A DYNAMIC MODEL AND AN ALGORITHM FOR SUPPLY CHAIN SCHEDULING PROBLEM SOLVING

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ABSTRACT
We present a new model and algorithm for optimal scheduling of a supply chain (SC) with multiple customers and suppliers. The formulation assumes that suppliers can dynamically allocate jobs and schedule their resources in a coordinated manner so that all the suppliers are equally utilized and jobs are accomplished without interruptions and scheduled subject to maximal customer service level and minimal delays. This problem is represented as a special case of the job shop scheduling problem with dynamically distributed jobs. The approach is based on a natural dynamic decomposition of the problem and its solution with the help of a modified form of continuous maximum principle blended with combinatorial optimization.

Keywords: supply chain, scheduling, dynamic model, optimal program control, continuous maximum principle, combinatorial optimization.

1. INTRODUCTION

We present the scheduling model and algorithm where an SC (Ivanov, Sokolov, 2010) is a networked controlled system that is described through differential equations based on a dynamic interpretation of the job execution. The studies by Holt, Modigliani, Muth, and Simon (1960), Hwang, Fan, and Erikson (1967), Zimin and Ivanilov (1971) and Moiseev (1974) were among the first to apply the optimal program control (OPC) and the maximum principle to multi-level and multi-period master production scheduling that determined the production as an optimal control with a corresponding trajectory of the state variables (i.e., the inventory). This stream was continued by Kimemia and Gershwin (1983), who applied a hierarchical method in designing a solution procedure to the overall model, and by Khmelnitsky, Kogan, and Maimom (1997) for planning continuous-time flows in flexible manufacturing systems.

The study (Sarimveis, Patrinos, Tarantilis & Kiranoudis, 2008) showed a wide range of advantages regarding the application of OPC to production and logistics. They include, first of all, a non-stationary process view and accuracy of continuous time. In addition, a wide range of analysis tools from control theory regarding stability, controllability, adaptability, etc. may be used if a schedule is described in terms of control. Recent studies (e.g., Subramanian, Rawlings, Maravelias, Flores-Cerrillo, & Megan 2013) discussed the possibilities to translate the MP scheduling models into a state-space form and the design of rescheduling algorithms with the desired closed-loop properties.

However, although the OPC was widely applied to flexible manufacturing system scheduling, it cannot be directly applied to the flow or job shop scheduling level as a computational procedure. The continuous time models are not applicable in their direct form to discrete assignment problems due to the continuous values of the control variables from 0 to 1. In addition, such problems as numerical instability, non-existence of gradients, and non-convexity of state space should be mentioned. The calculation of the OPC with direct methods of the continuous maximum principle has also not been proved efficient. It can be concluded that the application of OPC to scheduling is not a trivial problem for two reasons. First, a conceptual problem consists of the continuous values of the control variables. Second, a computational problem with a direct method of the maximum principle exists. In this paper we present a new model and algorithm for optimal scheduling of a SC with multiple customers and suppliers. In this case the job execution is characterized by (1) execution results (e.g., volume, time, etc.), (2) capacity consumption of the resources, and (3) supply flows resulting from the delivery to the customer. We propose to use a two-stage scheduling procedure in line with Chen and Pundoor (2006). A job control model (M_1) is first used to assign jobs to suppliers, and then a flow control model (M_2) is used to schedule the processing of assigned orders subject to capacity restrictions of the production and transportation resources. The basic interaction of these two models is that after the solving the job control model, the found control variables are used in the constraints of the flow control model. In additional models of resource and channel control, the material supply to resources and its consumption as well as setup times are represented.

2. PROBLEM STATEMENT AND MODEL

2.1. Dynamic Model of Job Control (model M_1)

We consider the mathematical model of job control. We denote the job state variable $x_{i\mu}^{(o)}$, where (o) — indicates the relation to jobs (orders). The execution dynamics of the job $D_\mu^{(i)}$ can be expressed as (1).

$$\frac{dx_{i\mu}^{(o)}}{dt} = \dot{x}_{i\mu}^{(o)} = \sum_{j=1}^{n} \varepsilon_{ij}(t) u_{i\mu j}^{(o)} \tag{1}$$

where $\varepsilon_{ij}(t)$ isan element of the preset matrix time function of time-spatial constraints, $u_{i\mu j}^{(o)}(t)$ is a 0–1 assignment control variable.

Remark 1. The economic sense of (1) consists of the job dynamics representation in which process non-stationary and time consumption are reflected.

Let us introduce equation (2) to assess the total resource availability time:

$$\dot{x}_j^{(o)} = \sum_{i=1}^{\bar{n}} \sum_{\substack{\eta=1 \\ \eta \neq i}}^{\bar{n}} \sum_{\mu=1}^{s_i} \sum_{\rho=1}^{p_i} (u_{i\mu j}^{(o)}) \tag{2}$$

Equation (2) represents resource utilization in job execution dynamics. The variable $x_j^{(o)}$ characterizes the total employment time of the *j*-supplier. The control actions are constrained as follows:

$$\sum_{i=1}^{\bar{n}} \sum_{\mu=1}^{s_i} u_{i\mu j}^{(o)}(t) \leq 1, \forall j; \ \sum_{j=1}^{n} u_{i\mu j}^{(o)}(t) \leq 1, \forall i, \forall \mu \tag{3}$$

$$\sum_{j=1}^{n} u_{i\mu j}^{(o)} [\sum_{\alpha \in \Gamma_{i\mu_1}^{-}} (a_{i\alpha}^{(o)} - x_{i\alpha}^{(o)}) + \prod_{\beta \in \Gamma_{i\mu_2}^{-}} (a_{i\beta}^{(o)} - x_{i\beta}^{(o)})] = 0 \tag{4}$$

$$u_{i\mu j}^{(o)}(t) \in \{0,1\}; \tag{5}$$

where $\Gamma_{i\mu_1}^{-}$, $\Gamma_{i\mu_2}^{-}$ are the sets of job numbers which immediately precede the job $D_\mu^{(i)}$ subject to accomplishing of all the predecessor jobs or at least one of the jobs correspondingly, and $a_{i\alpha}^{(o)}, a_{i\beta}^{(o)}$ are the planned lot-sizes. Constraint (3) refers to the allocation problem constraint according to the problem statement (i.e., only a single order can be processed at any time by the manufacturer). Constraint (4) determines the precedence relations; more over, this constraint implies the blocking of operation $D_\mu^{(i)}$ until the previous operations $D_\alpha^{(i)}, D_\beta^{(i)}$ have been executed. If $u_{i\mu j}^{(o)}(t) = 1$, all the predecessor jobs of the operation $D_\mu^{(i)}$ have been executed. Note that these constraints are identical to those in MP models.

Proposition 1. *The constraints (4) ensure that all the scheduled jobs from one customer should be fully fulfilled, i.e. the planned service level can be reached.*

Corollary 1. *The analysis of constraints (4) shows that control* $\mathbf{u}(t)$ *is switching on only when the necessary predecessor operations have been executed.* $\sum_{j=1}^{n} u_{i\mu_j}^{(o)} \sum_{\alpha \in \Gamma_{i\mu_1}^{-}} (a_{i\alpha}^{(o)} - x_{i\alpha}^{(o)}(t)) = 0$ *guarantees the total processing of all the predecessor operations, and* $\sum_{j=1}^{n} u_{i\mu_j}^{(o)} \prod_{\beta \in \Gamma_{i\mu_2}^{-}} (a_{i\beta}^{(o)} - x_{i\beta}^{(o)}) = 0$ *of at least one of the predecessor operations.*

According to equation (5), controls contain the values of the *Boolean variables*. In order to assess the results of job execution, we define the following initial and end conditions at the moments $t = T_0$, $t = T_f$:

$$x_{i\mu}^{(o)}(T_0) = 0; \ x_{i\mu}^{(o)}(T_f) = a_{i\mu}^{(o)}; \tag{6}$$

Conditions (6) reflect the desired end state. The right parts of equations are predetermined at the planning stage subject to the lot-sizes of each job.

According to the problem statement, let us introduce the following performance indicators (7)–(9):

$$J_1^{(o)} = \frac{1}{2} \sum_{i=1}^{\bar{n}} \sum_{\mu=1}^{s_i} [(a_{i\mu}^{(o)} - x_{i\mu}^{(o)}(T_f))^2] \tag{7}$$

$$J_2^{(o)} = \sum_{i=1}^{\bar{n}} \sum_{\mu=1}^{s_i} \sum_{j=1}^{n} \int_{T_0}^{T_f} \alpha_{i\mu j}^{(o)}(\tau) u_{i\mu j}^{(o)}(\tau) d\tau \tag{8}$$

$$J_3^{(o)} = \frac{1}{2} \sum_{j=1}^{n} (T - x_j^{(o)}(T_f))^2 \tag{9}$$

The performance indicator (7) characterizes the accuracy of the end conditions' accomplishment, i.e. the service level of an SC. The goal function (8) refers to the estimation of an job's execution time with regard to the planned supply terms and reflects the delivery reliability, i.e., the accomplishing the delivery to the fixed due dates. The functions $\alpha_{i\mu}^{(o)}(\tau)$, assumed to be known, characterizes the fulfilment of time conditions for different jobs and time points of the penalties increase due to breaking supply terms respectively. The indicator (9)estimates the equal resource utilization in the SC.

2.2. Dynamic Model of Flow Control (model M_2)

We consider the mathematical model of flow control in the form of equation (10):

$$\dot{x}_{i\mu j}^{(f)} = u_{i\mu j}^{(f)}, \ \dot{x}_{ij\eta\rho}^{(f)} = u_{ij\eta\rho}^{(f)} \tag{10}$$

We denote the flow state variable $\dot{x}_{i\mu j}^{(f)}$, where (f) indicates the relation of the variable *x* to flows.

__Remark 2.__The economic sense of the first part of equation (10) consists of the representation of flow consumption of the resource $C^{(j)}$. The second part of (10) describes the delivery to the customer $\bar{B}^{(\eta)}$.

The control actions are constrained by maximal capacities and intensities as follows:

$$\sum_{i=1}^{\bar{n}}\sum_{\mu=1}^{s_i} u_{i\mu j}^{(f)}(t) \le \tilde{\tilde{R}}_{1j}^{(f)},\ \sum_{\rho=1}^{p_i} u_{ij\eta\rho}^{(f)}(t) \le \tilde{\tilde{R}}_{1j\eta}^{(f)}, \tag{11}$$

$$0 \le u_{i\mu j}^{(f)}(t) \le c_{i\mu j}^{(f)} \cdot u_{i\mu j}^{(o)},\ 0 \le u_{ij\eta\rho}^{(f)}(t) \le c_{i\eta\rho j}^{(f)} \cdot u_{i\eta\rho j}^{(o)}, \tag{12}$$

where $\tilde{\tilde{R}}_{1j}^{(f)}$ is the total potential intensity of the resource $C^{(j)}$, $\tilde{\tilde{R}}_{1j\eta}^{(f)}$ is the maximal potential channel intensity to deliver products to the customer $\bar{B}^{(\eta)}$, $c_{i\mu j}^{(f)}$ is the maximal potential capacity of the resource $C^{(j)}$ for the job $D_\mu^{(i)}$, and $c_{i\eta\rho j}^{(f)}$ is the total potential capacity of the channel delivering the product flow $P_{<s_i,\rho>}^{(j,\eta)}$ of the job $D_\mu^{(i)}$ to the customer $\bar{B}^{(\eta)}$.

The end conditions are similar to those in (6) and subject to the units of processing time. The goal functionals of the flow control model are defined in the form of equations (11) and (12):

$$J_1^{(f)} = \frac{1}{2}\sum_{i=1}^{\bar{n}}\sum_{\mu=1}^{s_i}\sum_{j=1}^{n}[(a_{i\mu j}^{(f)} - x_{i\mu}^{(f)}(T_f))^2 + \sum_{\substack{\eta=1\\ \eta\ne i}}^{\bar{n}}\sum_{\rho=1}^{p_i}(a_{ij\eta\rho}^{(f)} - x_{ij\eta\rho}^{(f)}(T_f))^2], \tag{13}$$

$$J_2^{(f)} = \frac{1}{2}\sum_{i=1}^{\bar{n}}\sum_{\mu=1}^{s_i}\sum_{j=1}^{n}\int_{T_0}^{T_f}\beta_{i\mu}^{(f)}(\tau)u_{i\mu j}^{(f)}(\tau)d\tau. \tag{14}$$

The economic meaning of these performance indicators correspond to equations (7) and (8). With the help of the weighting performance indicators, a general performance vector can be denoted as (15):

$$J\left(x(t),u(t)\right) = \left\| J_1^{(o)}, J_2^{(o)}, J_3^{(0)}, J_1^{(f)}, J_2^{(f)} \right\|^T. \tag{15}$$

The partial indicators may be weighted depending on the planning goals and SC strategies. Original methods (Gubarev et al. 1988) have been used to transform the vector $\mathbf{J}$ to a scalar form J_G.

The job shop scheduling problem can be formulated as the following problem of OPC: this is necessary to find an allowable control $\boldsymbol{u}(t)$, $t \in (T_0, T_f]$ that ensures for the model (1)–(2), and (10) meeting the vector constraint functions $\boldsymbol{q}^{(1)}(\boldsymbol{x},\boldsymbol{u}) = 0$, $\boldsymbol{q}^{(2)}(\boldsymbol{x},\boldsymbol{u}) \le 0$ (3)–(5) and (10–11), and guides the dynamic system (i.e., job shop schedule) $\dot{\boldsymbol{x}} = \boldsymbol{\phi}(t,\boldsymbol{x},\boldsymbol{u})$ from the initial state to the specified final state. If there are several allowable controls (schedules), then the best one (optimal) should be selected in order to maximize (minimize) J_G. In terms of optimal program control (OPC), the program control of job execution is also the job shop schedule. We will refer to this problem of OPC as *PS*.

The formulated model is a linear non-stationary finite-dimensional controlled differential system with the convex area of admissible control. Note that the *PS*is a standard OPC problem; see (Lee and Markus 1967). In fact, this model is linear in the state and control variables, and the objective is linear. The transfer of non-linearity to the constraint ensures convexity and allows using interval constraints.

3. COMPUTATIONAL PROCEDURE AND ANALYSIS OF THE ALGORITHM

The computational procedure for the developed model is based on the integration of the main and conjunctive equation systems subject to the maximization of the following Hamiltonian (16)–(18):

$$H(\boldsymbol{x}^*(t), \boldsymbol{u}^*(t), \psi(t)^*) = \max_{\tilde{\boldsymbol{u}}\in\tilde{\tilde{Q}}(\boldsymbol{x})}\sum_{z=1}^{2} H_l(\boldsymbol{x}(t), \boldsymbol{u}(t), \psi(t)) \tag{16}$$

$$H_1 = \sum_{i=1}^{\bar{n}}\sum_{\mu=1}^{s_i}\sum_{j=1}^{n}[\psi_{i\mu}^{(o)} \cdot \varepsilon_{ij} + \psi_j^{(k)} + w_2^{(o)}\alpha_{i\mu j}^{(o)}]u_{i\mu j}^{(o)} \tag{17}$$

$$H_2 = \sum_{i=1}^{\bar{n}}\sum_{\mu=1}^{s_i}\sum_{j=1}^{n}[\psi_{i\mu j}^{(f)} + w_5^{(f)}\beta_{i\mu}^{(f)}]u_{i\mu j}^{(f)} \tag{18}$$

where $\boldsymbol{\psi}(t)$ is the conjunctive vector.

The maximization of the Hamiltonian H_1 for model (1) in combination with the constraints (3)–(5) solves the assignment problem. The maximization of the Hamiltonian H_2 for model (10) in combination with the constraints (11)–(12) solves the LP problem. At each time instant, only those jobs and constraints from the "active scheduling zone" are considered in the models M_o and M_f which meet the requirements (3)–(5), (11), and (12). By a dynamic switching of the constraints (4) from inequalities to equalities, the size of the scheduling problem at each time point is reduced. The Hamiltonians (17) and (18) can be maximized when the constraints (4) satisfy the corresponding variables $u_{i\mu j}^{(o)}$ and $u_{i\mu j}^{(f)}$. In this case, only a part of the constraints (4) is considered for the current assignment problem since, when the control in (4) is switched to zero, then it becomes active in the right-hand part of the equations (12). Therefore, the reduction of the problem dimensionality at each time instant in the calculation process is ensured due to the recurrent operation description.

A methodical challenge in applying the maximum principle is to find the coefficients of the conjunctive system, which change in dynamics. One of the contributions of this paper is that these coefficients can be found analytically (Boltyanskiy, 1973, Ivanov, Sokolov, 2010). The coefficients of the conjunctive system play the role of the dynamical Lagrange multipliers as compared with MP dual formulations.

In accordance with the maximum principle, the following conjugate system can be written:

$$\frac{d\psi_{i\mu}^{(o)}}{dt} = \dot{\psi}_{i\mu}^{(o)} = -\sum_{j=1}^{n}[\psi_{i(\mu+1)}^{(o)}\varepsilon_{ij} + \psi_{j}^{(k)} + \lambda_2^{(o)}\alpha_{i(\mu+1)j}^{(o)}]u_{i(\mu+1)j}^{(o)}, \quad (19)$$

$$\frac{d\psi_{j}^{(k)}}{dt} = \dot{\psi}_{j}^{(k)} = 0, \quad (20)$$

$$\frac{d\psi_{i\mu j}^{(f)}}{dt} = \dot{\psi}_{i\mu j}^{(f)} = 0. \quad (21)$$

The transversality conditions can be formulated in the following way:

$$\psi_{i\mu}^{(o)}(T_f) = \lambda_1^{(o)}(a_{i\mu}^{(o)} - x_{i\mu}^{(o)}(T_f)), \quad (22)$$

$$\psi_{j}^{(k)}(T_f) = \lambda_3^{(k)}(T - x_{j}^{(k)}(T_f)), \quad (23)$$

$$\psi_{i\mu j}^{(f)}(T_f) = \lambda_5^{(f)}(a_{i\mu j}^{(f)} - x_{i\mu j}^{(f)}(T_f)). \quad (24)$$

The basic peculiarity of the boundary problem considered is that the initial conditions for the conjunctive variables $\psi(t_0)$ are not given. At the same time, an OPC should be calculated subject to the end conditions. To obtain the conjunctive system vector, we use the Krylov–Chernousko method of successive approximations (MSA) for an OPC problem with a free right end, which is based on the joint use of a modified successive approximation method (Krylov&Chernousko, 1972). We propose to use a heuristic schedule $\bar{\boldsymbol{u}}(t)$ to obtain the initial conditions for $\psi(t_0)$. Then, the algorithm DYN can be stated as follows:

Step 1. An initial solution $\bar{\boldsymbol{u}}(t), t \in (T_0, T_f]$ (a feasible control, in other words, a feasible schedule) is selected and $r = 0$.

Step 2. As a result of the dynamic model run, $\boldsymbol{x}^{(r)}(t)$ is received. Besides, if $t = T_f$ then the record value $J_G = J_G^{(r)}$ can be calculated. Then, the transversality conditions (22)–(24) are evaluated.

Step 3. The conjugate system (19)–(21) is integrated subject to $\boldsymbol{u}(t) = \bar{\boldsymbol{u}}(t)$ and over the interval from $t = T_f$ to $t = T_0$. For the time $t = T_0$, the first approximation $\boldsymbol{\psi}_i^{(r)}(T_0)$ is obtained as a result. Here, the iteration number $r = 0$ is completed.

Step 4. From the time point $t = T_0$ onwards, the control $\boldsymbol{u}^{(r+1)}(t)$ is determined ($r = 0,1,2,...$ denotes the number of the iteration). In parallel with the maximization of the Hamiltonian, the main system of equations and the conjugate one are integrated. The maximization involves the solution of several MP problems at each time point.

The assignments (i.e., the control variables $u_{i\mu j}^{(o)}$) from the model M$_o$ are used in the flow control M$_f$ (10)–(12) by means of the constraints (12). At the same time, the model M$_f$ influences the model M$_o$ through the transversality conditions (22)–(24), the conjunctive system (19)–(21), and the Hamiltonian function (16).

4. EXPERIMENTAL ENVIRONMENT

Continuous optimization is a challenging calculation task. Thus any sensible judgments on the models and algorithms can be made only by application of special tools. For the experiments, we elaborated the model in a software package. Because of the limited size of this paper, we cannot describe this package in-depth here, but will sum up the main experiment design features. The software has three modes of operation with regard to scheduling and an additional mode to analyse stability of the schedules. This mode is beyond the scope of this paper.

The first mode includes the interactive generation/preparation of the input data. The second mode lies in the evaluation of heuristic and optimal SC schedules. The following operations can be executed in an interactive regime:

- multi-criteria rating, analysis, and the selection of SC plans and schedules;
- the evaluation of the influence that is exerted by time, economic, technical, and technological constraints upon SC structure dynamics control;
- the evaluation of a general quality measure for SC plans and schedules, and the evaluation of particular performance indicators.

The third mode provides interactive selection and visualization of SC schedule and report generation. An end user can select the modes of program run, set and display data via a hierarchical menu.

The first step is the input data generation. These data create SC structure and the environment on which scheduling will be performed. The data can also be input by a user. After setting up SC structures, planning goals and environment parameters (customer orders and possible uncertainty impacts), the scheduling algorithm is then run. The algorithm of dynamic control is programmed by us; for the optimization of problems (16)-(18) under the presence of constraints (3)-(5) and (11)-(12) at each time point by means of MP techniques, the OPC algorithm addresses the MP library of the MS Excel Solver.

The schedule can be analyzed with regard to performance indicators. Subsequently, parameters of the SC structures and the environment can be tuned if the decision-maker is dissatisfied with the values of performance indicators. More than 15 parameters can be changed to investigate different interrelations of schedule parameters and SC planning goals (e.g., service level and costs) achievement. E.g., there is an explicit possibility to change:

- the amount of resources, their intensity, and capacities;
- the amount and volumes of customers' orders and operations within these orders (including key customer orders and bottleneck operations);
- the priorities of orders, operations, and resources;
- the lead times, supply cycles, and penalties for breaking delivery terms;
- the perturbation impacts on resources and flows in the SC;
- the priorities of the goal criteria.

Of course, these 15 parameters should not be tuned all at once. The tuning depends a great deal on the SC strategy. In the case of a responsive strategy, the increase in the amount of resources and capacities leads in the direction to improving the values of service level and to increasing the amount and volumes of customers' orders. In the case of an efficient strategy, resource consumption and penalties should be reduced as much as possible even if the lead times and supply cycles would increase and the service level decrease. With regard to perturbation impacts, an SC planner can also analyse different alternative SC plans, fill these plans with reliability and flexibility elements to different extents, and then analyse how these changes influence the key performance indicators. In the current version of the software package, this tuning is still performed manually; hence we are still unable to provide either justified conclusions of recommended settings of parameters or established methods for tuning. However, the extension of the software prototype in this direction is under development. The conducted experiments showed that the application of the presented dynamic scheduling model is especially useful for the problems where a number of operations are arranged in a certain order (e.g., technological restrictions). This is the case in SC planning and scheduling.

The building of the scheduling model within the proved theorems and axioms of the optimal CT (Lee and Markus 1967) allows us to consider the found solutions as optimal (see the proofs of the maximum principle in Pontryagin et al. (1964) and the application of maximum principle to economic problems by Sethi and Thompson (2006). Based on the optimal solutions, we can also methodically justify the quality of different heuristics that have launched the optimization procedure (see Step 1 Section 3) (see Fig. 1).

Fig. 1 depicts the idea that having calculated optimal solutions for several points, it is possible to validate the decision to use either dynamic or one of the heuristic planning algorithms (for simplification, we consider here only FIFO, LIFO and Zimin-Ivanilov-Moiseev (ZIM) algorithms).In ZIM algorithm priority of each job depends on quantity of following jobs. It can be observed that, in the case of a number of processes between 10 and 12, the quality of the heuristic and optimal solutions does not differ by more than 4%. In area 2, the DYN algorithm is preferable to the heuristics. If still using the heuristics, the FIFO algorithm is preferable to the LIFO and ZIM. The most benefit from using the DYN algorithm is achieved in area 3. In this area, the ZIM algorithm is preferable to the LIFO and FIFO algorithms. These data are provided only to depict the idea of using an optimal solution for the evaluation of different heuristics. Of course, for other data structures, the interrelation may be different.

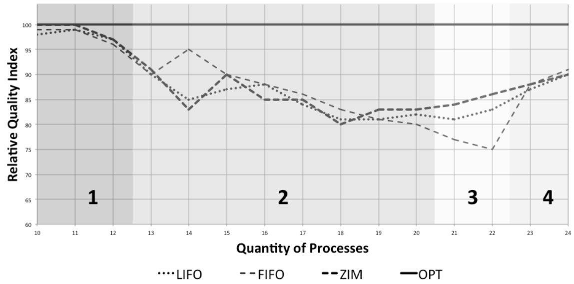

Fig. 1 Comparison of heuristic algorithms' quality with regard to an optimal solution

3. CONCLUSIONS

In this paper, we considered deterministic issues in SC scheduling where scheduling is interconnected to the control function.

In this study, an original approach to a dynamic decomposition of an NP-hard combinatorial SC scheduling problem has been presented. The decomposition is based on the developed model and an algorithm for the optimal control of the execution of the operations blended with mathematical programming (MP). The proposed dynamic decomposition is supported both with an algorithm of local coordination with the help of MP (i.e., at each time instant) and an algorithm of global optimization (i.e., upon the whole planning horizon). This results in the formulation and solution of partial combinatorial problems of lower dimensionality.

In light of this result, the theoretical contribution of this study is directed towards increasing the scheduling quality with the help of a sophisticated scientific methodology. The proposed novelty of this study consists of a detailed theoretical analysis of the time-based decomposition and computational complexity with an application to flow-shop scheduling with continuous flows and discrete assignments. A dynamic model and an algorithm have been developed for the simultaneous solution of the assignment and SC control tasks.

The main idea of the proposed modification of the classical OPC model is to implement and update (e.g., due to dynamic changes in capacity availability) non-linear constraints on a convex domain of feasible control inputs rather than in the right-hand sides of differential equations. In this case, the coefficients of the conjunctive system (i.e., the dynamic Lagrange coefficients), keeping the information about the operational and logical constraints, can be explicitly defined via the local cut method (Boltyanskiy, 1973).

Furthermore, we proposed to substitute the relay constraints by interval ones, i.e., instead of the relay constraints $u_{i\mu j}(t) \in \{0,1\}$ less strict ones $u_{i\mu j}(t) \in [0,1]$ can be considered. Nevertheless, the control takes Boolean values as it is caused by the linearity of the differential equations and the convexity. The proposed substitution enables us to use fundamental scientific results of the OPC theory in scheduling.

The formulated model is a linear non-stationary finite–dimensional controlled system of differential equations with a convex area of feasible control. This is the essential structural property of the proposed approach, which allows applying methods of discrete optimization for the OPC calculation and ensuring the required consistency between OPC and LP/integer programming (IP) models. Although the solver works in the space of piecewise continuous functions, the control actions (i.e., the assignments) can be presented in a discrete form as in LP/IP models.

The continuous time representation allows analyzing the execution of the operations at each time point, and therefore, to obtain additional information about the execution of the SC operations and the flow control. The analysis showed that, since the complexity of the IP/LP problem at each cut is polynomial and the number of integration steps and iterations increase linearly, the computational complexity of the proposed DYN algorithm is also polynomial.

Among the limitations of this study, the strong orientation on centralized SC control and the lack of software tools for a comparative analysis with the existing benchmark solutions can be mentioned. This is the focus of our future efforts. As the convergence speed of the proposed algorithm depends on the selection of the heuristic solution to the vector of the conjunctive system, further research in this direction is needed, e.g., an application of higher-level heuristics.

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REFERENCES

Boltyanskiy, B. (1973). *Optimal control of discrete systems*. Moscow: Nauka.

Chen, Z.-L., G. Pundoor. 2006. *Order assignment and scheduling in a supply chain*. Oper. Res. 54 555–572.

Gubarev, V. A., Zakharov, V. V., Kovalenko, A. N. (1988).*Introduction to systems analysis*. Leningrad: LGU.

Holt, C.C., Modigliani, F., Muth J.F., & Simon H.A. (1960).*Planning production, inventories and work force*. Englewood Cliffs, NJ: Prentice Hall.

Hwang, C.L., Fan, L.T., & Erikson L.E. (1967). *Optimum production planning by the maximum principle*. Management Science, 13(9), 751–755.

Ivanov, D.A., Sokolov B.V. (2010) *Adaptive supply chain management*. Springer, Wiley & Sons, New York.

Kimemia, J.G & Gershwin, T.I. 1983 *An algorithm for the computer control of a flexible manufacturing system*. *IIE Transactions*, 15, 353-362.

Khmelnitsky, E., Kogan K., &Maimom, O. (1997). *Maximum principle-based methods for production scheduling with partially sequence-dependent setups*. Int J Prod Res, 35(10), 2701–2712.

Lee, E.B., Markus, L. (1967). *Foundations of optimal control theory*. Wiley & Sons, New York.

Moiseev, N.N. (1974). *Element of the optimal systems theory* [In Russian].Moscow: Nauka.

Pontryagin, L.S., Boltyanskiy, V.G., Gamkrelidze, R.V., andMishchenko, E.F. (1964). *The mathematical theory of optimal processes*.Pergamon Press, Oxford.

Sarimveis, H., Patrinos, P., Tarantilis, C.D., &Kiranoudis, C.T. (2008). *Dynamic modeling and control of supply chain systems: A review*. Computers & Operations Research, 35, 3530–3561.

Sethi, S.P., Thompson, G.L. (2006). *Optimal Control Theory: Applications to Management Science and Economics*, Second Edition. Springer, Berlin.

Subramanian, K., Rawlings, J.B., Maravelias, C.T., Flores-Cerrillo, J., and Megan L. (2013). *Integration of control theory and scheduling methods for supply chain management*.Computers & Chemical Engineering, in press.

Zimin, I.N., &Ivanilov, Yu.P. (1971). *Solution of network planning problems by reducing them to optimal control problems*.Zh. Vychisl. Mat. Mat. Fiz., 11(3), 632–641.

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