A SEQUENTIAL HEURISTIC PROGRAMMING APPROACH FOR A CORRUGATED BOX FACTORY: TRADEOFF BETWEEN SETUP COSTS AND TRIM WASTE

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ABSTRACT

This paper considers a corrugator trim problem for a cardboard boxes manufacturing plant that produces boxes of various sizes for a finished goods inventory that services known customer demand. We present a suboptimal three-step procedure that considers minimizing both trim waste cost and setup time cost where setup cost is measured as the loss of production resulting from stock rolls size changes. The procedure leads to the least-cost method of combining customer orders on the corrugator and the optimal corrugator width to use over an entire shift. The proposed method was motivated by our analysis of the day-to-day scheduling of the corrugator at the UniPaK facility one of the leading manufacturer of cardboard boxes in Lebanon. This paper concludes with an application of the proposed method to derive a lower cost corrugator schedule at UniPak.

Keywords: Corrugator scheduling, heuristic, decision support system

1. INTRODUCTION

The corrugator trim problem is defined in the literature as the problem of determining the least-cost method of combining customer orders on a corrugator where one of the major costs to avoid is waste or excess trim lost from the material used.

The corrugator trim problem is a well studied problem in the literature. There have been many attempts to solve this problem using computer models, heuristics and optimal methods (Haessler and Talbot 1983). Early work was done by Eismann (1957) and Gilmore and Gomory (1961) which proposed linear programming formulations for the general trim problem in the paper industry. It was recognized later that because of the nature of the production process, the corrugator problem could not be completely modeled by means of linear programming which led to the development of sequential heuristic procedures. Viswanthan and Bagchi (1993) developed a best-first tree search algorithm to solve a constrained two-dimensional cutting stock problem where constraints are set on the number of sheets of given dimensions to be cut using only orthogonal guillotine cuts only. Minimizing trim loss is only one of several major concerns that arise in drawing the optimal corrugator schedule. Other concerns are corrugator width utilization, cutting pattern changes (order changes), avoidance of split orders, and shutdowns costs. The problem becomes far more complex if the corrugator scheduling problem is integrated with other problems of the multi-stage production process, machine failures and unpredictability of customer behavior to ensure “on-time and in-full” deliveries of customer orders (Darley and Sanders 2004).

Krichagina et. al (1998) considered the cutting-stock problem subject to random customer demand and where the objective is to minimize long run expected average costs related to paper waste, shutdowns, backordering, and holding finished good inventory. They used a 2-step procedure with a linear programming model in the first step and a Brownian control in the second step to generate a suboptimal solution to the problem. Simplifying assumptions such as aggregating machines and dedicating machines to the production of a single grade single color papers, and not explicitly modeling shutdown and startup times were used to be able to find a good solution to the problem.

Given the complexity of the corrugator scheduling problem, it would be impossible to make optimal decisions that will achieve all the desired objectives. Thus a hierarchical heuristic approach has been typically adopted to solve the corrugator scheduling problem and this by decomposing it into smaller problems that are solved sequentially. This is why the corrugator trim problem is still for the most part solved manually.

The approach proposed in this paper was motivated by our observations at UnipaK, one of the largest cardboard manufacturers in Lebanon and UnipaK’s interest in developing a method for scheduling jobs on the corrugator that would minimize both trim waste cost and setup cost resulting form roll width and cutting pattern changes on the corrugator. More specifically, this paper looks at the tradeoff between minimizing trim loss and maximizing roll width utilization in order to minimize waste in material and setup times incurred by changing the roll width and the cutting patterns at every order run. The approach proposed in this paper is a sequential heuristic programming one where a 3-step sequential procedure is used to draw an optimal daily schedule for UniPak.

2. PRACTICAL CONSIDERATIONS IN SCHEDULING JOBS ON CORRUGATORS

The manufacture of corrugated cardboard boxes involves fabricating a continuous strip of corrugated board then cutting it into sheets of customer-specified dimensions. The corrugator forms a linerboard strip into a fluted shape then sandwiches it between two liners to produce a single-wall board. There are a number of common flute styles. The corrugated strip is next passed over a long set of rollers to allow sufficient time for drying. It is here that the first trim waste is incurred because the board edges are rough and irregular, about 1 cm is removed from each side of the strip. Then the corrugator board is cut into smaller strips corresponding to the specified sheet width, and cut-off to yield the sheet length. Cut-off knives and slitting knives are used to make horizontal cuts and vertical cuts respectively. Most trim waste occurs during the slitting/cut-off stage. The amount of waste is determined by the width of the corrugated strip being produced. This is why linerboard rolls exist in a number of different sizes. Changing from a narrower roll width to a larger roll width normally slows down the machine.

Corrugated boxes are bulky and subject to weather damage, hence the manufacturer would want to have low inventory levels and frequent production runs to ensure on-time delivery. To this end, the production scheduler typically goes through the following steps to schedule customer orders on the corrugator in order to minimize excess trim waste.

- ***Selection and classification of Jobs:*** the planner divides and arranges the factory tickets received from the customer service department first based on the fluting type of each order and on the liner (paper) type and color. The scheduler then arranges the different sets of factory tickets in ascending order of the different layers GSM (gram per square meter) and not the overall GSM and further group them into sets of orders that have the same GSM requirements within a 5% tolerance (difference). It is a common practice in the corrugated cardboard industry to quality-upgrade orders for practical considerations.
- ***Combining Jobs:*** the planner combines jobs within a group that can be processed simultaneously on the corrugator in order to minimize trim waste. In combining jobs, the planner has to determine the roll width based on the number of sheets that can be produced for each job per horizontal cut. The roll length is determined later based on the total number of sheets to be produced for each job to satisfy the order quantity. The planner can increment or decrement the order quantity of any job in order to adjust the total length of roll needed to meet the demand of both jobs in a combination that minimizes linear trim waste. This is another common practice in the corrugated cardboard industry where orders specify the quantity with overrun and underrun tolerances, typically a 10% tolerance. Also in combining jobs, the planner has to accommodate many practical considerations in combining jobs. For example, a maximum of four sheets can be generated per cut on one cutoff and a maximum of six to seven sheets can be generated as a total on both cutoffs. This is important to control serious bottleneck problem down the line at the finishing stage.
- ***Sequencing Jobs on the corrugator:*** the last step is to schedule the jobs on the corrugator based on the availability of the finishing machines and the due date priority of the orders.

Setup times are incurred when changing rolls and in axle changeovers. Setup times vary with the type of change; from our observation of Unipak operations, an average12 minutes are needed to change the linerboard roll width and 10 minutes to change the fluting type to the next thicker or thinner fluting type. As for the triplex or triple axle changeovers, 5 minutes are needed on average to fix and set the next order cutting pattern. In this paper an average of 12 minutes setup time is used for roll-width changes including triplex changeovers if any.

It has also been noted that the corrugator speed varies between a maximum of 160 Mpm (Meters/min) and a minimum of 60 Mpm depending on the order length. The smaller the order length is, the slower the corrugator speed. In this paper and for the purpose of assessing the savings in the proposed scheduling approach, we shall assume an average corrugator speed of 100 Mpm. Consequently the money value of the unit setup time could be estimated by multiplying the corrugator speed by the product of the setup time required and the unit cost of paper which is assumed in this paper to be 0.07$ per meter

3. CORRUGATOR SCHEDULING APPROACH FOR MINIMIZING WASTE AND SETUP TIME

This section presents a three-step suboptimal procedure for drawing the daily corrugator schedule. The procedure returns the least-cost combination of orders to be scheduled on the corrugator over a given shift along with the optimal corrugator roll width to be used to produce all customer orders scheduled for production over that shift at minimal trim loss. Fixing the roll width over the whole shift will reduce setup time incurred by changing roll width for different order runs. The output is a roll width used over an entire production shift and the optimal combination of customer orders that will be processed during that shift that minimizes trim loss.

The following is a sequential heuristic programming approach for scheduling jobs on a corrugator while minimizing trim loss and maximizing roll width utilization.

1. Solve the ***Job Selection problem*** to identify the set S of all possible pairs of jobs (i, j) that can be combined based on paper type, color, fluting and GSM requirements.
2. Fix the roll width k and solve the ***Job Matching problem*** to find, for each roll width and for each pair of jobs (i, j) in S, the optimal number of horizontal and vertical cuts that minimize total trim waste W_{ijk}.
3. For each roll width k, solve the ***Roll Width Optimization problem*** to find the optimal combination of customer orders that minimizes trim waste for roll width k, W_k. Then select the best roll width that minimizes geometric waste and setup time based on equivalent dollar values.

3.1. Step 1: Solve Job Selection Problem

The corrugator under consideration has 2 cutoffs and thus can process 2 different orders at the same time. Jobs are first classified as primary or secondary depending on their due date. Primary jobs are jobs that should be produced now in order to meet their delivery date. Secondary jobs are those that can be run along a primary job in order to minimize trim waste. Orders are paired if they can be processed simultaneously on the corrugator. A pair of orders is formed by combining a primary job with another primary or secondary job provided that the second job has:

- The same fluting type and paper color as the first job
- A gram per square meter (GSM) requirement within 5% of the first job GSM requirement.

Figure 1 shows the algorithm used for solving the *Job Selection* problem. The Job Selection problem is solved on Excel. The program returns the set of paired orders S that can be processed simultaneously, where $S = \{(i, j); i \in POrders; j \in AOrders; P_{ij} = 1\}$. Each pair (i, j) is a possible solution for the Job Matching problem.

3.2. Step 2: Solve Job Matching Problem

Given S the set of paired orders generated in step 1 and R the set of roll widths available, where R for the case of Unipak is R = {1.8, 1.9, 2, 2.05, 2.1, 2.10, 2.15, 2.2} in meters, the Job Matching problem consists of finding, for a set roll width k, the optimal number of vertical and horizontal cuts for each combination of jobs (i, j) in S, that minimizes the geometric trim waste W_{ijk}.

```
Job_Selection(POrders, AOrders)
{
    • Get number (Np) of primary orders (POrders),
      and number (N) of all orders (AOrders).
      POrders is a subset of AOrders. AOrders is
      ordered by listing POrders first.
    • Get fluting requirements (F) of AOrders
    • Get paper color (C) of AOrders
    • Get GSM requirements (Gsm) of AOrders
    For i = 1 to Np do
    {
        For (j = i+1 to N; j++)
        {
            if OR$\{(F_i \neq F_j);(C_i \neq C_j)\}$
                then $P_{ij} = 0$
                else if
                    $\text{ABS}\left(\frac{gsm_j - gsm_i}{\text{Max}(gsm_j, gsm_i)}\right) \geq 0.05$
                    then $P_{ij} = 0$
                    Else $P_{ij} = 1$
        }
    }
}
```

Figure 1: Job_Selection Algorithm

There are a number of practical considerations and machine limitations that should be observed in qualifying paired jobs in S for further consideration. Some of these practical considerations and limitations are

- the total width required for combining the two orders should not exceed the corrugator roll width size
- the two orders combined would need the same run length with a ±10% margin.
- the total number of sheets produced on the two cut-offs should not exceed seven sheets for the case considered in this paper
- the number of sheets produced per cut on each of the upper and lower cut-off should not exceed four sheets also for the case considered in this paper.

Figure 2 shows the algorithm used for solving the *Job Matching* problem. The algorithm returns for each roll width k in R the set S_k of paired jobs (i, j) from S that can be processed together on the corrugator using a roll width k and the minimum trim waste W_{ijk} resulting from running them together on the corrugator. Note that if the two jobs in a pair cannot be processed simultaneously on the corrugator for a given roll width because of roll width limitations and thus the number of vertical cuts for either one of the 2 jobs is zero, the pair (i, j) is removed from the set *S* and thus from further consideration.

```
Job_Matching (S, R, S_k)
{
  Forall $k \in R$, do
  {
    Forall $(i, j) \in S$ , do
    {
    Solve nb_verticalcuts to find optimal number of vertical cuts per horizontal cut for order i ($V_{ij}$) and order j ($V_{ji}$) that minimize linear trim waste
    If $V_{ij} \neq 0$ and $V_{ji} \neq 0$
        then
        • Solve nb_horizontalcuts to find total length of roll needed to satisfy demand of primary order i while minimizing geometric trim waste $W_{ijk}$ for the combination (i, j, k)
        • Append $(i, j, W_{ijk})$ to the set $S_k$
    }
    Forall $i \in POrders$, do
    {
    • Compute number of vertical cuts per horizontal cut for order i by dividing k with the sheet width of order i
    • Compute total length of roll needed to satisfy demand of order i by dividing order quantity of i by the number of vertical cuts found
    • Compute geometric trim waste $W_{iik}$ for the combination (i, i, k)
    • Append $(i, i, W_{iik})$ to the set $S_k$
    }
  }
}
```

Figure 2: Job_Matching Algorithm

3.2.1. Get Optimal Number of Vertical Cuts Per Horizontal Cut (nb_vertical cuts)

Given a roll width *k* and a given combination of jobs (i, j), let W_i, L_i, and D_i be the width, length, and demand in number of sheets for order *i*. Similarly let W_j, L_j, and D_j represent the requirements for order *j*. The following program finds the optimal number of sheets for orders *i* (V_{ij}) and *j* (V_{ji}) that can be generated per horizontal cut in order to minimize linear trim waste generated LW_{ijk}.

$$\text{Min } LW_{ijk} = k - V_{ij}W_i - V_{ji}W_j$$

Subject to

$$\begin{aligned}
&V_{ij}W_i - V_{ji}W_j \leq K - 0.025 \\
&D_j L_j V_{ij} \geq 0.9 D_i L_i V_{ji} \\
&D_j L_j V_{ij} \leq 1.1 D_i L_i V_{ji} \\
&V_{ij} + V_{ji} \leq 7 \\
&V_{ij} \leq 4 \\
&V_{ji} \leq 4 \\
&V_{ji} \text{ and } V_{ij} \text{ integers}
\end{aligned} \tag{1}$$

The constraints in (1) take into account some practical considerations and limitations of the corrugator machine highlighted earlier. In particular the first and second constraints limit possible job combinations to those that would require the same run length plus or minus a 10% acceptable margin. The above program could be easily adjusted to allow the production of partial orders by relaxing this constraint and updating the list of orders in *S* by adding the residual orders with updated remaining demand. This program was solved using LINGO and the optimal number sheets, V_{ij} and V_{ji} that minimize the linear trim waste for the combination (i, j, k) is exported to excel for further processing.

3.2.2. Get Optimal Number of Horizontal Cuts (nb_horizontalcuts)

Given a roll width *k* and a job combination (i, j), and given the optimal number of sheets per horizontal cut V_{ij} and V_{ji} for orders *i* and *j* respectively, the following program returns the optimal number of horizontal cuts H_{ij} and H_{ji} for orders *i* and *j* respectively that minimize the geometric trim waste W_{ijk} for the combination (i, j, k)

$$\text{Min } W_{ijk} = (k - V_{ij}W_i - V_{ji}W_j)H_{ij}L_i$$

Subject to

$$\begin{aligned}
&0.9D_i \leq V_{ij}H_{ij} \leq 1.1D_i \\
&0.9D_i L_i \leq H_{ji} L_j V_{ij} \leq 1.1 D_i L_i \\
&H_{ij} \text{ and } H_{ji} \text{ integers}
\end{aligned} \tag{2}$$

The first constraint in (2) ensures that the total number of sheets produced for each order meets the required demand of order *i* within a 10% tolerance range. The second constraint ensures that the total vertical length of the paper roll required is the same for the two orders combined within the 10% acceptable margin. The above model is solved using LINGO. The data is imported from excel and the solution was exported back to the same excel sheet.

3.3. Step 3: Solve Roll Width Optimization Problem

The ***Roll Width Optimization problem*** consists of finding, for a set roll width, the optimal combination of orders from S_k that minimize total trim waste W_k and where all primary orders are processed either separately or paired with other primary or secondary orders. This process is repeated for all roll widths available and the total trim waste W_k for each roll width *k* is computed. The roll width with minimum W_k is selected and used for production during the day/shift under consideration. Note that S_k is the set of paired jobs that was computed in step 2 where

$$S_k = \{(i, j, W_{ijk}); i \in POrders, j \in AOrders, j \geq i, V_{ij} \neq 0, V_{ji} \neq 0\}$$

The roll width optimization problem is modelled and solved as an assignment problem. Let *O* be the set of all orders *i* and *j* such that $(i, j) \in S_k$. The cost matrix would consist of the waste values W_{ijk} and W_{iik} found in step 2. Note that the cost matrix is symmetrical because $W_{ijk} = W_{jik}$ for a pair of jobs *i* and *j*. The optimal combination of jobs to run on the corrugator for a set roll width *k* is found by solving (3).

$$W_k = \sum_{i \in O} \sum_{j \in O} 0.5 Z_{ij} W_{ijk}$$

$$\text{Subjectto} \tag{3}$$

$$\sum_{i \in O} Z_{ij} = 1; \forall j$$

$$\sum_{j \in O} Z_{ij} = 1; \forall i$$

$$Z_{ij} = \{0,1\}$$

(3) is solved for every roll width *k* and the roll width that result in the minimum W_k is set for use during the shift/day under consideration.

4. THE UNIPACK CASE

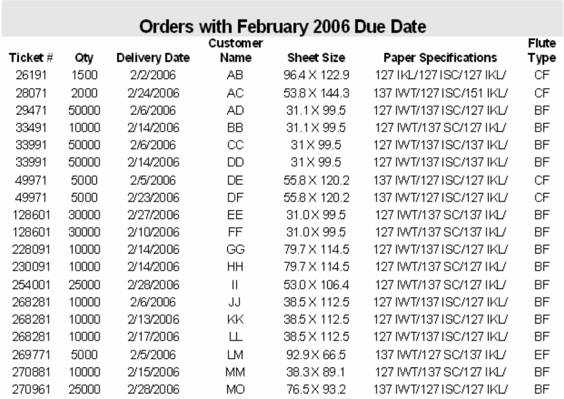
Motivated by our analysis of the day-to-day scheduling of the corrugator at the UniPaK facility, one of the largest packaging industries in Lebanon, we automated the three-step suboptimal procedure outlined in section 4 to provide UnipaK with a decision support tool that enables them to find a lower cost corrugator schedule. The tool is built in Excel and calls LINGO for solving programs (1), (2) and (3). Excel is used as a platform for preprocessing data files that come from the company and for post-processing results obtained from LINGO.

Customer order data comes in the form of an excel sheet including data such as

- Order name
- Factory Ticket Number
- Order sheet width
- Order sheet length
- Order sheet demand
- Order sheet GSM, fluting, and paper type

A small sample of actual orders that come with due dates in February 2006 are shown in Table 1 for illustration purposes. Customer names and other proprietary information is not shown. Orders are referred to using anonymous customer names.

Table 1: Small Sample of customer Orders for the Month of February

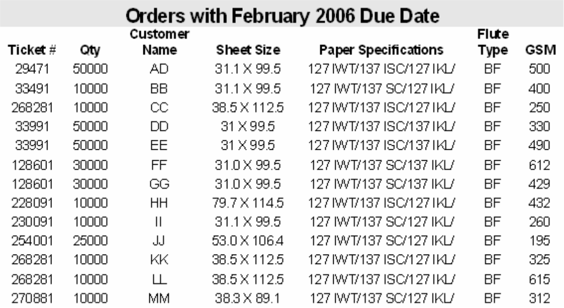


Orders with February 2006 Due Date

Ticket #	Qty	Delivery Date	Customer Name	Sheet Size	Paper Specifications	Flute Type
26191	1500	2/2/2006	AB	96.4 X 122.9	127 IKL/127 ISC/127 IKL/	CF
28071	2000	2/24/2006	AC	53.8 X 144.3	137 IWT/127 ISC/151 IKL/	CF
29471	50000	2/6/2006	AD	31.1 X 99.5	127 IWT/137 ISC/127 IKL/	BF
33491	10000	2/14/2006	BB	31.1 X 99.5	127 IWT/137 SC/127 IKL/	BF
33991	50000	2/6/2006	CC	31 X 99.5	127 IWT/137 ISC/137 IKL/	BF
33991	50000	2/14/2006	DD	31 X 99.5	127 IWT/137 ISC/137 IKL/	BF
49971	5000	2/5/2006	DE	55.8 X 120.2	137 IWT/127 ISC/127 IKL/	CF
49971	5000	2/23/2006	DF	55.8 X 120.2	137 IWT/127 ISC/127 IKL/	CF
128601	30000	2/27/2006	EE	31.0 X 99.5	127 IWT/137 SC/137 IKL/	BF
128601	30000	2/10/2006	FF	31.0 X 99.5	127 IWT/137 SC/137 IKL/	BF
228091	10000	2/14/2006	GG	79.7 X 114.5	127 IWT/137 ISC/127 IKL/	BF
230091	10000	2/14/2006	HH	79.7 X 114.5	127 IWT/137 SC/127 IKL/	BF
254001	25000	2/28/2006	II	53.0 X 106.4	127 IWT/137 SC/127 IKL/	BF
268281	10000	2/6/2006	JJ	38.5 X 112.5	127 IWT/137 ISC/127 IKL/	BF
268281	10000	2/13/2006	KK	38.5 X 112.5	127 IWT/137 ISC/127 IKL/	BF
268281	10000	2/17/2006	LL	38.5 X 112.5	127 IWT/137 ISC/127 IKL/	BF
269771	5000	2/5/2006	LM	92.9 X 66.5	137 IWT/127 SC/137 IKL/	EF
270881	10000	2/15/2006	MM	38.3 X 89.1	127 IWT/137 SC/127 IKL/	BF
270961	25000	2/28/2006	MO	76.5 X 93.2	137 IWT/127 ISC/127 IKL/	BF

Table 2 shows those jobs from Table 1 that have the same fluting type and paper type and color, namely, it shows the list of jobs requiring B fluting and White 1st grade type paper top.

Table 2: Orders sorted by fluting and paper type

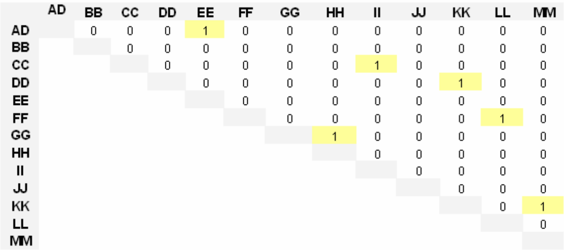


Orders with February 2006 Due Date

Ticket #	Qty	Customer Name	Sheet Size	Paper Specifications	Flute Type	GSM
29471	50000	AD	31.1 X 99.5	127 IWT/137 ISC/127 IKL/	BF	500
33491	10000	BB	31.1 X 99.5	127 IWT/137 SC/127 IKL/	BF	400
268281	10000	CC	38.5 X 112.5	127 IWT/137 ISC/127 IKL/	BF	250
33991	50000	DD	31 X 99.5	127 IWT/137 ISC/137 IKL/	BF	330
33991	50000	EE	31 X 99.5	127 IWT/137 ISC/137 IKL/	BF	490
128601	30000	FF	31.0 X 99.5	127 IWT/137 SC/137 IKL/	BF	612
128601	30000	GG	31.0 X 99.5	127 IWT/137 SC/137 IKL/	BF	429
228091	10000	HH	79.7 X 114.5	127 IWT/137 ISC/127 IKL/	BF	432
230091	10000	II	31.1 X 99.5	127 IWT/137 SC/127 IKL/	BF	260
254001	25000	JJ	53.0 X 106.4	127 IWT/137 SC/127 IKL/	BF	195
268281	10000	KK	38.5 X 112.5	127 IWT/137 ISC/127 IKL/	BF	325
268281	10000	LL	38.5 X 112.5	127 IWT/137 ISC/127 IKL/	BF	615
270881	10000	MM	38.3 X 89.1	127 IWT/137 SC/127 IKL/	BF	312

For the purpose of illustration the Primary orders (*POrders*) are identified to be AD through MM and the *AOrders* are the same list of orders. Solving the *Job Selection problem* (step 1) on the data in Table 1 gave the paired jobs shown in Table 3 and the set $S = \{(AD, EE)\ (CC, II)\ (DD, KK)\ (FF, LL)\ (GG, HH)\ (KK, MM)\}$. Jobs in *S* can be paired because they have GSM requirements within acceptable tolerance.

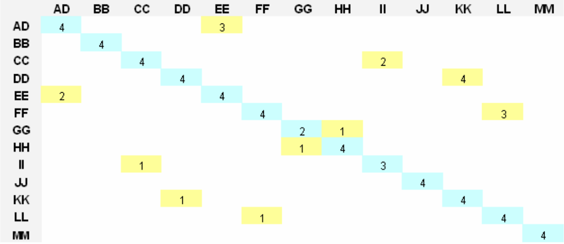
Table 3: Solution of Job Selection Problem



	AD	BB	CC	DD	EE	FF	GG	HH	II	JJ	KK	LL	MM
AD		0	0	0	1	0	0	0	0	0	0	0	0
BB			0	0	0	0	0	0	0	0	0	0	0
CC				0	0	0	0	0	1	0	0	0	0
DD					0	0	0	0	0	0	1	0	0
EE						0	0	0	0	0	0	0	0
FF							0	0	0	0	0	1	0
GG								1	0	0	0	0	0
HH									0	0	0	0	0
II										0	0	0	0
JJ											0	0	0
KK												0	1
LL													0
MM													

The following shows the computations for a selected roll width of 1.8 m. The same steps are carried for all available roll widths but are not presented in this paper. Tables 4 and 5 show the results of solving the *Job Matching problem* (step 2) for the selected roll width of 1.8 m. The same computations are carried on all available roll widths and the results saved for the next step (step 3 of the proposed procedure). In particular, Table 4 shows the optimal number of vertical cuts per horizontal cut found by solving *nb_verticalcuts* for each pair of jobs in *S* in addition to the optimal number of vertical cuts found if each job is to be processed individually on the corrugator for the same roll width of 1.8 m (shown on the diagonal). For example, for the pair (*AD*, *EE*) in *S*, the number of vertical cuts of order *AD* is 3 cuts or 3 sheets per horizontal cut and the number of sheets for order *EE* is 2. The number of cuts or sheets per horizontal cut can be read similarly for the remaining pairs of orders in *S*. Note that the pair *(KK, MM)* of *S* was eliminated from further consideration as a result of applying *nb_verticalcuts* for a roll width of 1.8m and this is due to the fact that the run length of both jobs does exceed the 10% accepted tolerance. Hence, the set $S_{1.8}$ formed at the end of this step include the remaining jobs in S in addition to individual jobs.

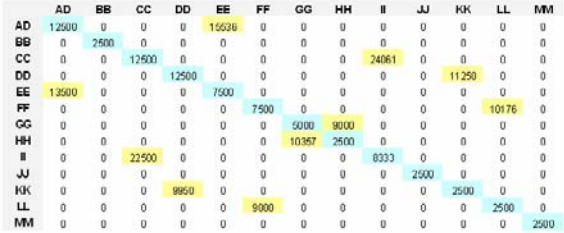
Table 4: Solution of *nb_verticalcuts* for a Roll width of 1.8 m



	AD	BB	CC	DD	EE	FF	GG	HH	II	JJ	KK	LL	MM
AD	4				3								
BB		4											
CC			4						2				
DD				4							4		
EE	2				4								
FF						4						3	
GG							2	1					
HH							1	4					
II			1						3				
JJ										4			
KK				1							4		
LL						1						4	
MM													4

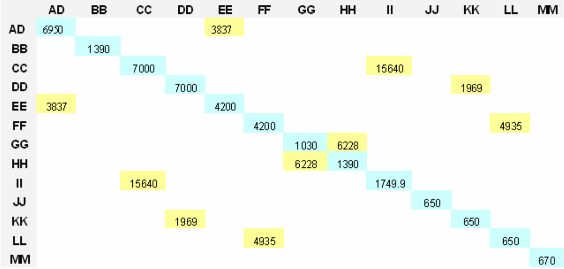
Table 5 shows the results of solving nb_horizontal cuts for each pair of jobs in $S_{1.8}$. In particular it shows the number of horizontal cuts or sheets that will be generated for each job in a pair to satisfy the demand per job shown earlier in Table 1. Table 6 shows the geometric waste in square meters generated from running jobs in the combinations shown in Table 5 for a roll width of 1.8 m. The geometric waste that would be generated from running jobs individually on a roll width of 1.8 m is shown on the diagonal of Table 6.

Table 5: Solution of nb_horizontalcuts for a Roll width of 1.8 m



	AD	BB	CC	DD	EE	FF	GG	HH	II	JJ	KK	LL	MM
AD	12500	0	0	0	15636	0	0	0	0	0	0	0	0
BB	0	2500	0	0	0	0	0	0	0	0	0	0	0
CC	0	0	12500	0	0	0	0	0	24061	0	0	0	0
DD	0	0	0	12500	0	0	0	0	0	0	11250	0	0
EE	13500	0	0	0	7500	0	0	0	0	0	0	0	0
FF	0	0	0	0	0	7500	0	0	0	0	0	10176	0
GG	0	0	0	0	0	0	5000	9000	0	0	0	0	0
HH	0	0	0	0	0	0	10357	2500	0	0	0	0	0
II	0	0	22500	0	0	0	0	0	8333	0	0	0	0
JJ	0	0	0	0	0	0	0	0	0	2500	0	0	0
KK	0	0	0	9950	0	0	0	0	0	0	2500	0	0
LL	0	0	0	0	0	9000	0	0	0	0	0	2500	0
MM	0	0	0	0	0	0	0	0	0	0	0	0	2500

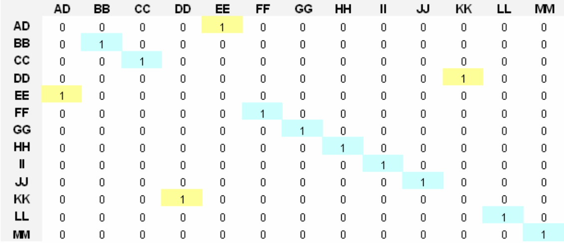
Table 6: Waste in square meters generated for a roll width of 1.8 m



	AD	BB	CC	DD	EE	FF	GG	HH	II	JJ	KK	LL	MM
AD	6950				3837								
BB		1390											
CC			7000						15640				
DD				7000							1969		
EE	3837				4200								
FF						4200						4935	
GG							1030	6228					
HH							6228	1390					
II			15640						1749.9				
JJ										650			
KK				1969							650		
LL						4935						650	
MM													670

Table 7 shows the results obtained by solving the *Roll Width Optimization* problem (Step 3 of the procedure) for a roll width of 1.8 m. The results indicate the combination of jobs that would result in minimal total geometric waste. Note that the combinations *(CC, II)* and *(FF, LL)* were not optimal and thus only *(AD, EE)*, and *(DD, KK)* were retained from *S* the remaining jobs are to be run individually if total geometric waste is to be minimized.

Table 7: Solution of *Roll Width Optimization* problem for a roll width of 1.8 m

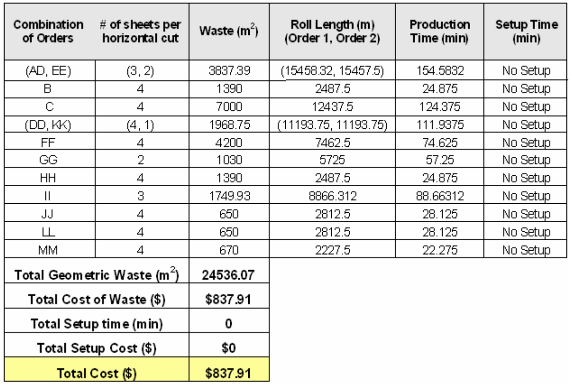


	AD	BB	CC	DD	EE	FF	GG	HH	II	JJ	KK	LL	MM
AD	0	0	0	0	1	0	0	0	0	0	0	0	0
BB	0	1	0	0	0	0	0	0	0	0	0	0	0
CC	0	0	1	0	0	0	0	0	0	0	0	0	0
DD	0	0	0	0	0	0	0	0	0	0	1	0	0
EE	1	0	0	0	0	0	0	0	0	0	0	0	0
FF	0	0	0	0	0	1	0	0	0	0	0	0	0
GG	0	0	0	0	0	0	1	0	0	0	0	0	0
HH	0	0	0	0	0	0	0	1	0	0	0	0	0
II	0	0	0	0	0	0	0	0	1	0	0	0	0
JJ	0	0	0	0	0	0	0	0	0	1	0	0	0
KK	0	0	0	1	0	0	0	0	0	0	0	0	0
LL	0	0	0	0	0	0	0	0	0	0	0	1	0
MM	0	0	0	0	0	0	0	0	0	0	0	0	1

Table 8 summarizes the findings for a roll width of 1.8 m. The least cost corrugator schedule for this roll width calls for combining jobs AD and EE and jobs DD and KK together and running the remaining jobs separately. The resulting total waste for this job combination and roll width is found to be 24536.07 m^2 or the equivalent of $ 837.91.

The same steps were carried for all available roll widths namely 1.9, 2, 2.05, 2.1, 2.10, 2.15, and 2.2 meters and the total waste value was found for each roll width and resulting corrugator schedule. The minimum total waste among all roll widths did correspond to the roll width of 1.8 m and thus the schedule shown in Table 8 is the final schedule returned by the proposed method. Note that no setup time is recorded for the proposed schedule because the roll width is fixed over the entire production.

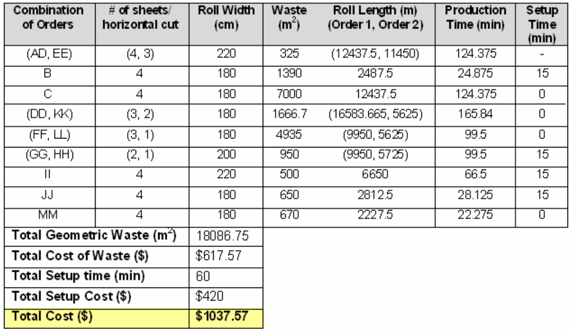
Table 8: Optimal Schedule for a roll width of 1.8 m



Combination of Orders	# of sheets per horizontal cut	Waste (m^2)	Roll Length (m) (Order 1, Order 2)	Production Time (min)	Setup Time (min)
(AD, EE)	(3, 2)	3837.39	(15458.32, 15457.5)	154.5832	No Setup
B	4	1390	2487.5	24.875	No Setup
C	4	7000	12437.5	124.375	No Setup
(DD, KK)	(4, 1)	1968.75	(11193.75, 11193.75)	111.9375	No Setup
FF	4	4200	7462.5	74.625	No Setup
GG	2	1030	5725	57.25	No Setup
HH	4	1390	2487.5	24.875	No Setup
II	3	1749.93	8866.312	88.66312	No Setup
JJ	4	650	2812.5	28.125	No Setup
LL	4	650	2812.5	28.125	No Setup
MM	4	670	2227.5	22.275	No Setup
Total Geometric Waste (m^2)		**24536.07**			
Total Cost of Waste ($)		**$837.91**			
Total Setup time (min)		**0**			
Total Setup Cost ($)		**$0**			
Total Cost ($)		**$837.91**			

Table 9 shows the actual schedule for the job selection shown in Table 1. The roll width was changed depending on the combination of jobs to minimize geometric waste or material scrap.

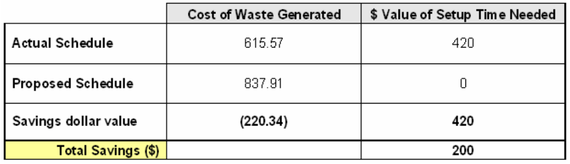
Table 9: Summary of waste and other operations information of actual corrugator schedule



Combination of Orders	# of sheets/ horizontal cut	Roll Width (cm)	Waste (m^2)	Roll Length (m) (Order 1, Order 2)	Production Time (min)	Setup Time (min)
(AD, EE)	(4, 3)	220	325	(12437.5, 11450)	124.375	-
B	4	180	1390	2487.5	24.875	15
C	4	180	7000	12437.5	124.375	0
(DD, KK)	(3, 2)	180	1666.7	(16583.665, 5625)	165.84	0
(FF, LL)	(3, 1)	180	4935	(9950, 5625)	99.5	0
(GG, HH)	(2, 1)	200	950	(9950, 5725)	99.5	15
II	4	220	500	6650	66.5	15
JJ	4	180	650	2812.5	28.125	15
MM	4	180	670	2227.5	22.275	0
Total Geometric Waste (m^2)		18086.75				
Total Cost of Waste ($)		$617.57				
Total Setup time (min)		60				
Total Setup Cost ($)		$420				
Total Cost ($)		**$1037.57**				

The actual schedule as shown has a lower total materials waste cost, however if we consider the additional setup time associated with changing roll widths and its equivalent dollar value, we find that the total cost of the actual schedule is higher than the cost of the proposed schedule as shown in Table 10 below. The savings for the small selection of 20 jobs presented in this paper is around $200. This figure is much more significant if all orders over the month of February were considered and even more significant if we were to consider the cost of waste over a year of production.

Table 10: Comparison of proposed vs. actual corrugator schedule



	Cost of Waste Generated	$ Value of Setup Time Needed
Actual Schedule	615.57	420
Proposed Schedule	837.91	0
Savings dollar value	**(220.34)**	**420**
Total Savings ($)		**200**

5. CONCLUSION

This paper presented a three-step suboptimal procedure that draws a daily corrugator schedule while minimizing both trim waste cost and setup time cost where setup cost is measured as the loss of production resulting from stock rolls size changes. The procedure yields the optimal roll width to use over an entire shift. The proposed method was motivated by our analysis of the day-to-day scheduling of the corrugator at the UniPaK facility one of the leading manufacturer of cardboard boxes in Lebanon. An example application of the proposed method to UniPak operations showed that the proposed procedure yields a lower-cost daily schedule of jobs on the corrugator when compared to the cost of the actual schedule for the same day. Indeed and although the actual schedule had a lower total materials waste cost, if we consider the additional setup time associated with changing roll widths and its equivalent dollar value, the proposed procedure gave significant improvements. By fixing the corrugator roll width for a whole shift of eight hours, we were able to achieve significant improvement and savings in terms of the dollar value of paper loss.

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