AN EFFICIENT CONSTRAINT PROGRAMMING MODEL FOR COOPERATIVE FLIGHT DEPARTURES

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ABSTRACT
Nowadays, air transportation is considered the fastest way to transport people and goods within the shortest time over long distances. Due to growing air traffic trends, the development of advanced Decision Support Tools (DSTs), based on technological advances in communications and navigation in Air Traffic Management (ATM), is important to guarantee sustainable transport logistics to balance airspace capacity with user demands. In this paper, the tuning of Calculated-Take-Off Times (CTOTs) as a tool for mitigating the propagation of perturbations between trajectories in dense sectors is analysed. The proposed methodology uses a powerful tool for predicting potential spatio-temporal concurrence events between trajectories over the European airspace that are removed by considering bounded time stamp adjustments on strategic agreed points of the aircraft trajectory. The proposed approach is based on a robust Constraint Programming model aimed to determine the feasible time stamp agreements considering the Trajectory Based Operation (TBO) interdependencies.

Keywords: Air transportation, Constraint programming, Air traffic management, Decision support tools

1. INTRODUCTION
Air transport is an integral part of transport infrastructure and a significant sector of the economy predicted next decades with steady growth. Therefore, the identification of operational and managing policies for better performance of existing airspace procedures is important in order to cut European Air Traffic Management (ATM) costs, increase capacity and operational safety and decrease the environmental impact. The intention of this innovative approach is to design a competitive ATM system, supporting up to a certain extent the Airspace User (AU's) demands at the right time (i.e. departure slots), at the right cost (i.e. suitable level of Air Traffic Control (ATC) service) at the right place (i.e. AU's preferred trajectories) and at the right service quality (i.e. safety) without extra investments, just by removing the ATM non-added-value operations that indirectly impact on present ATM capacity.

By empowering the concept of Trajectory-based operations (TBO) as a flexible synchronization mechanism towards an efficient and competitive ATM service a precise description of an aircraft path in space and time can be retrieved. Under this TBO approach, airspace users should fly precise 4-dimensional trajectories (4DTs), previously agreed upon with the network manager.

Europe has some of the busiest airspace in the world, managed by a network covering 11.5 million km² of airspace (SESAR 2015). The Network Manager Operations Centre receives, processes and distributes up to 35,000 flight plans a day (Eurocontrol 2016). This concerns over 500 European airports and airfields. To safely operate this demand any AU intending to depart from, arrive at or overfly one of the 42 countries which form part of the EUROCONTROL operations area must submit a flight plan that has to be approved in advance. Once the flight plan has been approved, the Reference Business Trajectory (RBT) is agreed and the aircraft is authorized to proceed in accordance with the RBT by defined conflict free segments. This set of business objectives may be updated or revised.

Although the ATM network is becoming designed to be robust and resilient to a whole range of disturbances, due to its dynamic and complexity unforeseen disruption can occur at any time and influence the functionality. Delay causes can be found for example in the rotation of aircrafts, in the turnaround processes, in Air Traffic Flow Management (ATFM) and Air Traffic Control (ATC) restrictions, in maintenance problems and weather conditions. When the delays exceed the agreed *green delay* of [-5,10] minutes, the extant aircraft must be rescheduled (Nosedal 2016).

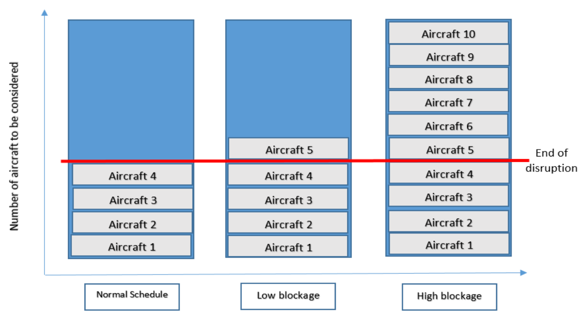

Figure 1: Example of delay and resulting combinatorial possibilities

As it can be seen in Figure 1, low blockages might occur when for example one aircraft is delayed due to maintenance problems whereas high blockages might result due to weather problems or strikes.

The presented approach supports the recovery process by rescheduling the sequence of delayed aircraft for takeoff in a way that the departure-time-bounded adjustment process that preserves the scheduled slots will be used while relaxing tight 4DT interdependencies to mitigate demand-capacity imbalances.

Small adjustments within the $[-5,10]$ interval around the Calculated Take-Off Time, along with bounded modifications on the flight duration, will be considered as the actions to be taken considering the trajectory and the impact on potential ATC interventions. Using this approach, the decision variables and their domains are included in the Constraint Programming (CP) model which leads to more combinatorial possibilities to find a more robust solution in which tight interdependencies can be removed. For instance, the Figure 2 illustrates how after a high blockage the combinatorial possibilities to adjust the departure sequence are much higher, which brings the opportunity to likely find optimal solutions avoiding tight interdependencies.

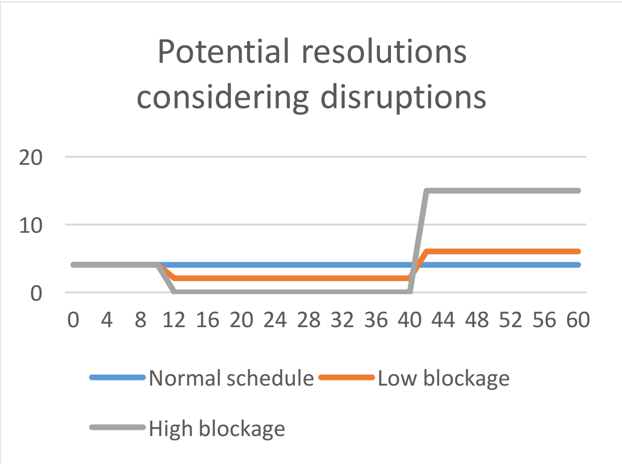

Figure 2: Potential resolutions considering disruptions (x-axis time in minutes; y- axis sequencing alternatives)

Whether by huge delays, slight delay impacts or by regular running of operations, whenever the complexity becomes too high, the proposed approach introduces small time adjustments on the aircraft to remove the detected tight interdependencies.

Two kind of time adjustments are considered:

- Introduce ground delays with a time offset of $[-5,10]$ minutes, remaining in the boundary of *green delays* to achieve fairness between airlines, since greater delays are quite unpopular as they can be very costly and would affect the strategic airspace configuration.
- Issue slight modifications on time stamps for relevant trajectory points. Since a simple shifting of the whole trajectory does not contribute in preserving the Target Time of Arrival (TTA), a more flexible approach is proposed considering additional small time adjustments at strategic points to have a control over the Time-To-Overfly (TTO) on relevant points close to the hotspot areas.

Using this approach, the goal of reaching the TTA on time as expressed in the ATM concept could be guaranteed by combining the relaxation of the CTOT and the total flight duration see Figure 3. The relevant points are identified according to the potential concurrence events that are computed in the detection of tight trajectory interdependencies and the TTO stamp of these points is calculated by the conjunction of both, the CTOT and the duration of the segments that separate these concurrence events. This approach leads to the resolution of tight interdependencies maximizing the adherence to the RBT.

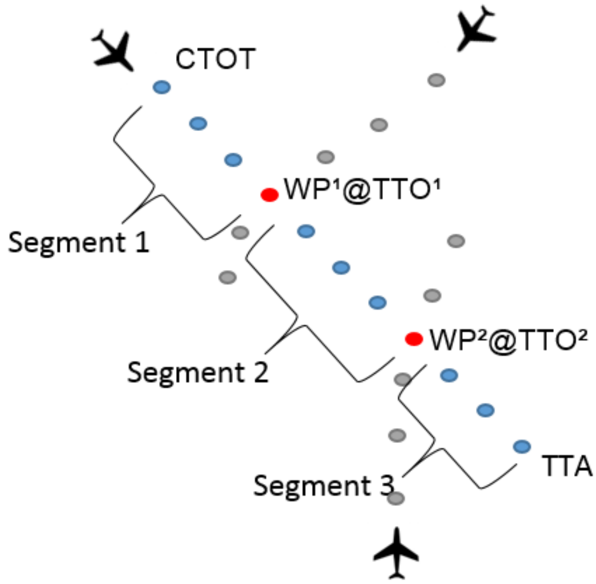

Figure 3: Dynamics of CTOT, segments, points with a TTO stamp and TTA

The depicted problem belongs to the class of Constraint Satisfaction Problems (CSP). During the mapping process, a list of proximate events is detected: two or more aircrafts losing the separation minima. This situation can be represented in terms of constraints able to describe the conditions to be met in order to avoid such proximate event. For removing these interdependencies, the approach proposes two action mechanisms, that is, two kind of decision variables for adjusting the relevant time stamps in a way that the constraints modeling the interdependencies are satisfied.

Moreover, this problem can be considered as an optimization problem, since maximizing the adherence to the RBT will be included as a goal. Since the decision variables belong to the integer domain, the overall problem falls in the category of combinatorial problems.

The paper is organized as follows: Section 2 explains the methodology description to retrieve the input for the CP Model, section 3.1 describes the constraint model that has been developed to tackle the problem considering one degree of freedom and section 3.2 outlines the extension of the model introducing speed changes between segments. The validation of results, conclusions and opportunities for further work are discussed in section 4.

2. METHODOLOGY DESCRIPTION

The detection of tight trajectory interdependencies is realized in four constituent processes which will be presented in the following. The output of the detection of tight trajectory interdependencies allows the resolution of these tight trajectory interdependencies using CP.

To identify tight trajectory interdependencies, the entire European Airspace is classified into so called collective microregions. Based on the TBO concept the en-route trajectories are initially projected on a discrete grid by flight level covering the European Airspace (longitude -20 and 30 degrees and latitude of 0 to 80 degrees). The trajectories and relevant flight information must be supported by computational efficient algorithms and databases.

2.1. Macro-mapping process

One objective when developing the search algorithm to detect tight trajectory interdependencies is to solve the scalability problem and to design a computational efficient algorithm. Therefore, the airspace is first divided into macrocells with a size of 12NM (22,224 km). The position tracking is stored as a vector. Each position in the vector can assume a binary value of 0 or 1. Presence in a cell is represented by 1 and absence by 0 (see Figure 4). The entry and exit times of an aircraft into a cell are registered and stored in a vector.

2.2. Micro-mapping process

After the initial mapping, the macrocells with an occupancy rate equal or greater than two are partitioned for the identification of collective microregions, that is the set of cells showing potential concurrent events. The microcells represent square cells of 6NM that are in use by at least two aircraft simultaneously (Barnier and Allignol 2012). The size 6NM (11.112 km) has been chosen with respect to the safety distance two aircrafts always have to respect. For collective microregions, entry times and exit times are used to determine the size of the overlap or clearance between aircraft pairs. As it can be seen in Figure 4, the process is identical to the previous presented macro-mapping process considering smaller cells. To improve the reliability of the collective microregion identification, four areas located on the boundaries of surrounding cells, macro- and micro-mapping processes are applied in order to detect any concurrence event between trajectories neighbor cells.

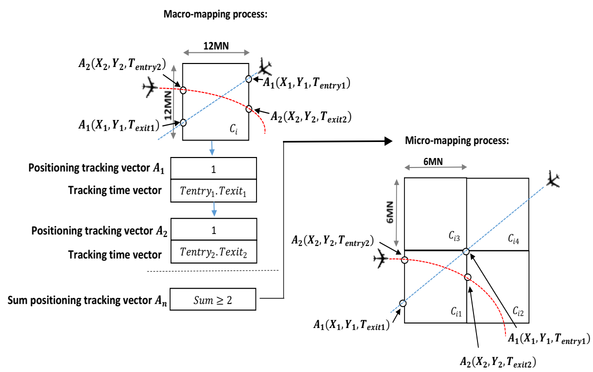

Figure 4: Macro- and micro mapping process

2.3. Filtering process

Finally, the detected concurrence events are filtered for each pair of aircraft. The outcome after the filter are ''tightest'' potential concurrence events for each pair of aircraft (see Figure 5), since aircraft that have enough clearance to guarantee the safety minimum do not have to be considered in the resolution of tight trajectories that will be explained in the following section.

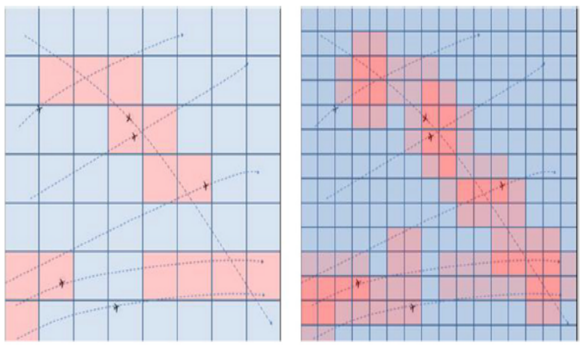

Figure 5: Detection of "collective microregions"

3. CONSTRAINT MODEL

CP is a powerful paradigm for representing and solving a wide range of combinatorial problems. In the last few decades it has attracted much attention among researchers due to its flexibility and its potential for solving hard combinatorial problems in areas such as scheduling, planning, timetabling and routing. CP combines strong theoretical foundations (e.g. techniques originated in different areas such as Mathematics, Artificial Intelligence, and Operations Research) with a wide range of application in the areas of modelling heterogeneous optimization and satisfaction problems. Moreover, the nature of CP provides other important advantages such as fast program development, economic program maintenance and efficient runtime performance. Problems are expressed in terms of three entities: variables, their corresponding domains, and constraints relating them.

The presented approach recognizes the synchronization problem as a scheduling problem, similar to some extend to the well-known Job Shop Scheduling Problem (JSSP). Roughly, this problem consists in allocating the proper resources to the list of jobs facing an optimization goal to minimize some temporal, productivity or efficiency cost function.

Drawing lines to the JSSP, the available cells as portions of the airspace can be considered as the existing resource and the aircraft as the jobs that are performed requiring the resource.

3.1. Tight trajectory interdependencies resolution

In this CP model version, the tight trajectory resolution is modeled using one control action: shifting the entire trajectory by the delay applied on the CTOT as is can be seen in Figure 6. The CTOT of aircraft 2 is shifted ahead of its original schedule and the CTOT of aircraft 3 is delayed in order to guarantee that all three aircraft arrive to the cell in conflict at different time windows.

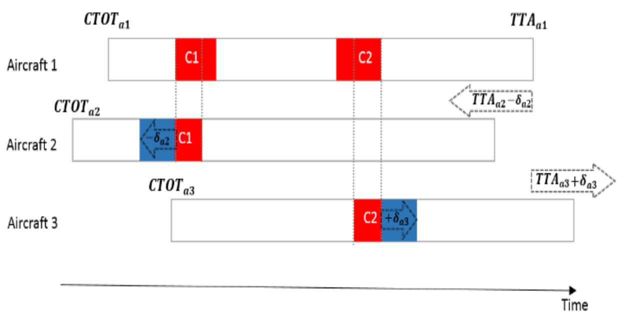

Figure 6: Resolution of tight trajectory interdependencies with one freedom degree (C=Conflict)

After the mapping and filtering process we obtain a representation of all the conflicts that must be removed by the optimization model. This information is then processed in order to define the following data structures.

Let A be the set of aircrafts, C the set of cells belonging to one collective microregion and $c_a =< c, a >$ the pairing between the aircraft a using a given cell c at the microregions. The pairings $c_a \in C_A$ is defined as:

$$C_A = \{< c, a > \mid \forall c \in C, \forall a \in A\}$$

Finally, the time occupancy of the cell c by aircraft a is defined by the two parameters:

$$c_a^{te} \equiv entry\ time \qquad (1)$$
$$c_a^{ts} \equiv exit\ time$$

3.1.1. Decision variables

To ensure that the departure adjustment of the aircraft remain in the defined timeframe of [-5,10] minutes, the integer decision variable δ_a is defined as the delay applied to the CTOT of aircraft a:

$$\delta_a \in [-\delta_{min}, \delta_{max}],$$

where $\delta_{min} = 5$ and $\delta_{max} = 10$, expressed in minutes, sets the domain for the delay decision variable.

The use of a cell by an aircraft is modeled by means of interval decision variables. Interval decision variables represent time periods whose duration and position in time are unknown in the optimization problem. The interval is characterized by a start value, an end value and a size. Addressing this concept as a scheduling problem, the interval is the time during which something happens (e.g. an activity is carried out). In this case, it is the occupancy of the cell c by an aircraft a is modeled by the interval decision variable:

$$P_{c_a} = [s_{c_a}, e_{c_a}), \qquad \forall\ c_a \in C_A \qquad (2)$$

and the size:

$$sz(P_{c_a}) = e_{c_a} - s_{c_a} (= c_a^{ts} - c_a^{te})$$

where s_{c_a} and e_{c_a} are the interval start and end time respectively.

Since the shifting applied to the trajectory to avoid the proximate events is determined by the delay δ_a and no speed adjustment are accepted, the domain of the interval variable can be defined as (see also Equation 1):

$$P_{c_a} \in [c_a^{te} - \delta_{min}, c_a^{ts} + \delta_{max}], \forall c_a \in C_a \qquad (3)$$

As illustrated in Figure 6, the time occupancy of the cell that is involved in a concurrent event remains constant. The aircraft takeoff time instants are shifted according to the delay δ that is applied to avoid the concurrent event in the cell.

Each of the cells can be occupied by one aircraft at a time, so the aircrafts going through the cell must be sequenced accordingly. The decisions on the use of conflicting cells are modeled by sequence variables, which are defined as:

$$F_c = \{P_{c_a} | c_a \in C_A\}, \qquad \forall c \in C \qquad (4)$$

with the permutation π of the sequence variable F_c as the function

$$\pi: F_c \rightarrow [1, m]$$

where $m = |F_c|$ is the number of aircrafts going through the cell c. The elements of the sequence meet the following conditions:

$$P_{c_{ai}} \neq P_{c_{aj}} \Rightarrow \pi(P_{c_{ai}}) = \pi\left(P_{c_{aj}}\right), \forall\ P_{c_{ai}}, P_{c_{aj}} \in F_c$$

3.1.2. Constraints

Two constraints are identified in order to define the space of feasible solutions. The first constraint aims to model the shifting of every interval variable according to the applied delay:

$$s(P_{c_a}) = c_a^{te} + \delta_a, \forall c_a \in C_A \qquad (5)$$

where the function $s(\cdot)$ is defined as the interval start time (aircraft entry to cell c):

$$s(P_{c_a}) = s_{c_a} \qquad (6)$$

The second constraint is the *no overlap* constraint that imposes a set of interval variables to not overlap each other in time. In this case, all aircraft in a cell c with proximate events should have no overlap:

$$\forall P_{c_i}, P_{c_j} \in F_c$$
$$NO(F_c) \Leftrightarrow \pi(P_{c_i}) < \pi(P_{c_j}) \Rightarrow e(P_{c_i}) \leq s(P_{c_j}) \qquad (7)$$

where the function $e(\cdot)$ are defined as the interval end time (aircraft exit from cell c):

$$e\left(P_{c_a}\right) = e_{c_a} \qquad (8)$$

and the no overlap is guaranteed for the proximate event P_{c_i} at a position prior to any P_{c_j} by constraining its exit time to be lower or equal to the entry time of the subsequent proximate events P_{c_j}.

3.1.3. Optimization goal

The objective function was chosen to enhance adherence with a synchronization mechanism, though flexible, does not preserve the TTA at destination airport. Therefore, it aims to minimize the differences between actual takeoff times and the planned or CTOTs.

The optimization goal of the solution is to minimize the total aircraft delays, and it is formulated as follows:

$$\sum_{a=1}^{n} |\delta_a| \qquad (9)$$

where a refers to the aircraft and δ_a is the delay applied.

The whole optimization model is listed here:

A set of aircrafts
C set of cells at a collective microregion
$C_A = \{< c, a > \mid \forall c \in C, \forall a \in A\}$
d.v. $\delta_a \in [-\delta_{min}, \delta_{max}], \forall a \in A$
d.v. $P_{c_a} \in [c_a^{te} - \delta_{min}, c_a^{ts} + \delta_{max}], \forall c_a \in C_a$
d.v. $F_c = \{P_{c_a} | c_a \in C_A\},\ \forall c \in C$

minimize $\sum_{a=1}^{n} |\delta_a|$
subject to {
$s\left(P_{c_a}\right) = c_a^{te} + \delta_a, \forall c_a \in C_A$
$\forall P_{c_i}, P_{c_j} \in F_c$
$NO(F_c) \Leftrightarrow \pi(P_{c_i}) < \pi(P_{c_j}) \Rightarrow e(P_{c_i}) \leq s(P_{c_j})$
}

This model was applied to successfully solve an over-stressed realistic scenario. The scenario was composed of a set of 4010 real 4D trajectories in the European airspace for a time window of 2 h, showing more than 65.000 proximate events. Nevertheless, the modified trajectories do not meet the TTA, since no speed adjustment possibility is included in this model. Next section extends the model in order to improve the RBT adherence of the modified trajectories.

3.2. Tight trajectory interdependencies resolution with speed adjustments

TTA adherence is a main objective to enhance capacity at arrival airports. Clearly, the TTA cannot be preserved by shifting the CTOT and therefore, the full trajectory. The TTA in ATM has a small margin of [-1,1] minute. Therefore, its compliance is of high importance. To meet these conditions, the model described in section 3.1 has been extended by introducing the concept of segments for describing the full trajectory from departure (CTOT) until the arrival time to the destination (TTA). The Figure 7 illustrates this concept. For instance, aircraft 1 in the figure is divided into five segments: C1 and C2 represent the concurrence events while S1, S2 and S3 are the segments between the concurrence events. In the modified trajectory, the segment S1' is shifted according to the applied delay on the CTOT to avoid the first concurrence event while S3' is shortened in time by speed change in order to preserve the TTA within the margin. The intermediate segment S2' is extended in time by flying with reduced speed to avoid concurrence event C2.

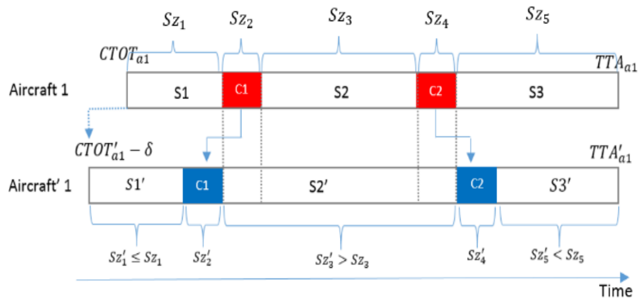

Figure 7: Resolution of tight trajectory interdependencies with speed change (C=Conflict; S=Segment; Sz=Size)

The speed adjustments are realized under the condition that the segment between proximate events are of a certain minimum duration. That allows to introduce a speed change that is efficient in the sense of fuel consumption and in the effect on the resolution of the conflict while trying to preserve the TTA.

New data structures are included to model the trajectory segments for speed adjustments. Let $\hat{g}_i^a$ be a segment of the aircraft a trajectory. Therefore, the RBT can be noted as:

$$RBT_a = \{\hat{g}_i^a\}, \qquad i = 1..p(a)$$

where $p(\cdot)$ is the number of segments required for describing the trajectory. For instance, the Figure 7 shows the trajectory segments of aircraft 1, represented as $RBT_a = \{S1, C1, S2, C2, S3\}$ with

$$s(\hat{g}_i^a) = \text{start time of } \hat{g}_i^a,$$
$$e(\hat{g}_i^a) = end\ time\ of\ \hat{g}_i^a$$

where the functions $s(\cdot)$ and $e(\cdot)$ yield the start and end times of the corresponding RBT segments (see Equation 6 and 8 for the function definition).

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Finally, the concept of segment elasticity $l(\hat{g}_i^a)$ is introduced to denote the allowed speed variation as a percentage of the $\hat{g}_i^a$ segment duration $sz(\hat{g}_i^a)$.

3.2.1. Additional decision variables

In this new CP model approach, the duration of the entire flight becomes an unknown itself, since CTOT can be delayed while keeping the intend to preserve the TTA.

A decision interval variable G_a is introduced for representing the entire flight:

$$G_a = [s_a, e_a)$$

where s_a will be the takeoff time and e_a the arrival time in the solution.

Secondly, the interval variables representing the segments of the G_a solution trajectory are modeled. Let g_i^a be the interval variable:

$$g_i^a = [s(g_i^a), e(g_i^a))$$

and the size of the g_i^asegment is

$$sz(g_i^a) = e(g_i^a) - s(g_i^a)$$

The domain of the g_i^a segment can be defined as:

$$sz(g_i^a) \in [sz(\hat{g}_i^a) - l(\hat{g}_i^a), sz(\hat{g}_i^a) + l(\hat{g}_i^a)] \qquad (10)$$

Note that in this model version, interval duration can differ from RBT segment duration, since some elasticity is enabled by the bounded speed changes, whereas the domain for the interval start and end time cannot be specified, since their values at the solution are a combination of the takeoff delay and the bounded speed adjustments.

Finally, a sequence variable T_a is introduced to set the relationship between the trajectory segments g_i^a and the entire trajectory G_a:

$$T_a = \{g_i^a \mid \forall a \in A, i \in 1..p(a)\}$$
$$\pi: T_a \to [1, n] \qquad (11)$$
$$g_i^a \neq g_j^a \Rightarrow \pi(g_i^a) \neq \pi(g_j^a), \forall g_i^a, g_j^a \in T_a$$

3.2.2. Additional Constraints for speed change

The duration of the flight is determined by the constraint of the takeoff time and the time to arrival.

$$s(G_a) = CTOT_a \pm \delta_a \qquad (12)$$

$$e(G_a) \in [TTA_a - 1, TTA_a + 1] \qquad (13)$$

The relationship between the flight interval variable and its segments is modeled by the following *span* condition:

$$span(G_a, \{g_i^a\}), \forall a \in A, \forall g_i^a \in T_a$$

This constraint sets the following time relationship among the interval variables:

$$\begin{cases} s(G_a) = \min\limits_{i \in [1,p(a)]}(\{s(g_i^a)\}) \\ e(G_a) = \max\limits_{i \in [1,p(a)]}(\{e(g_i^a)\}) \end{cases} \qquad (14)$$

The constraint *span* states that the interval flight spans over all present intervals from the set segments. That is, interval flight G_a starts together with the first present segment interval and ends together with the last one.

Additionally, the following three constraints are set to order the trajectory segments:

1. The *no overlap* constraint to ensure that interval variables to not overlap each other.

$$NO(G_a) \Leftrightarrow \pi(g_i^a) < \pi(g_j^a) \Rightarrow e(g_i^a) \leq s(g_j^a) \qquad (15)$$

2. The constraint that one segment has to start before the next:

$$e(g_i^a) \leq s(g_j^a), \forall i, j: i \leq j \qquad (16)$$

3. The constraint that ensure that the start of segment j results after the end of segment i.

$$e(g_i^a) = s(g_j^a), \forall i, j: j = i + 1 \qquad (17)$$

The graphical representation of this three constraints is shown in Figure 8. Aircraft 1 has a flight duration and the projection of the segments onto the flight duration with the three conditions is shown.

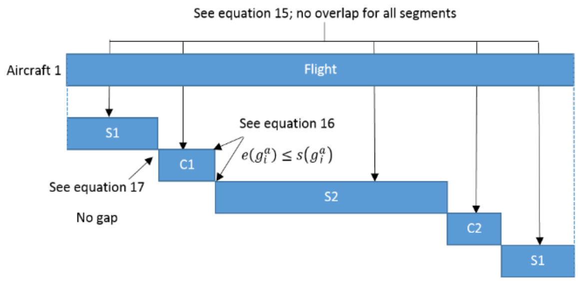

Figure 8: Representation of function flight and RBT segments

Finally, the P_{c_a} interval variable, that is used in combination with the sequence variable F_c to remove the concurrence events at cell c, must be linked with the concurrence segments of the trajectory T_a (e.g. C1 and C2 in Figure 8), since they are representing the same time windows. This is accomplished by the following constraint:

$$\begin{cases} s(g_i^a) = s(P_{c_a}) \\ e(g_i^a) = e(P_{c_a}) \end{cases} \Leftrightarrow \begin{matrix} s(\hat{g}_i^a) = c_a^{te} \\ e(\hat{g}_i^a) = c_a^{ts} \end{matrix}, \forall c_a \in c_A$$

3.2.3. Objective function

The constraint in Equation 9 binds to the TTA attainment, but it might happen that no solution is found because time adjustment is bounded so it is possible that the required delays δ_a cannot be compensated by the speed adjustments. For this reason, the TTA constraint is relaxed. The following logical function is added:

$$L(G_a) = \begin{cases} 1, & e(G_a) \notin [TTA_a - 1, TTA_a + 1] \\ 0, & \text{otherwise} \end{cases}$$

With this function, the number of TTA violations can be counted for introducing its minimization as an objective

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that can be combined with the objective function stated in Eq. 4 to minimize the total delay of the aircraft takeoffs. The following equation weights both objectives to get the optimization goal:

$$\min w_1 \sum_{a=1}^{n} |\delta_a| + w_2 \sum_{a=1}^{n} L(G_a) \tag{18}$$

The extended optimization model is listed here:

A set of aircrafts
C set of cells at a collective microregion
$C_A = \{< c, a > \mid \forall c \in C, \forall a \in A\}$
$RBT_a = \{\hat{g}_i^a \mid \forall a \in A,\ i = 1..p(a)\}$
d.v. $\delta_a \in [-\delta_{min}, \delta_{max}], \forall a \in A$
d.v. $P_{c_a} \in [c_a^{te} - \delta_{min}, c_a^{ts} + \delta_{max}], \forall c_a \in C_a$
d.v. $F_c = \{P_{c_a} | c_a \in C_A\},\ \forall c \in C$
d.v. $G_a,\ \forall a \in A$
d.v. $g_i^a,\ \forall a \in A,\ \forall i \in 1..p(a):$
$\quad sz(g_i^a) \in [sz(\hat{g}_i^a) - l(\hat{g}_i^a), sz(\hat{g}_i^a) + l(\hat{g}_i^a)]$
d.v. $T_a = \{g_i^a\}, \forall a \in A$

minimize
$w_1 \sum_{a=1}^{n} |\delta_a| + w_2 \sum_{a=1}^{n} L(G_a)$
subject to {
$s(g_i^a) = CTOT_a \pm \delta_a\ \forall a \in A$
$\forall P_{c_i}, P_{c_j} \in F_c$
$\quad NO(F_c) \Leftrightarrow \pi(P_{c_i}) < \pi(P_{c_j}) \Rightarrow e(P_{c_i}) \le s(P_{c_j})$
$span(G_a, \{g_i^a\}), \forall a \in A, \forall g_i^a \in T_a$
$\forall a \in A,\ \ \forall i, j \in 1..p(a)$
$\quad NO(G_a) \Leftrightarrow \pi(g_i^a) < \pi(g_j^a) \Rightarrow e(g_i^a) \le s(g_j^a)$
$\quad e(g_i^a) \le s(g_j^a) :\ i \le j$
$\quad e(g_i^a) = s(g_j^a) : j = i + 1$
$\quad \begin{cases} s(g_i^a) = s(P_{c_a}) \\ e(g_i^a) = e(P_{c_a}) \end{cases} \Leftrightarrow \begin{matrix} s(\hat{g}_i^a) = c_a^{te} \\ e(\hat{g}_i^a) = c_a^{ts} \end{matrix}, \forall c_a \in c_A$
}

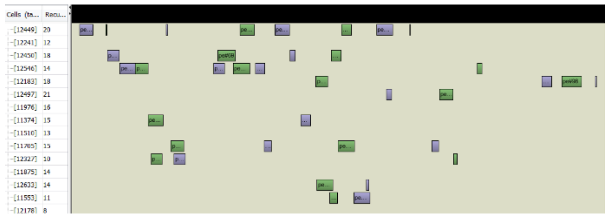
4. RESULTS

The model was applied to an over-stressed realistic scenario. The scenario was composed of a set of 4010 real 4D trajectories in the European airspace for a time window of 2 h. In this work, we assumed TBO without uncertainties. In this context, the trajectories were discretized at each second, and each position was specified in terms of geographic coordinates and a time stamp. This scenario was designed and analyzed in the STREAM project (Ranieri et al. 2011), a EUROCONTROL SESAR WP-E project. The CP model has been implemented with the ILOG Optimization Suite (IBM 2015) and the following results were obtained.

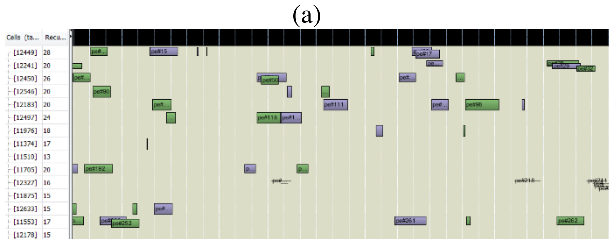
4.1. Macro and Micro Mapping

The detection of the concurrence events in this paper is based on the algorithms and results presented at (Nosedal et al. 2014) and (Nosedal et al. 2015). The aforementioned scenario is analyzed in these works, leading to the detection of the collective micro-regions that have been used in this work to find the optimal adjustments on CTOT and speed changes to reduce proximate events and, therefore, ATC interventions.

In Figure 9 (a) the en-route traffic thought the collective micro-regions is shown. The cells with potential concurrence events are detected based on the RBT trajectories of those aircrafts ready to depart, but still on ground, according to their CTOT. Therefore, en-route trajectories are conflict free at the given time instant.



(a)



(b)

Figure 9: Gantt diagrams showing the traffic through the cells with potential concurrence events. Diagram (a) shows the conflict free en-route traffic and (b) shows the emerging conflict after inserting the departing traffic for the same time period.

The Figure 9 (b) shows the situation found when the grounded aircrafts depart according to their RBT CTOT. As it can be seen, for instance, at cells 12241, 12449 and 12450 among others, concurrent events will appear between several aircraft if they depart according to their CTOT. In this case, aircraft regulations could be issued by ATM or, later on, ATC interventions would be needed to remove the proximate events caused by the inserted traffic.

4.2. Trajectory adjustments

The proposed CP model is used to determine the proper adjustments on the CTOT and aircraft trajectories to remove the potential concurrence events.

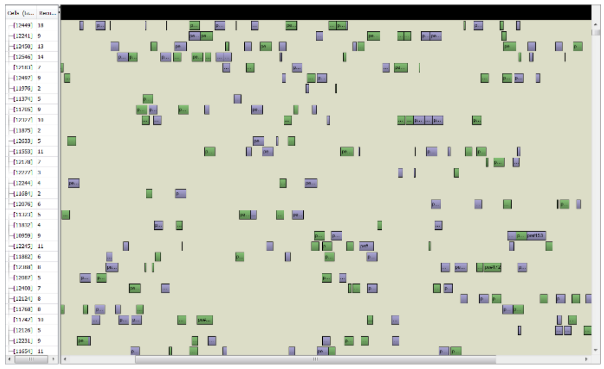
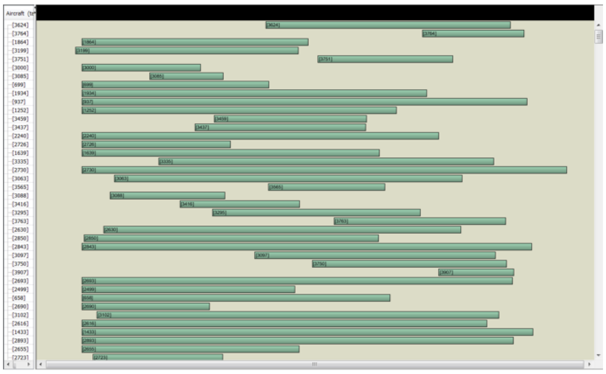
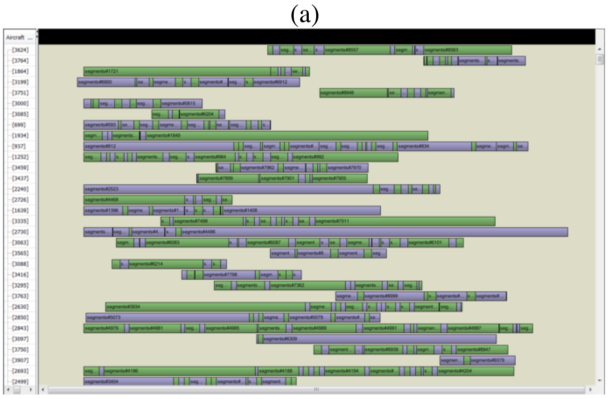


Figure 10: The diagram shows the conflict free solution after applying small adjustments on CTOT and segment' speed.



(a)



(b)

Figure 11: The (a) Gantt diagram shows the full flight interval of the aircrafts in potential concurrence events; the (b) diagram shows their segment structure.

As Figure 10 illustrates, all the potential concurrence events are removed by applying a combination of bounded delays on CTOT and/or speed adjustments, leading to a conflict free scenario. The bounded adjustments impose the actual takeoff time to be within the $[-5,10]$ minutes of the aircraft CTOT (see Equation 12) and the speed adjustments to be less than 10% of the RBT proposed by the airline (see Equation 10). The ILOG CP solver was limited to 180 seconds to get the best suboptimal solution. All the experiments were performed on a Window 10 computer with an Intel Core I7 CPU 2,30 GHz and 16GB RAM.

4.3. Solution analysis

Since the adjustments on CTOT and speed changes are bounded, the TTA fulfilment cannot be ensured. As stated at Equation 18, the TTA requirement was relaxed, and its fulfilment was included in the optimization goal. The used weights were $w_1 = 10\%$ and $w_2 = 90\%$, so giving priority to the TTA preservation.

The Figure 12 shows the correlation between the actual time of arrival (ATA) compared to the TTA with respect to the applied CTOT delay. As it can be observed, in most of the cases the bounded speed adjustments are not enough to recover the effect of the applied delays. In Figure 13 it is shown the absolute numbers of aircrafts not able to meet their TTA with respect to the applied delay. There are two main reasons explaining this results.

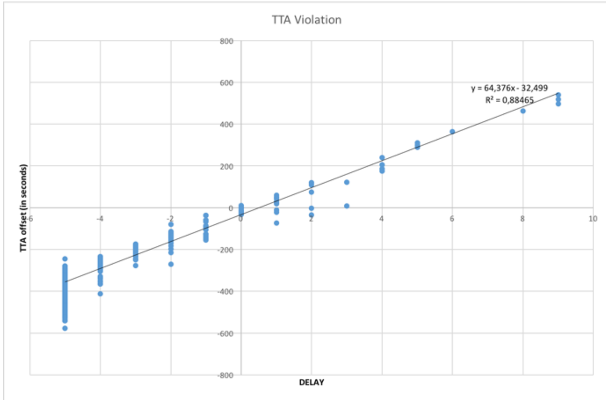


Figure 12: Correlation between TTA violation and the delays applied to the aircraft takeoff times.

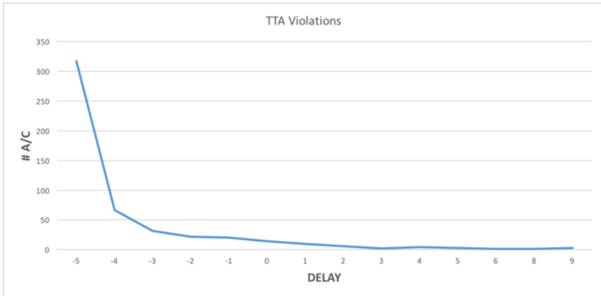


Figure 13: A/C not meeting their TTA with respect to the applied CTOT delay.

The first observable one is that most of the aircrafts are moved ahead of their CTOT. This is a consequence of the solver search strategy (Van Beek 2006), since time to get the suboptimal solution was limited to 180 seconds. This strategy is the default one and first takes the smallest values in the decision variable domains. In this case, this value is -5 minutes for the δ_a delay. Further research is required to define search strategies leading to better solutions.

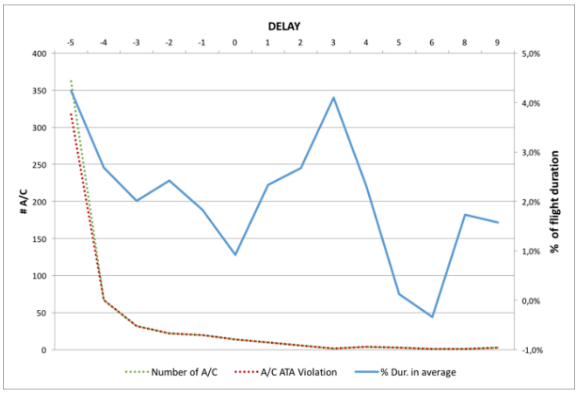


Figure 14: blue line represents the average modification of flight duration with respect to the applied delay; dotted lines represent the number of A/C with the respective delay and the number of A/C not meeting their TTA.

The second reason can be explained from the curves at Figure 14. The number of aircrafts not meeting their TTA is tightly related to the applied delay, as it can be observed from dotted curves. However, the average modification on flight duration is not related to the applied delay. The margins enabled by the bounded speed adjustments are not enough for compensating the applied delays. This fact could be overcome only if, first, solutions with lower absolute delays can be found (better search strategy) and, second, if the aircraft trajectory allows a bigger absolute elasticity. The latest does not depend on the solution method, but on the duration of the flight and on the number and relative position of the proximate events where it is involved.

5. CONCLUSIONS

In this work a CP model is presented for solving the concurrence events that might happen when the departure traffic is inserted into the en-route traffic. The model has been proved in a realistic and overstressed scenario and it has been able to find suboptimal solutions in a timeframe of 180 seconds for all the performed experiments.

The model constraints ensure that all the proximate events are resolved by introducing small time adjustment both on the CTOT and relevant TTO's while maximizing the adherence to the RBT's. Although the model is not able to ensure that the ATM concept of preserving the TTA in a strict time frame is met, the CP solver can find solutions that remove all the conflicts reducing the number of potential ATC interventions.

The concept of preserving the TTA has been relaxed and the objective function penalizes the TTA violation. The reason for this is the limit of the trajectory elasticity, since speed adjustments are bounded to a percentage of the total RBT duration.

Furthermore, the quality of the solution found so far is directly linked to the solver search strategy. In this work, default parameters for searching have been used, leading to a solution where the smallest domain values at the delay variable are tested first. The search starts with -5 minutes of adjustment on the CTOT and, due to time restriction for finding a solution, possible better solutions cannot be explored by the solver. In consequence, the obtained total delay requires extra effort for recovering the TTA and, since the trajectory elasticity is limited, no acceptable speed change can be found to meet the TTA. Further research is required to define search strategies favoring the selection of adjustments close to zero in first term. This way speed adjustment efforts are expected to be smaller.

6. ACRONYMS

ATC	Air Traffic Control
ATFM	Air Traffic Flow Management
ATM	Air Traffic Management
AU	Airspace User
CP	Constraint Programming
CSP	Constraint Satisfaction Problems
CTOT	Calculated-Take-Off Time
DST	Decision Support Tool
JSSP	Job Shop Scheduling Problem
TBO	Trajectory Based Operation
TTA	Target Time of Arrival
TTO	Time-To-Overfly
RBT	Reference Business Trajectory
4DT	4-dimensional trajectories

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- Production technologies//Logistics and Intelligent Transport Systems: modelling and simulation methodologies for production and logistics systems
- Information and Communication Technology: modelling language development for dynamical systems
- Information and Communication Technology: distributed systems and real time systems
- Industrial collaboration and Technology transfer: real-time decision making tools for logistics and transportation problems.

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