MRP-DRP MODEL AS A BASE FOR NEGOTIATIONS IN TIMES OF RECESSION

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ABSTRACT
In times of recession unemployment is increasing because of reduced demand, which influences the optimal production level. The impact of the production level on the cash flow, annuity stream and net present value generated by activities in a supply chain may be analysed in detail by employing MRP Theory.

In this paper we concentrate our attention on the question of (1) differences between planned production and realisation, which appear especially in the stochastic behavior of MRP-DRP systems, and (2) the oligopoly position of activity cells depending on the location and regional policies. Our extended MRP model enables us to derive consequences of these influences and interactions.

Keywords: MRP Theory, logistics, location, Net Present Value.

1. INTRODUCTION

The impact of the production level on the cash flow, annuity stream and net present value generated by activities in a supply chain may be analysed in detail by employing MRP Theory. MRP Theory has been developed in collaboration between Linköping Institute of Technology, Department of Production Economics, and other universities (in particular the University of Ljubljana) during the last two decades. The theory combines the use of Input-Output Analysis and Laplace transforms, enabling the development of a theoretical background for the dynamics of multi-level, multi-stage production-inventory systems together with their economic evaluation, in particular applying the Net Present Value principle (NPV) as the criterion function. In the late nineties, this theory has been extended from assembly to distribution (MRP-DRP) systems, and later also to include reverse logistics structures.

In this paper we concentrate our attention on the question of (1) differences between planned production and realisation, which appear especially in the stochastic behavior of MRP-DRP systems, and (2) the oligopoly position of activity cells depending on the location. A model is designed for predicting restructuring results and for the negotiation between regional authorities (where individual activity cells are located employing local human resources), and managers of the global supply chain. When in time of recessions the activity cells could be located at different regions, the regions differently participate to production level by their fiscal policies and level of subsidies mostly depends on the number of saved working places in their region, mostly proportional to the production level. But the policy of one region could influence the results of the total supply chain, also if the chain has activity cells allocated in several regions. Our extended MRP model enables us to derive consequences of these influences and interactions.

Among the elements that has a bearing on the suitability and viability of a community for capital investments in activity cells of the global supply chains are the following: (a) labour quality, availability and cost, (b) transportation cost and infrastructure, (c) labour union threats, (d) tax burden, (e) site and facility development and design, (f) development or acquisition cost and financing structure, (g) spatial planning restrictions and environmental legislation in region, (h) incentives, (i) access to infrastructure or other services, and other elements which influence profit and quality of life. Neoclassical theorists offer some insights into the spatial nature of industrial location. The more recent contributions of alternative location theorists explain the reasoning for such phenomena as decentralized production systems as a part of global supply chains. In our paper we wish to use some relevant pieces of neoclassical and modern theories to address the questions of industrial location, and decentralization applying MRP Theory when it is extended to supply chain models.

Central place theory, set forth by early location theorists like Weber, Christaller (Greenhut 1995), and Lösch (1954) is geometrically very simplified and based on the assumptions: (a) that population and resources are uniformly distributed over a homogeneous plane, (b) there exist free entry into the market, (c) the returns to scale are constant for all activities, (d) that perfect competition exists. In these models the production factors: labour and capital as well as transportation costs represent the keys to determine the optimal location. Firms locate in such a way that they maximize their profits. The models developed by these early location theorists fit reasonably well with observed reality. Lowry-like gravity models as an upgrade of these theories have been very well applied all over the world. A combination of this theory and the theory of land rent

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developed by von Thünen (Beckmann 1997) and later embedded in the theory of urban growth by Alonso (1964) provides a step further to the results of modern location theories, emerging towards MRP-DRP models.

2. NETWORK MODELS

Production –distribution – reverse logistics network models provide us with an effective tool to model manufacturing and logistics activities of a supply chain.

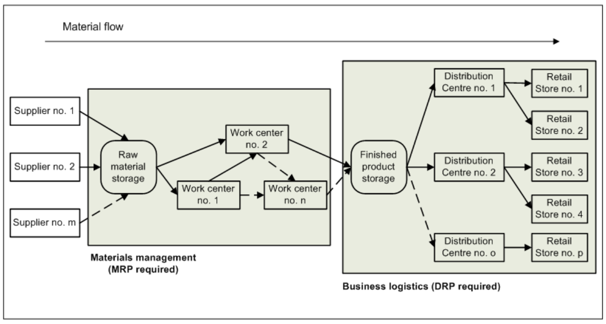

Figure 1: An example of material flow through many activity cells of a supply chain, (having different locations) divided into production and distribution segments. Between each pair of cells, there is transportation lead time.

In such a model, nodes represent vendors of raw materials or components, manufacturing and warehousing facilities in the production segment of a supply chain, and ports and distribution centres for semi-products and end items, warehouses and customers. We shall use the term "activity cell" for any of these. Arcs represent the infrastructure of flows between activity cells. The long-term performance goals for this production –distribution – reverse logistics system suggest strategic decision making regarding partners, playing different roles in the supply chain. The production –distribution – reverse logistics system design problem (PDRLSDP) involves the determination of the best configuration of the chain regarding location and capacity of the activity cells in the system. In such an attempt some activity cells have oligopoly positions and could be included in a network, which enable the flow of goods, or not. Here we will pay attention to the location of such activity cells and its impact on the net present value (NPV) and on a more general criterion function, when planning the flow could differ from its realisation.

The majority of analytical approaches for PDSDP utilizes discrete mixed integer programming models to represent facility design decision problems. Continuous models are successfully used in spatial economics and logistics, but there are only few papers that use continuous models for facility design (Daganzo 1998, Verter and Dincer 1995). Models of this type assume that customers are spread over a given market area and the optimal service region for each facility to be established is given. To develop a model for optimal strategic decisions on the location of activity cells, we shall start from MRP theory, developed by Grubbström and others.

3. LOCATION OF ACTIVITY CELL IN MRP THEORY

Optimal decisions (i) where to produce, (ii) where to locate distribution centres (which of them could be included in a network) and (iii) where to organise reverse activities in integrated supply chain can be successfully discussed and evaluated in a transformed environment, where lead times and other time delays can be considered in linear form. An integrated approach is needed especially when we consider reverse logistics as an extended producer responsibility (Grubbström, Bogataj, and Bogataj 2007).

The site and capacity selection, as for instance the problems where it is best to locate a facility and what capacity is needed to achieve the most rapid response, can be discussed more easily in transformed environment (Aseltine 1958), using MRP (Orlicky 1975) and I-O analysis (Leontief 1951) in Laplace transformed space, as previously presented by Grubbström (1996, 1998, 2007) and as it has been discussed in many other papers of his Linköping research group.

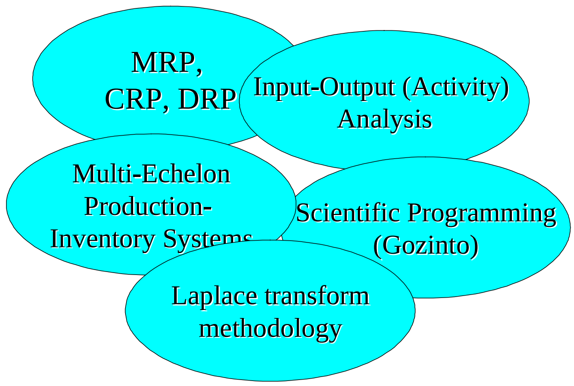

Figure 2: The ingredients of MRP Theory

MRP Theory has previously mainly dealt with *assembly structures* by which items produced downstream (on a higher level in the product structure) contain one or more sub-items on lower levels, but at each stage, the assembly activity produces only one type of output. This enables the *input matrix*, after enumerating all items suitably, to be organised as a triangular matrix, with non-zero elements only appearing below its main diagonal. The introduction of a diagonal *lead time matrix* capturing the advanced timing when required inputs are needed, enables compact expressions to be obtained, explaining the development of key variables such as available inventory and backlogs in the frequency domain. Central in this theory is the *generalised input* matrix showing when and how much the internal (dependent) demand amounts to for any production plan.

An extension of the production network has been made, including the distribution segment (Bogataj and Bogataj 2003) and later by Grubbström, Bogataj and Bogataj (2007) in close loop of product life cycle, including also reverse logistics. In these models transportation costs have been included in setup costs and transportation time lag was just extended production lead time, what is correct only if supply chain is linear or in radial form on the area described by Alonso's concentric models inherited from the Von Thünen model of agricultural land use, where all child nodes are equally remote from a certain activity cell. But this is very rear case. MRP theory was correct until transportation costs and transportation lead-time have been negligible and production lead time was the main reason for delays. Here we wish to improve MRP model to be able to use it for any supply chain evaluation, especially when we wish to study the impact of location and capacity of activity cells like ports are, on the certain objective function.

The labor cost and other costs of activities appear in every activity cell and depend on region where an activity cell is located. Together with transportation costs and costs of delay, which all depend on distances between two activity cells, (it means that it depends also on location of those cells), they influence NPV as the part of total of criterion function.

4. THE IMPROVED MRP THEORY FOR THE CASE WHEN REALISATION DO NOT FOLLOW THE PLANED ACTIVITIES IN A SUPPLY CHAIN

The line of research, now designated *MRP theory*, has attempted at developing a theoretical background for multi-level production-inventory systems, Material Requirements Planning (MRP) in a wide sense. Grubbström developed MRP theory on the basic methodologies of "Input-Output Analysis", (Leontief 1928) and *Laplace transform*. *Laplace transform* is a mathematical methodology dating back to the latter part of the 18th century and used for solving differential equations, for studying stability properties of dynamic systems, especially useful for evaluating the Net Present Value (NPV).

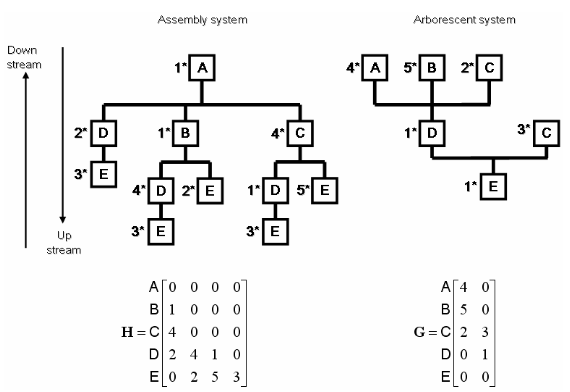

Figure 3. Examples of a pure assembly system and a pure arborescent system, in the form of product structures and their input and output matrices **H** and **G**, respectively (Grubbström, Bogataj, and Bogataj 2007).

Basic in MRP theory are the rectangular input and output matrices **H** and **G**, respectively, having the same dimension. Different rows correspond to different items (products) appearing in the system and different columns to different activities (processes). We let m denote the number of processes (columns) and n the number of item types (rows). If the jth process is run on activity level P_j, the volume of required inputs of item i is $h_{ij}P_j$ and the volume of produced (transformed) outputs of item k is $g_{kj}P_j$. The total of all inputs may then be collected into the column vector **HP**, and the total of all outputs into the column vector **GP**, from which the net production is determined as **(G - H)P**. In general **P** (and thereby net production) will be a time-varying vector-valued function of realized intensity of flows through the activity cells in a supply chain. In case when the plan of this intensity $\mathbf{P}_0$ is not equal to $\mathbf{P}$, we have to write the total of inputs by $\mathbf{HP}_0$, from which the net production is determined by $\mathbf{GP} - \mathbf{HP}_0$.

In MRP systems, lead times are essential ingredients. The lead time of a process is the time in advance of completion that the requirements are requested. If $P_j(t)$ is the volume (or rate) of item j planned to be completed at time t, then $h_{ij}P_j(t)$ of item i needs to be available for production (assembly) the lead time τ_j in advance of t, i.e. at time $(t-\tau_j)$. The volume $h_{ij}P_j$ of item *i*, previously having been part of *available inventory*, at time $(t-\tau_j)$ is reserved for the specific production $P_j(t)$ and then moved into *work-in-process* (*allocated component stock*, *allocations*). At time *t*, when this production is completed, the identity of the items type *i* disappear, and instead the newly produced items $g_{kj}P_j(t)$ appear. This approach has been developed for production systems, when transportation time did not influence lead time substantially. In case of transportation and production lead time τ_j should be split on two parts: production part of lead time τ_j^{pr} and transportation part τ_{ij}^{tr}. Therefore $h_{ij}P_j(t)$ of item *i* needs to be available for production (assembly) the lead time $\tau_{ij} = \tau_j^{pr} + \tau_{ij}^{tr}$ in advance of *t*, i.e. at time $(t - \tau_j^{pr} - \tau_{ij}^{tr})$.

In order to incorporate the lead times for assembly and arborescent processes in MRP systems without transportation time lags, Grubbström (1967, 1980, 1996, 1998, 2007) suggested transforming the relevant time functions into Laplace transforms in the frequency domain.

5. WORKING IN FREQUENCY DOMAIN

When a time function repeats itself periodically, like it is often the case in the global supply chains, the length of a period being, says *T*, the transform of an infinite sequence of such time functions is:

$$£\left\{\sum_{k=0}^{\infty} f(t-kT)\right\} = \frac{\tilde{f}(s)}{1-e^{-sT}} \qquad (1)$$

For sequence of discrete events within continuous processes, there is a need to introduce *Dirac's delta function* (*impulse function*) $\delta(t-t')$, having Laplace transform $£\{\delta(t-t')\} = e^{-st'}$.

6. TRANSPORTATION LEAD TIME AND COST

Consider an *assembly* system, for which the components of process *j* need to be in place $\tau_{ij} = \tau_j^{pr} + \tau_{ij}^{tr}$ time units before completion according to the plan $\mathbf{P}_0$, applying the time translation theorem, the input requirements as transforms will be

1. in case of equal distances to the child nodes from i:

$$\mathbf{H}\begin{bmatrix} e^{s\tau_1^{pr}} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & e^{s\tau_m^{pr}} \end{bmatrix}\begin{bmatrix} e^{s\tau_1^{tr}} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & e^{s\tau_m^{tr}} \end{bmatrix}\tilde{\mathbf{P}}_0(s) =$$

$$= \mathbf{H}\tilde{\boldsymbol{\tau}}^{pr}(s)\tilde{\boldsymbol{\tau}}^{tr}(s)\tilde{\mathbf{P}}_0(s) = \tilde{\mathbf{H}}^{pt}(s)\tilde{\mathbf{P}}_0(s) \qquad (2)$$

where $\tilde{\boldsymbol{\tau}}^{pr}(s)$ and $\tilde{\boldsymbol{\tau}}^{tr}(s)$ are the so called production and transportation *lead time matrix*, and $\tilde{\mathbf{H}}^{pt}(s)$ the *generalised input matrix* capturing the volumes of requirements as well as their advanced timing. This vector describes in a compact way all component volumes that need to be in place for the production plan $\tilde{\mathbf{P}}_0(s)$ to be possible.

2. in case of different transportation time delays from the node i to its child nodes, we have to add to the components h_{ij} of matrix $\mathbf{H}$ corresponding delays in the form $h_{ij}e^{-s\tau_{ij}} = h_{ij}^{(\tau)}$ so that product of matrices

$$\begin{bmatrix} 0 & 0\,0 & \cdots & 0 \\ \ldots. & h_{ij} & \ddots & \vdots \\ h_{m1} & h_{m2} & \cdots & 0 \end{bmatrix}\begin{bmatrix} e^{s\tau_1^{pr}} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & e^{s\tau_m^{pr}} \end{bmatrix}\begin{bmatrix} e^{s\tau_1^{tr}} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & e^{s\tau_m^{tr}} \end{bmatrix}$$

(3)

is replaced by

$$\begin{bmatrix} 0 \;\; 0 \;\; 0\,0 & \cdots & 0 \\ \ldots. \;\; \ldots\ldots\ldots h_{ij}e^{s\tau_{ij}^{tr}} & \ddots & \vdots \\ h_{m1}e^{s\tau_{m1}^{tr}} \;\; h_{m2}e^{s\tau_{m2}^{tr}} & \cdots & 0 \end{bmatrix}\begin{bmatrix} e^{s\tau_1^{pr}} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & e^{s\tau_m^{pr}} \end{bmatrix} = \tilde{\mathbf{H}}^{tr}\tilde{\boldsymbol{\tau}}^{pr}(s)$$

(4)

and

$$\begin{bmatrix} 0 \;\; 0 \;\; 0\,0 & \cdots & 0 \\ \ldots. \;\; \ldots\ldots\ldots h_{ij}e^{s\tau_{ij}^{tr}} & \ddots & \vdots \\ h_{m1}e^{s\tau_{m1}^{tr}} \;\; h_{m2}e^{s\tau_{m2}^{tr}} & \cdots & 0 \end{bmatrix}\begin{bmatrix} e^{s\tau_1^{pr}} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & e^{s\tau_m^{pr}} \end{bmatrix}\tilde{\mathbf{P}}_0 =$$

$$= \tilde{\mathbf{H}}^{tr}\tilde{\boldsymbol{\tau}}^{pr}(s)\tilde{\mathbf{P}}_0(s) = \tilde{\mathbf{H}}^{pt}(s)\tilde{\mathbf{P}}_0(s) \qquad (5)$$

where in $\tilde{\mathbf{H}}^{pt}(s)$ all kind of delays are included.

The similar split of lead time has to be made in arborescent system. Here the output of item *k* from running process *j* on the level $P_j(t)$ in terms of volume is $g_{kj}P_j(t)$.

Therefore, if $P_j(t)$ refers to the start of the process (initiation time) and the time of distribution (extraction) is Δ_{kj}, then the extracted items will appear at $t+\Delta_{kj}$,

$$\begin{bmatrix} e^{-s\Delta_1^{pr}} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & e^{-s\Delta_n^{pr}} \end{bmatrix}\begin{bmatrix} 0 \;\; 0 \;\; 0 & \cdots & 0 \\ \vdots\ldots.. \quad g_{kj}e^{-s\Delta_{kj}^{tr}} & & \vdots \\ 0 & \cdots & 0 \end{bmatrix}\tilde{\mathbf{P}}(s) =$$

$$= \tilde{\boldsymbol{\Delta}}^{pr}(s)\tilde{\mathbf{G}}^{tr}(s)\tilde{\mathbf{P}}(s) = \tilde{\mathbf{G}}^{pt}(s)\tilde{\mathbf{P}}(s) \qquad (6)$$

Here the diagonal matrix

$\tilde{\boldsymbol{\Delta}}^{pr}(s)$, $\tilde{\boldsymbol{\Delta}}^{tr}(s)$

are the lead time matrices of outputs and

$$\tilde{\mathbf{G}}^{pt}(s) = \tilde{\boldsymbol{\Delta}}^{pr}(s)\tilde{\boldsymbol{\Delta}}^{tr}(s)\mathbf{G}$$

is defined as the generalized output matrix.

The net production of such a system will conveniently be written:

$$\tilde{\mathbf{G}}^{pt}(s)\tilde{\mathbf{P}}(s)-\tilde{\mathbf{H}}^{pt}(s)\tilde{\mathbf{P}}_0(s)=$$

$$=\begin{bmatrix} e^{-s\Delta_1^{pr}} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & e^{-s\Delta_n^{pr}} \end{bmatrix}\begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ \vdots\ldots\ldots & & g_{kj}e^{-s\Delta_{kj}^{tr}} & & \vdots \\ 0 & & & \cdots & 0 \end{bmatrix}\tilde{\mathbf{P}}(s)-$$
$$-\begin{bmatrix} 0 & 0 & 00 & \cdots & 0 \\ \ldots. & \ldots\ldots\ldots h_{ij}e^{s\tau_{ij}^{tr}} & & \ddots & \vdots \\ h_{m1}e^{s\tau_{m1}^{tr}} & h_{m2}e^{s\tau_{m2}^{tr}} & & \cdots & 0 \end{bmatrix}\begin{bmatrix} e^{s\tau_1^{pr}} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & e^{s\tau_m^{pr}} \end{bmatrix}\tilde{\mathbf{P}}_0(s) \quad (7)$$

Given a plan $\tilde{\mathbf{P}}_0(s)$, available inventory $\tilde{\mathbf{R}}(s)$ will develop according to:

$$\tilde{\mathbf{R}}(s)=\frac{\mathbf{R}(0)+\tilde{\mathbf{G}}^{pt}(s)\tilde{\mathbf{P}}(s)-\tilde{\mathbf{H}}^{pt}(s)\tilde{\mathbf{P}}_0(s)-\tilde{\mathbf{F}}(s)}{s} \quad (8)$$

where $\mathbf{R}(0)$ collect initial available inventory levels. The term $\tilde{\mathbf{G}}^{pt}(s)\tilde{\mathbf{P}}(s)$ is the inflow of purchasing, production, extraction, distribution etc. into available inventory, the term $\tilde{\mathbf{H}}^{pt}(s)\tilde{\mathbf{P}}_0(s)$ is the required outflow representing needs generated by all processes (internal demand, dependent demand), where in both cases location influence technology matrices and the term $\tilde{\mathbf{F}}(s)$ represents deliveries (exports) from the system to the users on their existing locations.

This is an instance of the fundamental equations of MRP theory in case of extension to MRP-DRP case, where transportation delays influence behavior of supply chain and plan differ realization. In order for the plan $\tilde{\mathbf{P}}_0(s)$ to be feasible, we must always have fulfilled $£^{-1}\left\{\tilde{\mathbf{R}}(s)\right\}\geq\mathbf{0}$. This is the *available inventory constraint*. If also capacity requirements are considered, a corresponding constraint for available capacities may be formulated like in some previous papers of Grubbström et al (Segerstedt 1996).

In the case that we wish to model *cyclical processes*, repeating themselves in constant time intervals T_j, j = 1, 2, … , m, we may write the plan $\tilde{\mathbf{P}}_0(s)$ in the following way, using two new diagonal matrices $\tilde{\mathbf{t}}(s)$ and $\tilde{\mathbf{T}}(s)$,

$$\tilde{\mathbf{P}}_0(s)=\tilde{\mathbf{t}}(s)\tilde{\mathbf{T}}(s)\hat{\mathbf{P}}_0$$

$$\tilde{\mathbf{t}}(s)=\begin{bmatrix} e^{-st_1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & e^{-st_m} \end{bmatrix} \quad (9)$$
$$\tilde{\mathbf{T}}(s)=\begin{bmatrix} \left(1-e^{-sT_1}\right)^{-1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \left(1-e^{-sT_m}\right)^{-1} \end{bmatrix}$$

where $\hat{\mathbf{P}}_0$ is a vector of constants, for instance describing the total amounts planned to be produced in (or delivered by) each process during one of the periods T_j, j = 1, 2, … , m, and where t_j, j = 1, 2, … , m, are the points in time when the first of each respective cycle starts. These latter times may be necessary in order for the system to have items on lower levels available as inputs on higher levels. And realization $\tilde{\mathbf{P}}(s)$ is going to be close to the plan as much as possible according to the criterion function.

Here a series expansion of $\tilde{\mathbf{T}}(s)$ leads to

$$\tilde{\mathbf{T}}(s)\hat{\mathbf{P}}=\begin{bmatrix} \dfrac{\hat{P}_1}{1-e^{-sT_1}} \\ \vdots \\ \dfrac{\hat{P}_m}{1-e^{-sT_1}} \end{bmatrix}=\frac{1}{s}\begin{bmatrix} \dfrac{\hat{P}_1}{T_1} \\ \vdots \\ \dfrac{\hat{P}_m}{T_m} \end{bmatrix}+\frac{1}{2}\begin{bmatrix} \dfrac{\hat{P}_1}{T_1^2} \\ \vdots \\ \dfrac{\hat{P}_m}{T_m^2} \end{bmatrix}+\mathbf{O}(s) \quad (10)$$

where O(s) is a vector vanishing at least with the speed of s.

7. CRITERION FUNCTIONS OF GLOBAL SUPPLY CHAIN AND SUBSIDIES IN TIME OF RECESSION

We now turn our attention to economic relationships. Activity cell j is assumed to produce item with value per item equal p_j. We collect these values per item into a price vector p being a row vector:

$$\mathbf{p}=\left[p_1,p_2,\ldots,p_n\right] \quad (11)$$

which could have different values at different locations. Because of state interventions in time of recession this values can be disturbed by

$$\Delta \mathbf{p} = \left[\Delta p_1, \Delta p_2, ..., \Delta p_n\right] \quad (12)$$

the values achieved on the market can be reduced by fees and taxes and increased by different kind of subsidies.

Although prices are normally positive, representing positive values to the holder of the asset, there may be instances when negative prices may be used. For instance, this is the case for waste items, which need to be disposed of at an expense, and having such items represents a negative value to the holder, which could differ from location to location. Prices p_i in general differ when changing location. Between location of activity cell i and following activity cell j it can differ for $b_{ij} \cdot \tau_{ij}^{tr}$ where b_{ij} presents transportation costs per item i per time unit, which we collect into a transportation price matrix per unit of product at j, $\mathbf{\Pi}_{G,}$ $\mathbf{\Pi}_{H,}$ so that the sum of transportation costs between activity cells TrC is equal to:

$$TrC = \mathbf{E}^T \left(\tilde{\mathbf{\Pi}}_G(s)\tilde{\mathbf{P}}(s) + \tilde{\mathbf{\Pi}}_H(s)\tilde{\mathbf{P}}_0(s)\right)$$

$$\tilde{\mathbf{\Pi}}_G(s) = \begin{bmatrix} e^{-s\Delta_1^{pr}} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & e^{-s\Delta_n^{pr}} \end{bmatrix} \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 \\ \vdots ..... & & g_{kj}d_{kj}\tau_{kj} & & \vdots \\ & 0 & & \cdots & 0 \end{bmatrix}$$

$$\tilde{\mathbf{\Pi}}_H(s) = \begin{bmatrix} 0 \quad 0 \quad 00 & & \cdots & 0 \\ .... \quad ....h_{ij}d_{ij}\tau_{ij} & & \ddots & \vdots \\ h_{m1}d_{m1}\tau_{m1}... & h_{mj}d_{mj}\tau_{mj} & \cdots & 0 \end{bmatrix} \begin{bmatrix} e^{s\tau_1^{pr}} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & e^{s\tau_m^{pr}} \end{bmatrix} \quad (13)$$

and $\mathbf{E}^T = (\sum_{j=1}^{m} \mathbf{e}_j)^T$ is an m-dimensional row vector of unit values.

When the processes take place in discrete batches at times t'_{jk}, $k = 1, 2, \dots$, for process j, we may also locate fixed costs (setup costs) at these times. Such setup times of process j are conveniently collected into a sequence of Dirac impulses $\delta\left(t - t'_{jk}\right)$ and the transform of such a sequence is $\tilde{v}_j(s) = \sum_k £\left\{\delta(t - t'_{jk})\right\} = \sum_k e^{-st'_{jk}}$. If there is a fixed out-payment attached to each such batch, say K_j, the NPV of these payments together will amount to $K_j \tilde{v}_j(\rho) = K_j \sum_k e^{-\rho t'_{jk}}$. In particular, if batches are completed in an infinite sequence and all batches are temporally located between constant time intervals of length T_j, the NPV of the setup (ordering) costs will be $\mathrm{NPV} = K_j e^{-\rho t_j} / \left(1 - e^{-\rho T_j}\right)$, assuming the first batch being timed at $t'_{j1} = t_j$. In our standard treatment, ordering costs are collected into the row vector $\mathbf{K} = \left[K_1, K_2, ..., K_m\right]$.

If all processes take place in discrete batches, letting $\tilde{\mathbf{v}}(s) = \begin{bmatrix} \tilde{v}_1(s) \\ \vdots \\ \tilde{v}_m(s) \end{bmatrix}$ denote the m-dimensional vector of all setup events, the NPV of all fixed ordering costs will be

$$\mathrm{NPV}_{\mathrm{ordering}} = = -\mathbf{K}\tilde{\mathbf{v}}(\rho) = -\mathbf{K} \begin{bmatrix} \tilde{v}_1(\rho) \\ \vdots \\ \tilde{v}_m(\rho) \end{bmatrix} = -\sum_{j=1}^{m} K_j \tilde{v}_j(\rho), \quad (14)$$

When all item flows in the system together with the parameters contained in $\mathbf{p}$, $\mathbf{\Pi}_{G,}$ $\mathbf{\Pi}_{H,}$ and $\mathbf{K}$ also accurately describe the relevant cash flow, the overall NPV may be written:

$$\mathrm{NPV} = \mathbf{p}\left(\tilde{\mathbf{G}}^{pt}(\rho)\tilde{\mathbf{P}}(\rho) - \tilde{\mathbf{H}}^{pt}(\rho)\tilde{\mathbf{P}}_0(\rho)\right) - \\ -\mathbf{E}^T \left(\tilde{\mathbf{\Pi}}_G(\rho)\tilde{\mathbf{P}}(\rho) + \tilde{\mathbf{\Pi}}_H(\rho)\tilde{\mathbf{P}}_0(\rho)\right) - \mathbf{K}\tilde{\mathbf{v}}(\rho)\mathbf{E}$$

$$\mathrm{NPV} = (\mathbf{p}\tilde{\mathbf{G}}^{pt}(\rho) - \mathbf{E}^T\tilde{\mathbf{\Pi}}_G(\rho))\tilde{\mathbf{P}}(\rho) - \\ -(\mathbf{p}\tilde{\mathbf{H}}^{pt}(\rho) + \mathbf{E}^T\tilde{\mathbf{\Pi}}_H(\rho))\tilde{\mathbf{P}}_0(\rho) - \mathbf{K}\tilde{\mathbf{v}}(\rho)\mathbf{E} \quad (15)$$

If we wish to control a supply chain system so, that the realization of the flow in the system is close to the planed one as much as possible, where each activity cell has individual importance when approaching to planned production or distribution intensity, we have to write the criterion:

$$\mathrm{Min}\ ((\tilde{\mathbf{P}}(\rho) - \tilde{\mathbf{P}}_0(\rho))^T \boldsymbol{\theta}(\tilde{\mathbf{P}}(\rho) - \tilde{\mathbf{P}}_0(\rho)) \ ) \quad (16)$$

and a diagonal matrix $\boldsymbol{\theta}$ gives the importance to each production or distribution activity cell to approach to planned intensity.

In time of recession different state plans are trying to keep the activity on the level as it was before the recession by subsidies, reducing liquidity problems in time of recession. Let us assume that there was realization equal to plan before recession and now the local authorities where the activity cell j of supply chain is located wish to push the production to planned one, to keep the human resources in region, where activity cell is located, close to the previous employment. At the same time the supply chain managers try to relocate activities to keep the NPV at reducing demand high as much as possible. Their NPV is described by (15). When demand is falling also production has to be reduced and therefore employment would be much lower if we are following only criterion (15). In the negotiation procedure between local authorities at different regions where activity cells are located, giving the subsidies to keep the production high as much as possible for avoiding unemployment in the region, and supply chain managers, who are still following equation (15) the main goal is to determine the ponderous $\boldsymbol{\Psi}$ and $\boldsymbol{\theta}$ when maximizing NPV. Therefore the ponderous should be find for the following :

$$\max(((\mathbf{p}+\Delta\mathbf{p})\tilde{\mathbf{G}}^{pt}(\rho)-\mathbf{E}^T\tilde{\mathbf{\Pi}}_G(\rho))\boldsymbol{\Psi}\tilde{\mathbf{P}}(\rho)-$$
$$-((\mathbf{p}+\Delta\mathbf{p})\tilde{\mathbf{H}}^{pt}(\rho)+\mathbf{E}^T\tilde{\mathbf{\Pi}}_H(\rho))\boldsymbol{\Psi}\tilde{\mathbf{P}}_0(\rho)$$
$$-\mathbf{K}\tilde{\mathbf{v}}(\rho)\mathbf{E}-\ (\tilde{\mathbf{P}}(\rho)-\tilde{\mathbf{P}}_0(\rho))^T\boldsymbol{\theta}(\tilde{\mathbf{P}}(\rho)-\tilde{\mathbf{P}}_0(\rho))) \tag{17}$$

What becomes the game between regional policies and global supply chains, especially needed to be consider in time of recession to determine acceptable production level, which is reducing unemployment. The approach given in Bogataj and Bogataj (2001) about supply chain coordination in spatial games can be used here in more general sense.

8. CONCLUSIONS

In this paper we have studied some aspects of differences between pairs of planned activities and realization in a global supply chain. In time of recessions this kind of difference appear in every region and mostly in all global supply chains.

To describe the approach to negotiations among regional authorities and managers in global supply chain extended MRP model, previously developed by Grubbström and later extended by distribution and reverse logistics component in a compact form, presented by Grubbström, Bogataj, and Bogataj (2007), has been suggested.

We have used the results of Bogataj, Grubbström, and Bogataj (2008) demonstrating the basic differences between MRP Theory describing the flows “Under the same roof” and model of global supply chain. At global supply chain lead time appear not only because of production and logistic activities. They also influence strongly NPV of supply chain activities because of transportation time delays. Therefore we have split lead time to production and transportation part, which appear on different ways in the model.

Negotiations among regional authorities and global supply chain managers about subventions to keep the human resources in a supply chain and therefore the production on the intensity level as planned, could base on the criterion function (17), where $\Delta\mathbf{p}, \boldsymbol{\Psi}, \tilde{\mathbf{P}}(\rho)$ and $\boldsymbol{\theta}$ are subject of negotiation. MRP Theory approach with MRP-DRP extension makes a supply chain more visible and more controllable. While in market economy $\boldsymbol{\Psi}$ is supposed to play the most important role, in eastern economies of last century $\boldsymbol{\theta}$ had been over-weighting $\boldsymbol{\Psi}$. The influence of this extreme policies as well as any combination of it can be well describer and studied using MRP-DRP approach presented above.

REFERENCES

Alonso, W., 1964. *Location & Land Use: Toward a General Theory of Land Rent*, Thesis (PhD Harvard.

Aseltine, J.A., 1958. *Transform Method in Linear System Analysis*, McGraw-Hill, New York, N.Y.

Beckmann, M.J., 1999. *Lectures on Location Theory*, Springer, Berlin.

Bogataj, D. and Bogataj, M., 2007. Measuring the supply chain risk and vulnerability in frequency space. *Int. j. prod. econ.*. [Print ed.], vol. 108, Issues 1-2, pp. 291-301.

Bogataj, M. and Bogataj, L., 2003. On the compact presentation of the lead-time perturbations in distribution networks, *Int.j.Prod. Econ*, Volume 88, Issue 2, pp 145 – 155.

Bogataj, M., 1996. Inventories in spatial models. *Int. j. prod. econ.*. vol. 45, pp. 337-342.

Bogataj, M. and Bogataj, L., 2001. Supply chain coordination in spatial games. *Int. j. prod. Econ.*, vol. 71, no. 1-3, pp. 277-286.

Bogataj, M., 1999. Inventory allocation and customer travelling problem in spatial duopoly. *Int.j. Prod. Econ.*59, pp.271-279.

Bogataj, M., Grubbström, R.W., Bogataj, L (2008), *Location of activity cells in distribution and reverse logistics*, ISIR Symposium 2008, Budapest, to appear: *Int.j. Prod. Econ.*

Daganzo, C.F., 1991. *Logistics Systems Analysis*, Springer, Berlin.

Greenhut, M. L., 1995. *Location Economics: Theoretical Underpinnings and Applications*. Edward Elgar Publ.Co .

Grubbström, R.W. 1967, On the Application of the Laplace Transform to Certain Economic Problems, *Management Science*, Vol.13, No.7, pp.558-567.

Grubbström, R.W., 1998. A Net Present Value Approach to Safety Stocks in Planned Production, *Int.j. Prod. Econ.* Vol. 56-57, pp. 213-229.

Grubbström, R.W., 1996. Material Requirements Planning and Manufacturing Resources Planning, in Warner, M., (Ed.), *International Encyclopedia of Business and Management,* Routladge, London,

Grubbström, R.W., 1980. A principle for determining the correct capital costs of work-in-progress and inventory, *International Journal of Production Research*, 18, 259-271.

Grubbström, R.W., Tang, O., 2000. An Overview of Input-Output Analysis Applied to Production-Inventory Systems, *Economic Systems Review*, 12, pp.3-25.
Grubbström, R.W., Bogataj, L., (Eds), 1998. *Input-Output Analysis and Laplace Transforms in Material Requirements Planning*. Storlien 1997. FPP Portorož.
Grubbström, R.W., 2007. Transform Methodology Applied to Some Inventory Problems, *Zeitschrift für Betriebswirtschaft*, 77(3), pp. 297-324.
Grubbström, R.W., Bogataj, M., Bogataj, L, *A compact representation of distribution and reverse logistics in the value chain*, WP, Ljubljana: Faculty of Economics, KMOR, 2007.
Leontief, W.W., 1951. *The Structure of the American Economy*, Oxford University Press, London.
Lösch, A., 1954. *The Economics of Location*, Yale University Press, New Haven, CT,
Orlicky, J.A., 1975. *Material Requirements Planning*, McGraw-Hill, New York, N.Y.
Puu, T., 1997**.** *Mathematical Location and Land Use Theory*, Springer, Heidelberg.
Segerstedt, A., 1996. A Capacity- Constrained Multi-level Inventory and Production Control Problem, *International Journal of Production Economics*, Vol. 45, pp.449-461.
Verter, V., Dincer, M.C., 1995. Facility location and capacity acquisition: An integrated approach, *Naval Research Logistics* 42, pp. 1141-1160.

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