbouRizk 2006) and calculate the total amount of cash generated over the lifetime of the project by summing up all the cash flows compounded by the corresponding forward rate,

$$FV = \sum_{t=1}^{T} E\left[C_t\right] F^c(0, t_0, T-t_0). \tag{5}$$

The continuously compounded forward rate, $F^c(0, t_0, T-t_0)$, is the rate as seen from time $t=0$ that starts at time t_0 with residual maturity T-t_0. The total return rate over the period is obtained by solving

$$(1+IRR)^T = FV / C_0. \tag{6}$$

The two measures, NPV and IRR, require a continuous set of interest rates. A continuous function representing the spot rates, r_t, for different times to maturity is referred to as term structure of interest rates. There is a limited number of fixed income securities, and only a few of them are zero-coupon bonds, i.e. can provide spot rates. The rest are coupon-bearing bonds, and their yield to maturity depends on the coupon rates.

The zero-coupon bonds are typically represented by the Treasury bills, available with maturities up to one year. The remaining portion of the spot rate curve has to be estimated from the available longer term Government bonds, decomposing each coupon bond into a portfolio of zero-coupon bonds, with each zero-coupon bond corresponding to a coupon payment. The implied forward rates can be obtained from the spot rate curve by representing each forward rate as a synthetic portfolio of two bonds with equivalent cash flows (Martellini et al. 2003).

The most flexible estimation for the spot and forward rates is given by the Extended Nelson–Siegel form (Svensson 1994). Introducing, for readability, the auxiliary functions

$$A(\theta_k) = exp(-\theta_k), \qquad B(\theta_k) = \frac{1 - exp(-\theta_k)}{\theta_k}, \tag{7}$$

the fitting function that represents the instantaneous forward rate for the time θ is defined as:

$$F^c(0,\theta) = \beta_0 + \beta_1 A(\theta_1) + \beta_2 \theta_1 A(\theta_2) + \beta_3 \theta_2 A(\theta_3) \tag{8}$$

Here, $\theta_k = \theta / \tau_k$ is the scaled time, and the parameters are two groups: scaling parameters τ_1, τ_2, and shape parameters, $\beta_k, k = 0:3$. The implementation of the method involves fixing a priori the scaling parameters and obtaining the shape parameters through optimization.

The continuously compounded spot rate $R^c(t,T)$ at trade time t and maturity time T can be interpreted as the average of the instantaneous forward rates with settlements between time t and T, hence

$$R^c(t,T) = \frac{1}{T-t}\int_t^T F^c(t,\theta)d\theta. \tag{9}$$

The spot rate is derived from Eq.(8) using Eq.(9) as follows:

$$\begin{aligned} R^c(0,\theta) = \beta_0 + \beta_1 B(\theta_1) + \beta_2[B(\theta_1) - A(\theta_2)] \\ + \beta_3[B(\theta_2) - A(\theta_2)].. \end{aligned} \tag{10}$$

One problem with using the discounted cash flow method for project valuation is that the method is essentially static. First, it assumes that the state of the world remains unchanged during the life of the project and ignores the uncertainty of parameters, such as time and the size of the cash flows. More importantly, the DCF method assumes passive commitment to the project at the inception and ignores the ability of the management to make decisions that adjust project parameters in response to changing market conditions. These issues can be addressed by the *real options method* (Ronn 2003). In this work, we use only the traditional DCF measures, NPV and IRR, because of their simplicity and ease of communication with decision makers.

3. MOTIVATION FOR THE STUDY

The motivation for this study lies in the idea of using small scale power generating units as commercial electrical generators. Typically such PGUs will be part of the Emergency (standby, backup) Power Systems (EPS), alternate reserve sources of electric power that provide energy to critical loads in case of power outages.

Depending on the load, power sources for EPS can be batteries, uninterruptible power systems (UPSs) or PGUs, also called *gensets*. PGUs are electric power generators driven by engines or turbines. Most widely used are diesel engine generator sets with sizes in the range from 100 *kW* to 2,000 *kW* (Kusko 1989). Because PGUs vary in both size and efficiency, we take as a representative example a mid-range genset manufactured by SDMO with a Volvo Penta engine yielding 500 *kW* stand-by power. The fuel consumption at this level is 122.1*L/h*. Although the efficiency slightly increases at lower levels of utilization, the difference does not justify a separate treatment in this case.

The requirements for the design and implementation of EPS's are specified in the national standards and codes of good practices (NFPA 2005). One of the requirements of the National Fire Protection Association (NFPA) limits the capacity of unenclosed day tanks to 660 *gal* (2,498*L*) (NFPA 1998). In accordance with this rule, we assume that the genset for our example is supplied with a tank with a capacity of 2,442 *L*, which is equivalent to 20 hours of uninterrupted work. At the end of the period, the tank is refilled at the average retail price of diesel at the time.

4. MODELING ENERGY PRICES

The two commodities involved in this project valuation are diesel and electricity. As with any commodity, their prices are determined by the interplay of the forces of supply and demand, but both retail diesel and electricity exhibit quite specific characteristics that affect the price formation.

4.1. Characteristics of Electricity Prices

4.1.1. Electricity Supply

In the deregulated electricity market design, supply meets demand on a short-term spot market that is organized either in the form of a pool or as a power exchange. In both cases, the price formation is realized via a bidding process for each hour or half-hour of the following day 0.

Electricity has some features that make it quite distinct from other commodities. Electricity is a non-storable commodity and the matching of supply and demand has to be done in real time. Both the supply and demand are subject to unexpected shocks from weather changes, congestion, unit failures, etc. Demand fluctuations are exacerbated by the fact that the deregulated producers have no incentives to keep excess capacity idle, except for old and inefficient generating facilities with high production costs.

One additional factor that affects the supply is the electricity transportation. Electricity is transported in a single transmission network, usually referred to as the *grid,* that in many cases is geographically localized. The lack of alternative transportation channels increases the risk of failure and introduces an upper limit of the transmission capacity.

4.1.2. Electricity Demand

The demand side is characterized by recurring patterns over multiple time scales. The longest time scale is the one that reflects the seasonal changes in the specific geographic region (including demands related to heating and cooling). The second time scale deals with weekly variations due to industrial activity and thirdly, there are intraday patterns of variation.

The intraday variability is reflected in the terminology used by the exchanges. The 24 hour day is divided into three time periods: one *on-peak* and two *off-peak* periods. The on-peak period encompasses the time of the day with higher electricity demand and its boundaries vary with the geographic region. For Alberta, on-peak is defined as the period from HE 9 to HE 21 inclusive, Mountain Prevailing Time during a business day. Here “HE” (hour ending) refers to the 60-minute period ending that hour. For example HE 21 includes the time interval between 20:00 and 21:00. The average price over the whole 24-hour daily interval is referred to as the *baseload*. Figure 1 illustrates the weekly variability of the three different time periods – on-peak load, off-peak load, and baseload – for 2005. The different time periods are preserved even for the prices over the weekends and holidays, denoted by 'Hol'.

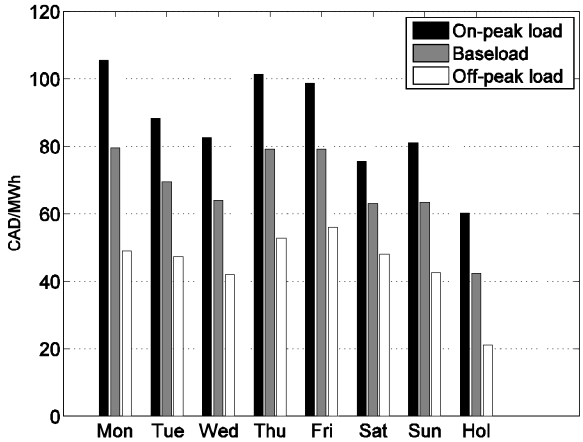

Figure 1: Average weekday and holiday prices for 2005.

Because the baseload is, in fact, the average of the off– and on–peak period prices we can use it as a representative of the daily price variations. Figure 2 shows the typical pattern of baseload price changes for Alberta.

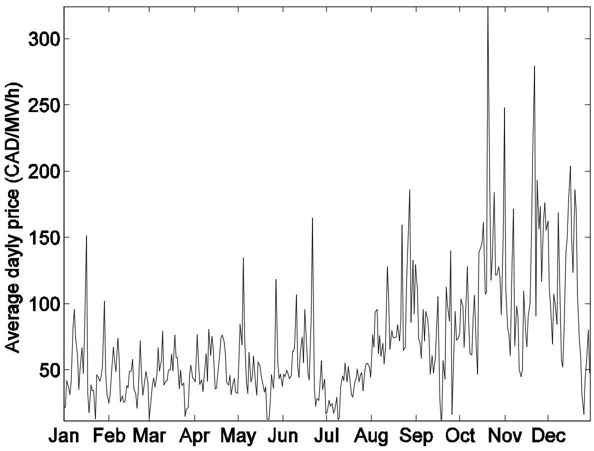

Figure 2: Baseload electricity prices for Alberta, 2005

In engineering terms, the realized current demand is referred to as the *load* on the system and represents the total amount of electricity consumed by the customers at any given moment. There are two main categories of load: base load and peak load. This differentiation attains importance because of its relation to the corresponding types of generating stations. *Base load* is the minimum amount of electricity that is pulled out of the grid by the consumers. This type of demand is covered by nuclear and hydro-power units that typically have lower marginal production costs, but also longer dispatch and response times. *Peak load* is the maximum amount of electricity demanded, usually understood as the marginal difference over and above the normal base load. It is typically covered by oil and gas power plants.

The demand is usually categorized as short-run and long-run. The *short-run demand* refers to time scales during which there is no change in the existing loads, such as residential appliances, industrial infrastructure, technological processes, etc. Currently, it is accepted that the short-run demand for electricity is income and price inelastic (Lafferty et al. 2001).

Ordering the generating units by their size and marginal production cost yields the supply stack. In the short-run, peak level electricity demand can only be matched by the remaining generating units from the far right side of the stack. Since these units are typically inefficient and have high marginal costs, the supply also becomes inelastic.

4.1.3. Features of Electricity Prices

The system exhibits significant uncertainty due to unexpected weather changes, generation failures, transmission congestion, etc. The combination of inelastic demand with inelastic supply in an environment with high levels of uncertainty determines the following unique features of electricity prices:

- **Cyclical seasonality patterns** on three different time scales – yearly, weekly, and daily – driven by the regular variation of the demand,
- **Random price spikes** – sudden jumps in prices relative to the average level for the season, due to unanticipated shocks in either demand or supply, e.g. unusually hot weather or congested transmission,
- **Mean reversion** – the tendency of the prices to revert to a dependent on the season average price level.

Figure 2 illustrates both the mean reversion and the price spikes, as well as the yearly seasonal dependence. Figure 3 clearly shows both the weekly patterns and the difference in the peaks intensity over two different representative months: May and November.

4.2. Alberta Energy Market

The design of a typical electrical system involves coordination between the independent power producers and importers on the supply side and the retailers and exporters on the demand side, including the interests of the owners of the transmission facilities and the owners of the distribution system. All transactions are channelled through the Power Pool operated by Alberta Electric System Operator (AESO), a statutory corporation. It has about 200 participants and over $7 billion in annual energy transactions. The Pool's Operating Policies and Procedures (AESCO 2006) require pool participants to submit their hourly bids and offers for the next seven days every day before noon. Distributors and exporters submit bids specifying the amount of energy they are willing to buy at specified prices. Power suppliers submit offers for blocks of power at prices they are willing to accept. The quotes are binding only for the next trading day. All valid bids and offers for a settlement interval are sorted in ascending order of how expensive they are and generation and load units are dispatched according to this merit order. There is a price cap currently set at *CAD*\$1,000*MWh*. The *System Marginal Price* (SMP), i.e. the price of the last block of energy dispatched to meet the load, is updated every minute in real time. The ex-post time-weighted average of the SMPs gives the market pool price for that hour.

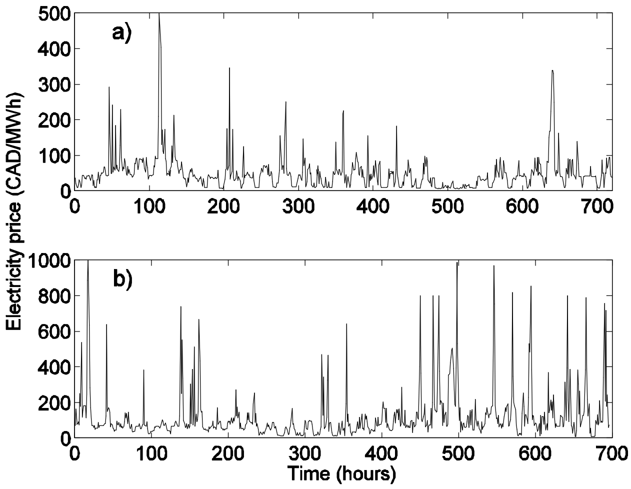

Figure 3: Hourly electricity prices for Alberta for two periods of 2005, a) May 01 – Jun 01, and b) Nov 01 – Dec 01.

4.3. Price Models

The taxonomy of electricity price models includes various approaches broadly classified in six groups depending on the mathematical methods used for process description, namely: time series models, game theory models, structural models, non-parametric models, statistical models, and production cost models (Gonzalez et al. 2005). For the purposes of the Monte Carlo simulations, the most appropriate models are the ones based on financial time series methods.

Modern finance theory postulates that price follows specific random process X_t and uses the methods of stochastic calculus for its description (see e.g. Bjork 2004). Here the subscript, *t*, shows time dependence. The main building block in the theory of stochastic processes is the (standard) Brownian motion (BM) and its mathematical formulation, *Wiener process*, W_t. BM is the continuous limit of the random walk and is the basic model of the effect of continuous noise over time *t*. The process has independent increments, and its change, dW_t, over a short period of time Δt is normally a distributed random variable with zero mean and a variance proportional to the length of the time interval, i.e.

$$W_{t+\Delta t} - W_t \sim N(0, \sqrt{\Delta t}). \tag{11}$$

If we denote the random draws from the standardized normal distribution $N(0,1)$ as ε_t the discrete version of the increments of the Wiener process can be written as $\Delta W_t = \varepsilon_t \Delta t$, which is the expression used in the Monte Carlo simulations.

The simplest non-trivial generalization of the Wiener process is the arithmetic Brownian motion, which has a deterministic term with *drift*, ν, and a random term that is multiplied by a scale factor, σ, that has the meaning of standard deviation of the process

$$dX_t = \nu dt + \sigma dW_t. \tag{12}$$

This equation already allows for the modeling of the variable drift and variance and is appealing due to its simplicity, but because the random variable X_t can take negative values it cannot be interpreted as price. The simplest solution is to make the instantaneous mean and standard deviation proportional to the random variable, S_t, formulating, in fact, the most popular model for description of price evolution, the *geometric Brownian motion* (GBM)

$$dS_t = \mu S_t dt + \sigma S_t dW_t. \tag{13}$$

Using Ito's lemma, it is quite easy to show 0 and Pindyck 1994) that this is equivalent to setting $X_t = \ln S_t$ and defining the drift as $\nu = \mu - \sigma^2/2$. Thus the distribution of the price return over a time interval Δt is normal with mean $\mu\Delta t$ and variance $\sigma^2 \Delta t$

$$X_{t+\Delta t} - X_t \sim N(\nu\Delta t, \sigma^2 \Delta t) \tag{14}$$

i.e. the random variable S_t from Eq.13 can be interpreted as (log-normally distributed) price.

GBM is commonly accepted as a model for stock price, but in its basic form it cannot adequately describe random variables with mean reverting behaviour, such as electricity prices. The first mean-reverting models were developed to describe the *term structure of interest rates*, i.e. the interest rate yield curve for different times to maturity (Vasicek 1977, Hull and White 1990). The simplest approach is to modify the drift term of the basic model Eq.13 as follows:

$$dS_t = \lambda(\alpha - S_t)dt + \sigma dW_t. \tag{15}$$

The first term of this equation changes its sign depending on the difference between the magnitude of the price, S_t and the long-term mean-reversion level α thus ensuring reversion towards this level with speed proportional to λ. The second term is the usual normally distributed stochastic term. This one-factor model is known as the the Vasicek model, or for the special case of zero level mean reversion, i.e. $\alpha = 0$ as the Ornstein-Uhlenbeck process. Still, it does not have a mechanism to describe price seasonality.

Price seasonality is usually modeled by adding a deterministic term to the mean-reverting part of the model, S_t^{MR} as

$$S_t = S_t^{MR} + f_t. \tag{16}$$

The most common choice of the deterministic function f_t is some combination of sinusoids (Roncoroni and Geman 2003), sometimes with the addition of deterministic linear trend and/or terms to account for the seasonalities at different time scales (Lucia and Schwartz 2002). One fairly general form of the deterministic function is (Escribano et al. 2002)

$$f_t = \alpha + \beta t + \gamma D_t + \sum_n \zeta_n sin\left((t+\theta_n)\frac{2\pi n}{365} \right), \tag{17}$$

for a set of constant parameters $\alpha, \beta, \gamma, \zeta_n, \theta_n$. Usually the sum is restricted to only two terms, which is enough to describe two annual maxima. The term γD_t is sometimes used to capture the day-of-week variability. With this adjustment for the existence of deterministic periodicity the stochastic differential equation describing the process takes the form

$$dS_t = \lambda\left(\alpha_t - S_t\right)dt + \sigma dW_t, \tag{18}$$

with the deterministic function α_t defined as

$$\alpha_t = f_t + f_t'/\lambda. \tag{19}$$

A comparison with Equation 15 shows that the model specified by Equation 18 can be regarded as an extended Vasicek model (Hull and White 1990) with a time dependent mean-reverting level given by the deterministic function α_t.

None of the models considered so far exhibit spikes, which are so prominent in the Alberta electricity market. The most common approach is the formulation of a jump-diffusion model (Eydeland and Geman 1999) by adding a jump component $J_t dq_t$ to Equation 18, with jump size J_t and intensity given by q_t. The problem with these models is that the duration of the electricity spikes is rarely longer than one hour, while jumps have much longer durations. Different approaches of forcing the jump back (Roncoroni and Geman 2003) increase model complexity and introduce statistical distortions. Therefore, an alternative set of approaches has been tested. In the hidden Markov models (HMM) an approach towards the price process is split in two regimes: the stable regime and the spike regime, and the spot price switches between the regimes (Gonzalez et al. 2005). The drawback of HMM is the difficulty of incorporating the seasonality. Yet another approach involves direct modeling of the supply and demand processes (Eydeland and Geman 1999, Burger at al. 2004).

For the purposes of this study, where the importance of the price spikes on hourly resolution scale is crucial, the model must incorporate the features of both the base price (Equation 18), and the frequency and intensity of the spikes.

5. EXTREME VALUE THEORY

Extreme Value Theory (EVT) addresses the behavior of stochastic processes that exhibit heavy tails. The classical EVT is based on the asymptotic Generalized Extreme Value (GEV) function that models the distribution of maxima over a specified period. An alternative approach, called 'peaks over threshold' (POT), models the observations that exceed a high threshold, the threshold exceedances. For a sequence $X_1, X_2, \cdots$ of independent and identically distributed random variables with an unknown underlying distribution function $F(x) = P\{X_i \le x\}$ we define the distribution of the excesses over a high threshold u as the conditional probability:

$$F_u(x) = P\{X - u \le x \mid X > u\} \tag{20}$$

It can be observed that the excess distribution can be written in terms of the underlying distribution F as:

$$F_u(x) = \frac{F(x+u) - F(u)}{1 - F(u)}, \tag{21}$$

which implies that if the underlying distribution is known, then its excess distribution is easily computed. It has been shown (Balkema and Haan 1974) that for a wide class of underlying distributions and sufficiently high threshold the excess distribution F_u can be approximated by the generalized Pareto distribution (GPD) given by:

$$G_{\xi,\beta}(x) = \begin{cases} 1 - (1 + \xi x/\beta)^{-1/\xi} & \xi \ne 0, \\ 1 - \exp(-x/\beta) & \xi = 0, \end{cases} \tag{22}$$

with a scale parameter $\beta > 0$. The value of the shape parameter ξ determines the particular type of distribution function subsumed in GPD. When $\xi = 0$, the GPD is the exponential distribution; when $\xi < 0$, it is known as Pareto type II distribution, and when $\xi > 0$, it is the ordinary Pareto distribution, which has a long history in actuarial mathematics as a model for large losses.

5.1. Parameter estimation

In order to take advantage of the extreme values distribution tail approximation

$$F_u(x) \approx G_{\xi,\beta}(x) \tag{23}$$

we need to estimate the parameters ξ and β for a properly chosen threshold u. The most common technique for parameter estimation is the Maximum Likelihood Estimation (MLE) method which is part of most statistical software packages. Figure 4 shows the MLE results for shape parameter ξ and scale parameter β of the GPD, with the 95% confidence intervals for different threshold values.

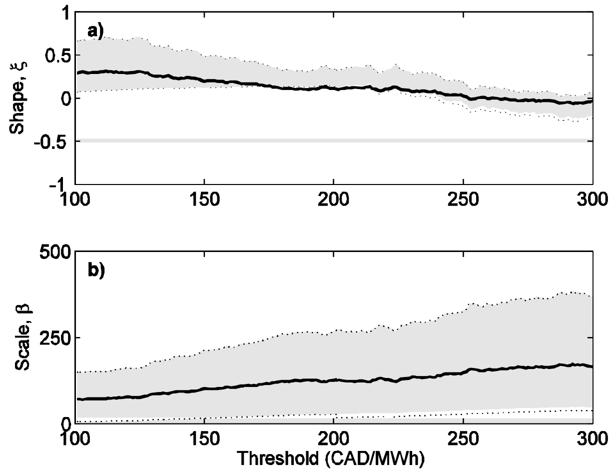

Figure 4: Maximum likelihood estimates of the parameters of GPD, a) shape parameter and b) scale parameter with 95% confidence intervals for different threshold values.

The choice of an appropriate threshold value is more difficult. On the one side, the threshold has to be high enough for the approximation of Equation 23 to hold, but on the other, we need a large enough number of exceedances to ensure good statistics. In addition to the QQ plots another useful tool is the conditional *sample mean excess* (SME) function defined as

$$e(u) = E(X - u \mid X > u), \quad u \geq 0, \tag{24}$$

An important property of GPD for values $\xi > -1$ is that SME is a linear function of the threshold, u

$$e(u) = \frac{\beta}{1+\xi} - \frac{\xi}{1+\xi} u. \tag{25}$$

SME can be empirically estimated as the total distance of exceedance over the threshold divided by the number of points exceeding that threshold, i.e.

$$e_n(u) = \sum_{k=1}^{N} (X_k - u)^+ \Big/ \sum_{k=1}^{N} I_{\{X_k > u\}}, \tag{26}$$

where n is the sample size, the '+' sign denotes the positive part of the expression in the brackets, i.e. $A^+ = max(A, 0)$, and the indicator function $I = 1$ if $X_k > 0$ and 0, otherwise. The empirical estimate of the SME, Equation 26 can be used to infer an appropriate threshold value by identifying a linear part of the function (Embrechts et al. 2004) according to the linearity property, Equation 25. Figure 5 shows the variation of the SME function on the electricity data for 2005.

At threshold, u=130 CAD/MWh, the maximum likelihood estimates for the shape and scale parameters of the GPD are $(\hat{\xi}, \hat{\beta}) = (0.257,\ 85.47)$ with confidence intervals: $CI(\hat{\xi}_{95\%}) = (0.167,\ 0.346)$ and $CI(\hat{\beta}_{95\%}) = (76.65,\ 95.31)$.

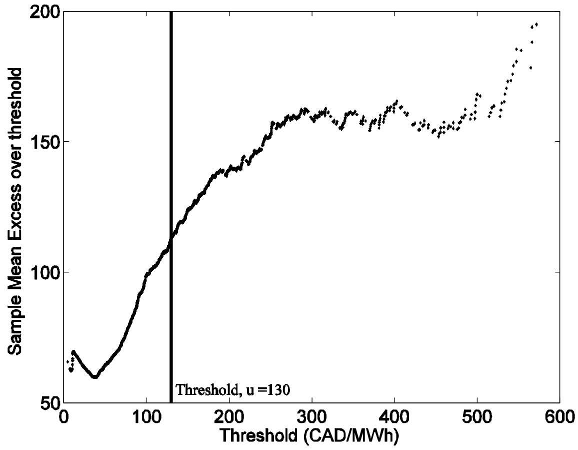

Figure 5: Sample Mean Excess function over threshold for hourly Alberta electricity prices for 2005

The graphical comparison between the GPD calculated with these parameters and the empirical histogram created on the basis of the same tail data is presented in Figure 6. Visual inspection confirms the goodness of fit for the chosen parameter.

In addition to the rather subjective visual inspection we applied the more formal goodness of fit test. We formulated the null hypothesis that the empirical tail distribution is a sample coming from the theoretical GPD against the alternative that it does not come from that distribution. We applied two-sample Kolmogorov–Smirnov tests to the data. Bearing in mind the limited power of the test (Choulakian and Stephens 2001, Zempleni 2004), it also confirmed the null hypothesis at 95% confidence level with p–value of $p \approx 0$.

Figure 7 shows the position of the threshold that separates the extreme values of the hourly electricity prices according to EVT. There are 937 extreme points over the threshold out of total of 8,471 observations, i.e. 11%.

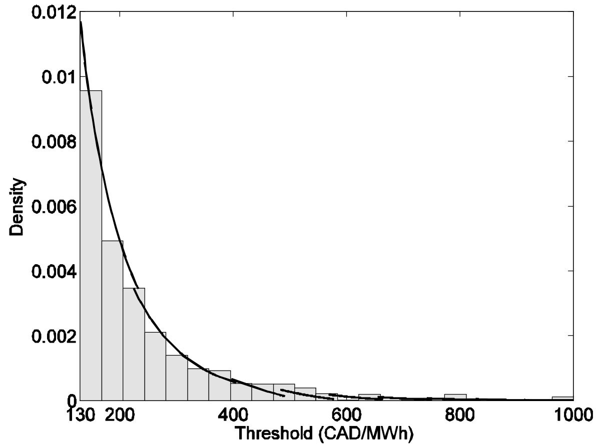

Figure 6: Diagnostic plot of GPD probability density and the empirical histogram for the values of Alberta hourly electricity prices for 2005 over the threshold u=130.

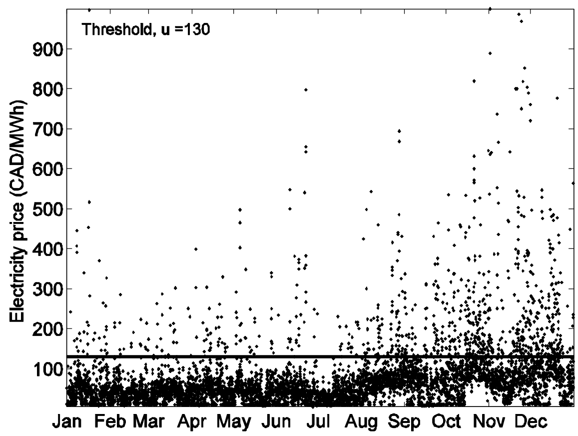

Figure 7: Threshold for the extreme value theory plotted against the hourly prices for 2005

6. MONTE CARLO SIMULATION

6.1. Algorithms

In this study, we apply Monte Carlo simulation within the framework of capital budgeting. The problem involves three sources of uncertainty, the price of fuel, the price of electricity, and the level of interest rate. The prices are modeled as stochastic processes. The hourly price of fuel is obtained by sequentially applying the discretized version of Equation 18. The hourly price of electricity is obtained from the same equation with the added 'spike' term.

The simulation is run N_r number of times. Each time, an hourly comparison of the simulated prices of fuel and electricity is performed. If the price of electricity is higher than the price of fuel used to produce it, the spark spread, the power generating unit is 'dispatched' for that hour. If the spark spread is negative, no energy is sold to the grid. The cash flows generated during the day are used to perform a DCF analysis of the investment.

The interest rate used in the DCF analysis, the third source of uncertainty, is modeled by fitting the extended Nelson-Siegel form to the data of tradable Canada Government bonds, as described below. The spot rate, Equation 10, is used to calculate the present value of the cash flow for every day, $PV(CF_t)$. The sum of all cash flows is used to determine the net present value for the particular realization NPV_r. In parallel, the forward rate, Equation 8, is used to calculate the future value of the cash flow for every day, $FV(CF_t)$ the sum of which is used to determine the total future value for the particular realization FV_r^{tot}. The latter is plugged into Equation 4 in order to calculate the internal rate of return from the asset for the particular realization IRR_r. The values of both NPV_r and IRR_r are retained at the end of each realization, thus at the end of the simulation we can calculate their average values (point estimators for the population means)

$$\hat{V} = \frac{1}{N_r}\sum_{r=1}^{N_r} V_r, \tag{27}$$

where V stands for both NPV and IRR. The standard error of the estimator $\hat{V}$ is calculated as (Hines et al. 2003)

$$\varepsilon(\hat{V}) = \sigma(\hat{V}) / \sqrt{N_r}, \tag{28}$$

where the sample standard deviation is given by

$$\sigma(\hat{V}) = \sqrt{\frac{1}{N_r - 1}\sum_{r=1}^{N_r}\left(V_r - \hat{V}\right)^2} \tag{29}$$

6.2. Data

The data for the electricity and fuel prices in Edmonton covers the period of 2001–2006, i.e. the period starting after the full deregulation of the electricity market in Alberta. The electricity data has an hourly frequency and some representative samples of the data are shown in Figure 2 and Figure 3.

The diesel price data refers to the weekly average prices at Exxon Mobil gas stations. Because the price differentials between the gas stations of the different oil companies is typically in the range of 1–2 cents, we take these prices as representative for the city. The price levels for the period are shown in Figure 8.

These weekly diesel prices were further re-sampled by linear interpolation in order to obtain hourly values. The low weekly variation allows obtaining the hourly price data by linearly interpolating between the weekly data points.

The data used to fit the extended Nelson-Siegel model consists of four Government of Canada Treasury bills with maturities of 1, 3, 6 and 12 months, and five selected Government of Canada benchmark bonds with maturities of 2, 5, 7, 10, and 30 years, summarized in Table 1.

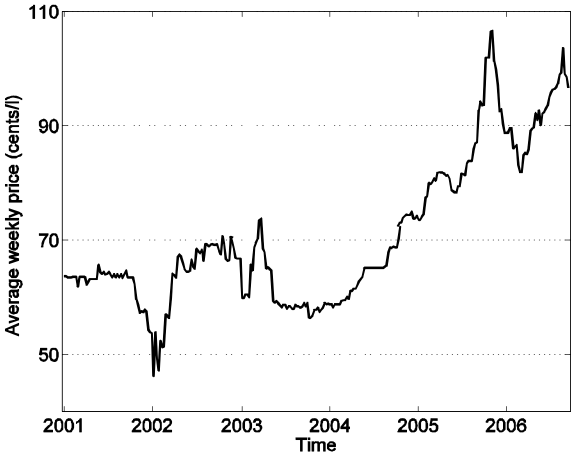

Figure 8: Weekly average retail prices of diesel in Edmonton for 2001-2006.

Table 1: Selected Government of Canada T-bills and benchmark bonds used to construct the term structure of the interest rates. The yields are based on mid-market closing values.

Maturity (years)	Yield (% per anum)	Coupon (% per anum)
0.08	4.1	0
0.25	4.15	0
0.50	4.12	0
1	4.00	0
2	3.785	4.25
3	3.76	4.25
5	3.745	3.75
7	3.79	5.25
10	3.86	4.00
30	3.96	5.75

6.3. Calibration

Diesel prices are modeled as a mean-reverting process described by Equation 18. In order to estimate the parameters of this model from data we write the equation in discretized form as

$$X_{k+1} - X_k \lambda(\alpha - X_k)\Delta t + \sigma\varepsilon_k . \tag{30}$$

where the stochastic term, $\varepsilon_k \sim X(0,1)$ are draws from standard normal distribution. This equation is recast as a linear function of the price levels:

$$X_{k+1} - X_k = a_0 + a_1 X_k + \sigma\varepsilon_k , \tag{31}$$

and the parameters $a_0 = \lambda\alpha\Delta t$, and $a_0 = -\lambda\Delta t$, a_1=-λΔt, are estimated by the least squares method. Using the log-prices data, the estimation yields $a_0 = 0.0419 \pm 0.0711$, and $a_0 = -0.0088 \pm 0.0163$. From here we determine the following parameters for the fuel model $\lambda_F = 0.4593$, $\alpha_F = 4.7393$, and $\sigma_F = 0.125$.

The calibration of electricity prices is similar to the calibration of the diesel prices, but involves additional steps because of the presence of spikes. Using the threshold u=4.8675, corresponding to logarithm of \$130\$/MWh, as determined in the previous section, we separate the 'normal' regime from the spike regime. The latter is modeled as a product of two distributions, one describing the frequency of the spikes, and the other describing their intensity. The frequency component was modeled with the Poisson distribution, and the intensity component was modeled with the Gamma distribution. The maximum likelihood estimates for the shape parameter, α_G and the scale parameter λ_G of the Gamma distribution are $(\hat{\alpha}_G, \hat{\lambda}_G) = (89.2453, 0.0576)$ with confidence intervals: $\hat{\alpha}_{G|95\%} = (85.1712, 93.5142)$ and $\hat{\lambda}_{G|95\%} = (0.0549, 0.0603)$ Similarly, for the Poisson distribution we obtain $\hat{\lambda}_P = 5.1392$ with values at 95% confidence interval $\hat{\lambda}_{P|95\%} = (5.0642, 5.2142)$.

The fitting of the term structure of interest rate, Equation 10 , to the data in

Table 1 was formulated as a minimization problem (Soderlind and Svensson 1997)

$$\min_{(\beta,\tau)\in R^4 \times R^2} \sum_{k=1}^{N_b} \left(R(t_k) - \hat{R}(t_k) \right)^2 , \tag{32}$$

where the squared price deviations are calculated for all bonds, $N_b = 10$, and the corresponding times to maturity t_k , subject to

$$\beta_0 > 0, \quad \beta_0 + \beta_1 > 0, \quad \tau_{1,2} > 0. \tag{33}$$

The solution of this nonlinear minimization problem is quite sensitive to the starting point. We used a Matlab implementation of the Nelder-Mead simplex method, and our choice of the starting points was guided by the interpretation of the parameters (β, τ). In practice, β_0 is interpreted as a long-term interest rate, β_1 is the spread between long- and short-term interest rates, β_2 and β_3 determine the height and the directions of the curvatures, and the scale parameters τ_1 and τ_2 determine curvature positions. The values of these parameters as determined from the optimization are shown in

Table 1. The continuous term structure of the spot and interest rates, calculated from formulas (Equation 8 and Equation 10) using the parameters from Table 2 are graphed in Figure 9.

Following the algorithm described in the previous subsection, we simulated $N_p = 500$, the number of paths for both the electricity prices and fuel prices for a period of 15 years. At each path, we calculated the NPV and the IRR of the cash-flows for positive price spread. Assuming an initial investment (hook-up cost) of $C_0 = \$25,000$ and no extra maintenance cost, the final estimates are: $NPV = \$605,686$ and $NPV = 248\%$. The standard error in the two estimates are respectively $SE(NPV) = 27\%$, and $SE(IRR) = 29\%$. One comment about the high value of IRR is that the calculation does not take into account the cost of the generation unit, only the part related to its connection to the grid. As noted before, such generators would already be in place as part of EPS. In addition, the value of the NPV is somewhat inflated and can only be used as a benchmark. The reason for this is the discount factor, which we assumed to be the risk free rate. For any other required rate of return this value would be smaller.

Table 2: Estimated extended Nelson-Siegel parameters used to calculate the continuous term structure interest rates curve

β_0	β_1	β_2	β_3	τ_1	τ_2
0.0454	-0.0044	-0.0149	-0.0145	2.5232	25.6014

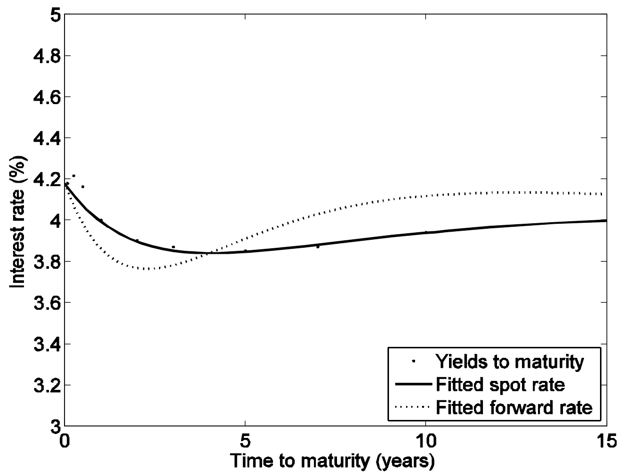

Figure 9: Extended Nelson-Siegel fit for the estimated duration of the project

7. CONCLUSION

In order to estimate the potential profitability of independent power producers, we performed the Monte Carlo simulations calculating two commonly accepted measures in the capital budgeting profitability: the net present value and the internal rate of return. These two measures were calculated on the basis of the cash flows generated by the power asset. A cash flow would be generated when the spread between the prices of the energy sold and the price of the fuel to produce this energy would be positive.

Our approach differs from most valuations of power generating assets in several aspects. First, we modeled the prices of electricity and fuel separately. This allowed for the incorporation of realistic sampling of fuel prices, subject to fuel tank capacity constraints. The tank would be "refilled" at the current fuel prices when emptied and the time between re-fills would be dependent on the time interval where the spark spread is positive.

Secondly, we paid close attention to the distribution of the spikes in electricity prices. Small power generating units with high production cost have high sensitivity to the extreme values as a result of their high production cost. We used the 'peaks over threshold' method to fit the tail distribution of the electricity prices and model more accurately the price spikes.

Finally, in order to account for the changes of the interest rates over time and their effect on the profitability measures we modeled the term structure of the interest rates by the extended Nelson-Siegel form.

Our results for the parameters of the chosen representative power generation unit show the viability using units that are currently part of the emergency power systems for peak hour power producing.

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