

SYSTEM IDENTIFICATION MODELING OF ROTORS BEHAVIOUR

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ABSTRACT

The prediction of the rotor behavior in its turbo machine final housing is a crucial problem for turbo machinery manufacturers and very complex to solve using classical approaches. The authors, in other works, proposed an innovative method for the determination of a transfer function (MTF) between the rotor responses on a balancing machine (HSB) and on turbo machinery (SP). The proposed method uses a particular formula (MTF), calculated with a black/box approach and based on the application of the theory of System Identification. MTF was determined by a regression analysis of the responses in HSB and SP of 10 rotors; subsequently it was tested and validated on other 15 rotors. The results demonstrate that the proposed MTF simulates a rotor behavior in SP with a satisfactory overlapping of the measured output. In the last presented works, only some results were shown. In this paper all results on prediction and simulation of rotors behavior are presented. An analysis on all graphs allows underlining the repeatability of the proposed method.

Keywords: Rotor Dynamics, Balancing, System Identification, Vibrations, Black-Box, Transfer Function, MTF

1. INTRODUCTION

The statoric parts and the rotors of turbo machines, such as compressors, steam, and gas turbines, are usually realized in an independent way. The rotor is balanced several times on a high speed balancing machine (HSB) in order to reduce the gap error between the theoretical simulation and the real behavior of the rotor during the high speed rotation, thus it is positioned within the turbo machine (SP). The vibration limits of rotors and turbo machinery for proper operation are defined by ISO (2002) and API (1996a; 1996b).

A very important problem, in the field of turbo machinery testing, is the prediction of rotor behaviour (Darlow 1989) in HSB and SP. Some rotors, with a stable behavior in rotation, during theoretical simulation and balancing in HSB, have an unstable behavior during rotation tests in SP.

In engineering problems, often the physical behavior of a system cannot be fully described (either for lack of baseline data or for excessive complexity of the system). In these cases it is necessary to use an experimental approach in order to define the mathematical model.

System identification is utilized to solve this kind of problems (Ljung 1994, 1999; Bittanti 1992a, 1992b; Söderström and Stoica 1989). The identification techniques generate a mathematical model using a regression analysis of the input or input-output data of a system (Natke 1988; Droper 1998).

In the papers (Muscolo 2008; Muscolo, Casadio, and Forte 2012), the authors proposed a new method based on a black-box approach to find a transfer function, called MTF, between the rotor responses on the High Speed Balancing machine (HSB) and in the turbo machine (SP). MTF allows to predict the vibration amplitude of the rotor in SP, already during the balancing steps in HSB. The black-box approach was used because of the high complexity of the two systems and for the unavailability of all the data required to define accurate and realistic white-box models using classical approaches. The research has been conducted in collaboration with a competitive Oil & Gas company, and the MTF represents a first relation between HSB and SP.

In this paper, the authors show other results obtained using the MTF formula and not presented in last papers. Graphs of measured, predicted, and simulated model output are shown in this paper underlining the repeatability of the MTF used model. The future planned steps are focused on the optimization of the formula (MTF) using a non-linear system identification approach.

The paper is so structured: section two presents the problem and the black-box used approach based on the system identification; section three shows the MTF formula and some graphs underlining the repeatability of the model. Section four presents the validation of the MTF and the paper ends with conclusions and future works.

2. SYSTEM IDENTIFICATION MODELING: BLACK-BOX APPROACH

Figure 1 shows a graph of vibration amplitude in micrometres peak-to-peak of an analysed rotor. One graph contains the vibration responses of four probes (A, B, C, D) in HSB overlapping to the four vibration responses in SP. A and B are the responses of the 2 probes mounted on the bearing of the support near to the motor transmission joint; C and D are the responses of the 2 probes mounted on the bearing on the other support (see Figure 2).

Table 1 shows the characteristics of the 25 rotors considered in the analysis. All of them respect the run-out limit. Run-out is a noise relative to probes (API 1996a) as defined in (Weaver, Timoshenko, and Young 1990) (Ehrich 1992; Bently, Hatch 2002).

The test performed in the turbo machinery lab (SP) is passed if the vibration amplitude of the rotor is below the threshold of 25 micrometres peak-to-peak in proximity of the first or second critical speed as in HSB (Darlow 1989).

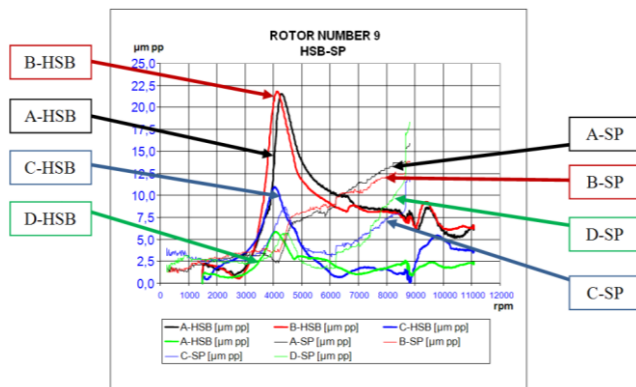


Figure 1: Vibration amplitude in micrometres peak-to-peak of rotor n° 9, in HSB and SP. A, B, C, D represent the vibration responses measured by 4 probes on bearings in HSB e in SP.

Table 1: Characteristics of the 25 analysed rotors.

Values	MAX	MIN
Rotor masses(kg)	1900	244
Diameter of impellers(mm)	600	350
Number of impellers	8	5
MCS = Maximum Continuous Speed (rpm)	13027	8167
OS = Over Speed (rpm)	14850	9310
TS = Trip Speed (rpm)	13678	8657

Statistical and physical approaches have been presented in (Muscolo 2008; Muscolo, Casadio, and Forte 2012), and have highlighted the difficulties of representing the complex relation between the HSB and SP systems and the need to use other tools to find a relation.

The new method proposed in the articles (Muscolo 2008; Muscolo, Casadio, and Forte 2012) allows

determining the transfer function between HSB and SP for rotors with the characteristics listed in Table 1, within the range of rotational speed from 1000 rpm to 12000 rpm.

A black-box approach, based on the theory of system identification (Ljung 1994, 1999; Bittanti 1992a, 1992b; Söderström and Stoica 1989), was utilized in order to determine the transfer function. In the proposed method, the rotor responses in HSB and SP were considered, respectively, as an input (H) and output (S) signal; a mathematical model, using a regression analysis of input-output data of the system H-S, was created. The transfer function linking these two signals (H and S), for each single rotor, was a particular solution. The general formula, valid for all rotors, relating H and S (and then HSB and SP), called MTF (Muscolo 2008; Muscolo, Casadio, and Forte 2012), was constructed from the subsequent analysis of many particular solutions.

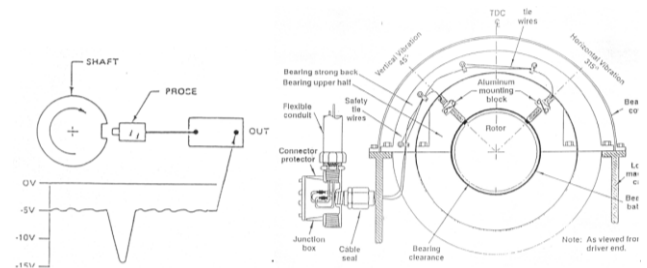


Figure 2: Classical position of the two probes in a machine with horizontal rotor axis

2.1. Analysis and Processing of Experimental Data

25 rotors different by weight, length, number of impellers, etc. (see Table 1) were considered for the determination and validation of MTF.

Instead of the rotational speed in rpm, the following variable was adopted:

$$i = \frac{rpm - 1000}{100} \quad (1)$$

For each rotor it is possible to get two signals, H and S (as input and output), for each probe (A, B, C, D), expressed as a function of variable i ranging from 0 to 110 corresponding to a range of rotational speed from 1000 to 12000 rpm.

The signal was sampled from 1000 rpm ($0i$) to 12000 rpm ($110i$) (period $T_i = 110$), with spaces of variable $\Delta i = 1$, satisfying the sampling theorem (Shannon theorem). Eight data vectors $[111 \times 1]$ were created for each rotor, corresponding to the eight signals of the probes (Figure 1). Four input data vectors (in HSB (H)) and four output data vectors (in SP (S)) were considered. Each vector is formed by a column of values and by 111 lines of sampling. The signal analysis was carried out on the data vectors $[111 \times 1]$. The process of analysis of 10 rotors is described in the following sections with reference to probe A; for the other probes the procedure is the same. 10 input and 10 output

vectors were constructed; three systems of data were built: one system was obtained without data filtering, one was obtained with constant detrend (removing mean value) and another system was obtained using linear detrend.

2.2. Identification Process, Optimal Model, and Its Validation

The first step of identification techniques should define what family models can describe the data. The analysis was limited to five linear family models: ARX, ARMAX, OE, BJ, PEM (Natke 1988). The second step of identification techniques is the determination of the complexity of the model by varying its order. The Final Prediction Error (FPE) and the Akaike Information Criterion (AIC) were used as prediction error (Ljung 1999; Natke 1988). The optimal order of a model corresponds to the lowest calculated values of AIC and FPE. Each rotor was analysed using Matlab System Identification Toolbox and the validation was done using Simulink.

In order to determine the transfer function, tests and simulations were performed on 10 rotors using parametric models (arx, armax, oe, bj) and models for signal processing (or prediction error (PEM)), with different polynomial order. For both models (parametric and pem) 3 types of analysis of values were carried out: without detrend, with average detrend (order 0) and with linear detrend (order 1).

The comparison of all models and the subsequent simulation allowed choosing model bj22221. Among all families the bj model best describes, with the lowest percentage error, the sequence of data of H and S (HSB and SP) for all 10 rotors. The bj22221 model, among the stable models, has the lowest values of AIC and FPE, respectively equal to 0.0963 and 2.3401.

3. MTF FORMULA: DETERMINATION

MTF is the proposed transfer function between HSB and SP defined by the determination of coefficients, $\alpha(q)$, $\beta(q)$, $\gamma(q)$ and $\delta(q)$, of model bj22221:

$$\underline{\underline{MTF}} : S(i) = \frac{\alpha(q)}{\delta(q)} \cdot H(i) + \frac{\beta(q)}{\gamma(q)} ; \quad (2)$$

where:

$$\alpha(q) = 2.12q^{-1} - 1.9q^{-2} ;$$

$$\beta(q) = 1 - 0.17q^{-1} + 0.39q^{-2} ;$$

$$\gamma(q) = 1 - 0.74q^{-1} - 0.27q^{-2} ;$$

$$\delta(q) = 1 - 0.35q^{-1} - 0.56q^{-2} ;$$

The elements of vectors $H(i)$ and $S(i)$ are the vibration amplitude values of 4 probes (see Figure 1) respectively obtained in HSB and SP varying the value of the variable i ($0 \leq i \leq 110 \Rightarrow 1000 \leq rpm \leq 12000$). Equation (2) is based on a BJ family model (Ljung 1994, 1999; Bittanti 1992a, 1992b; Söderström and Stoica 1989) and with MTF it is possible to obtain the simulation of vibration amplitude values of the rotor in

the turbo machinery bench SP ($S(i)$), giving vector $H(i)$ as input. In conclusion, by using MTF it is possible to obtain the simulation of the trend of vibration amplitudes in SP knowing the vibration amplitude values in HSB at the end of the rotor high speed balancing.

3.1. Results and Discussion

MTF was determined by analysing the trends of 10 rotors and it was validated on other 15 different rotors.

The following Figures show measured, predicted and simulated output graphs. The Figures predict the output of the identified model, MTF, 5 steps ahead using input-output data history, and simulate the output signal (MTF) using only input data history (H).

Figures also display the percentage of the output that the MTF reproduces (*Best Fit*), computed using the following equation:

$$BestFit = 100 \times \left(1 - \frac{\|MTF - S\|}{\|S - \bar{S}\|} \right) \quad (3)$$

MTF is the simulated or predicted model output, S is the measured output and \bar{S} is the mean of S . 100% corresponds to a perfect fit, and 0% indicates that the fit is no better than guessing the output to be a constant ($MTF = \bar{S}$).

Because of the definition of *Best Fit*, it is possible for this value to be negative. A negative best fit is worse than 0% and can occur for the following three reasons:

- The estimation algorithm failed to converge.
- The model was not estimated by minimizing $|S - MTF|$.
- The validation data set was not preprocessed in the same way as the estimation data set.

It was noted, by comparison between the predicted and simulated output, that with only 5 steps ahead using input-output data history, the goodness of the identified model using MTF respect to the measured one improves the Best Fit value from 10% to 40%.

Figures 3, 4, 5, 6, 7, 8, 9, 10, 11, and 12 show real (S) and simulated or predicted (MTF) responses respectively of rotors n° 1, 2, 3, 4, 5, 6, 7, 9, 10, and 11. Trends of 10 rotors, used in order to determine the MTF, have not all the same fit, but in all rotors the simulated signal with the MTF follows the real one. By analysis of graphs, the MTF is able to simulate the signal for its entire length.

From the graphs of the following Figures it is possible to note also the difference of rotors behavior in the SP: the rotor 1 (S1 in Figure 3) has a maximum limit at the first critical speed of 20 micrometers peak-to-peak and has a value lower than 5 micrometers peak-to-peak at the second critical speed; in the rotor 2 (S2 in Figure 4) the first critical speed has a value of 10 micrometer peak-to-peak and the second critical speed has a value of 20 micrometers peak-to-peak

The other rotors have different responses as index of the non-linearity of the system.

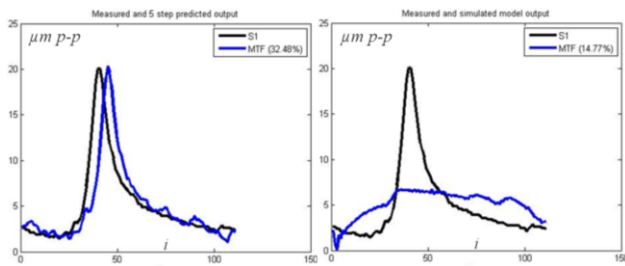


Figure 3: Determination of MTF: measured (S1), predicted and simulated model output (MTF) of the rotor 1.

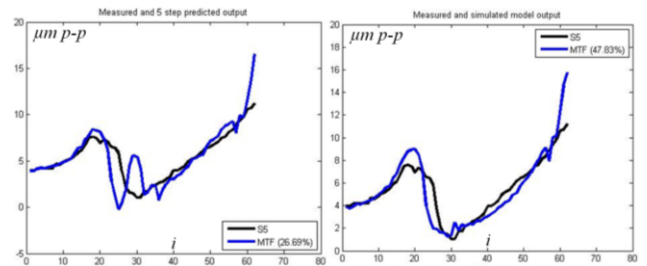


Figure 7: Determination of MTF: measured (S5), predicted and simulated model output (MTF) of the rotor 5.

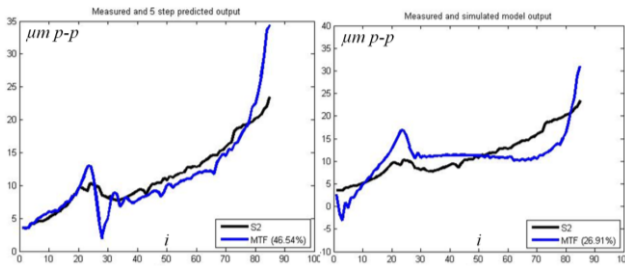


Figure 4: Determination of MTF: measured (S2), predicted and simulated model output (MTF) of the rotor 2.

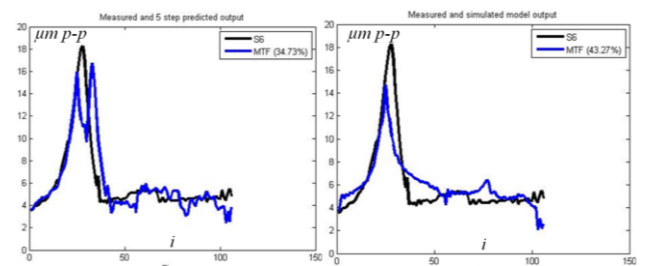


Figure 8: Determination of MTF: measured (S6), predicted and simulated model output (MTF) of the rotor 6.

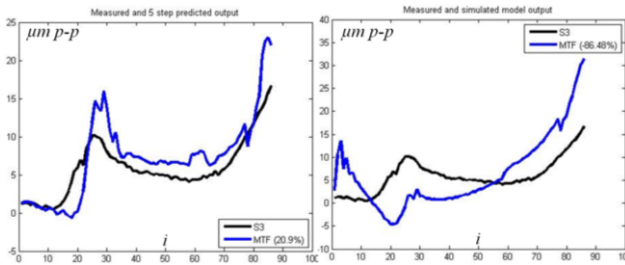


Figure 5: Determination of MTF: measured (S3), predicted and simulated model output (MTF) of the rotor 3.

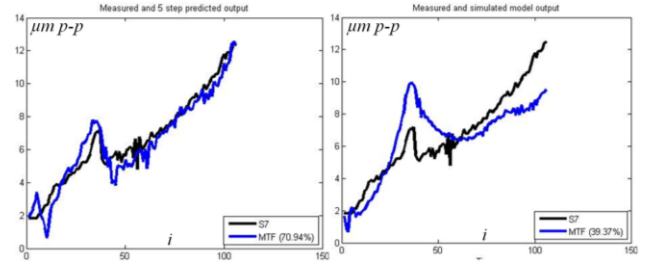


Figure 9: Determination of MTF: measured (S7), predicted and simulated model output (MTF) of the rotor 7.

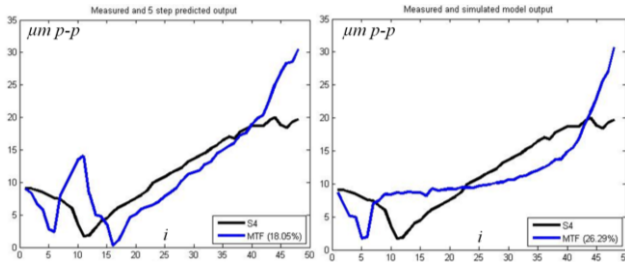


Figure 6: Determination of MTF: measured (S4), predicted and simulated model output (MTF) of the rotor 4.

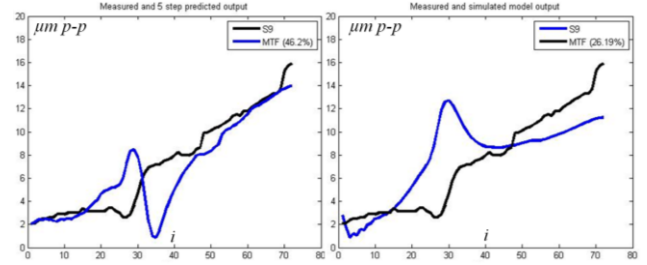


Figure 10: Determination of MTF: measured (S9), predicted and simulated model output (MTF) of the rotor 9.

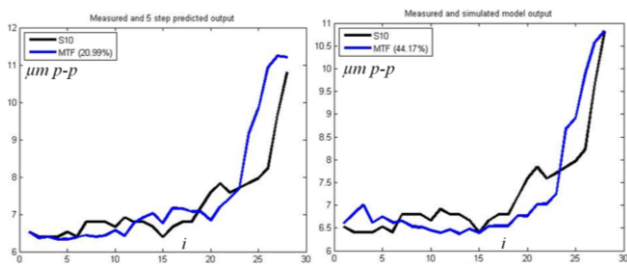


Figure 11: Determination of MTF: measured (S10), predicted and simulated model output (MTF) of the rotor 10.

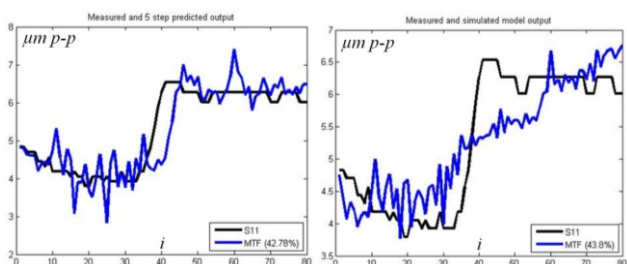


Figure 12: Determination of MTF: measured (S11), predicted and simulated model output (MTF) of the rotor 11.

The MTF was constructed for 10 rotors and from presented results is underlined as an optimization of the formula using non-linear identification systems is necessary. Figure 3 shows, in the simulation graph, a low value of best fit equal to 14.77%. However this value has been higher respect to the stable linear family models considered. Figure 4 shows a best fit in simulation of 26.91%. The rotor number 3 of the Figure 5 has a best fit predicted value of 20.9% and the simulated best fit value of -86.48%. The rotor number 4 (Figure 6) has 18.05% and 26.29% respectively in prediction and simulation. Figures 7, 8, and 9 show respectively a best fit in simulation of 47.83%, 43.27% and 39.37%. The best fit values of the other rotors are, respectively in prediction and simulation, equal to: 46.2% and 26.19% for the rotor number 9; 20.99% and 44.17% for the rotor number 10; 42.78% and 43.8% for the rotor number 11.

It is interesting to evidence that for rotor numbers 10 and 11 the MTF simulated values are bigger respect to the MTF predicted values. The best fit negative value of the rotor number 3 indicates that the estimation algorithm failed to converge using linear identification systems and it is necessary to explore a non-linear identification systems field.

4. MTF FORMULA: VALIDATION

The transfer function MTF was tested on 10 rotors (rotors number: 1, 2, 3, 4, 5, 6, 7, 9, 10, 11) and was validated on the other 15 different rotors (rotors number: 8, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24 and 25) with characteristics presented in Table 1.

Figures from 13 to 22 show the measured, predicted and simulated model output using the MTF. Predicted and simulated best fit values of the rotors are respectively shown in the Table 2.

The rotors with negative predicted and simulated best fit values are shown in the Table 3

Table 2: Predicted and simulated Best Fit values

Predicted %	Simulated %	Rotor number	Figure
41.53	22.37	12	13
43.6	36.11	13	14
56.2	57.48	15	15
30	47.95	17	16
67.54	32.85	22	17
43.66	23.13	25	18

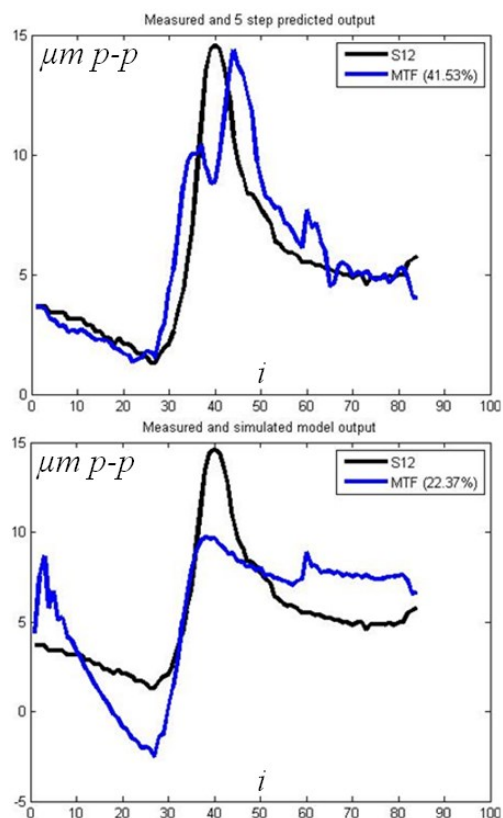


Figure 13: Validation of MTF: measured (S12), predicted and simulated model output (MTF) of the rotor 12.

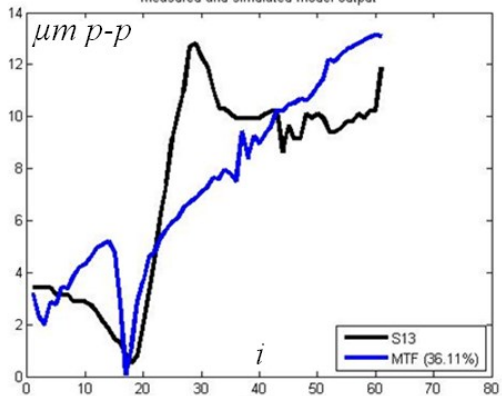
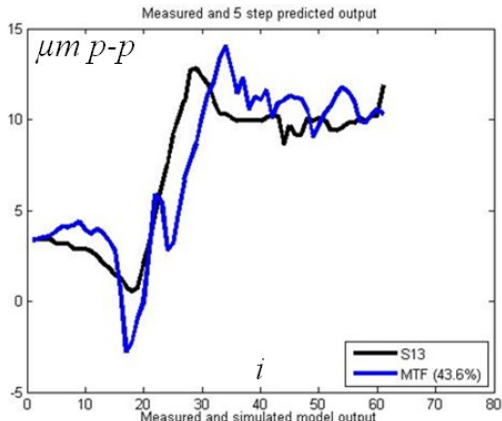


Figure 14: Validation of MTF: measured (S13), predicted and simulated model output (MTF) of the rotor 13.

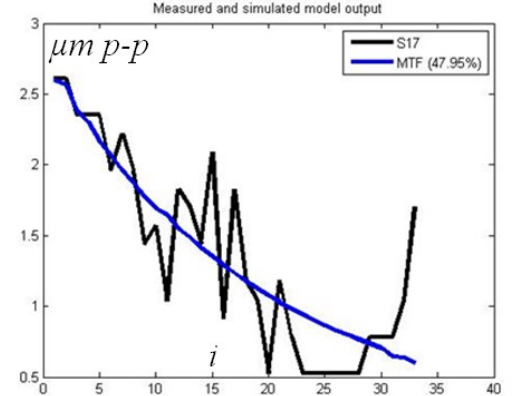
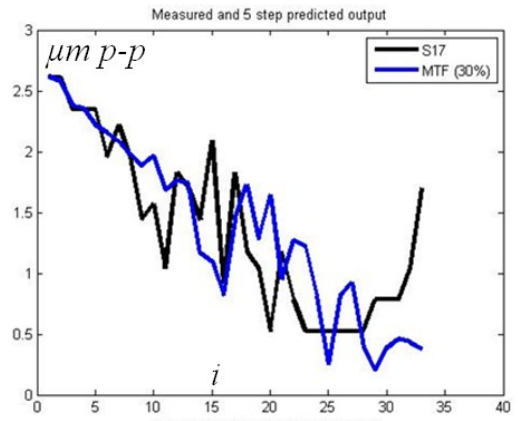


Figure 16: Validation of MTF: measured (S17), predicted and simulated model output (MTF) of the rotor 17.

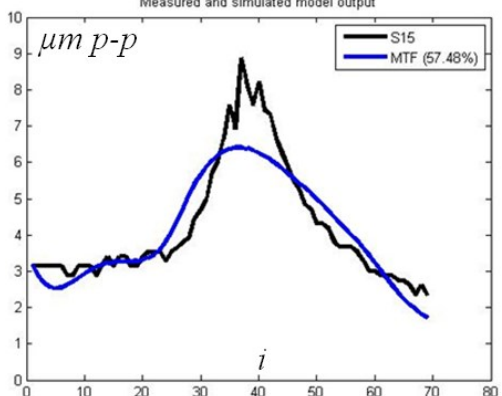
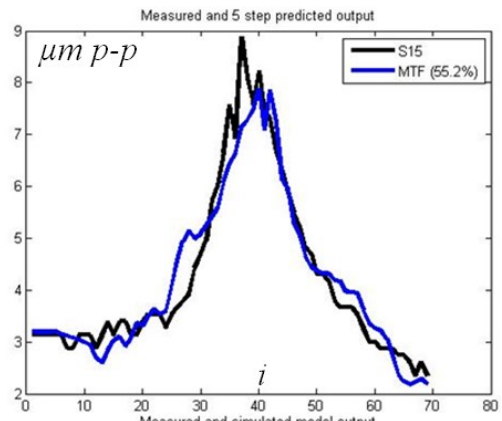


Figure 15: Validation of MTF: measured (S15), predicted and simulated model output (MTF) of the rotor 15.

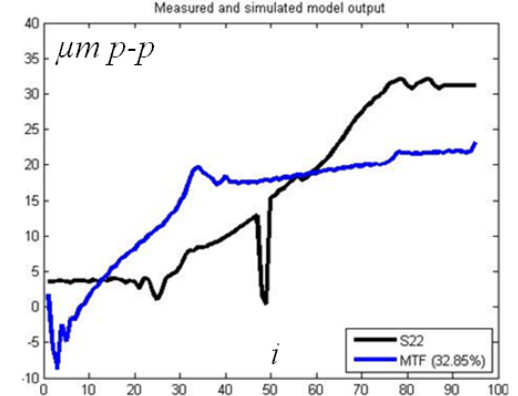
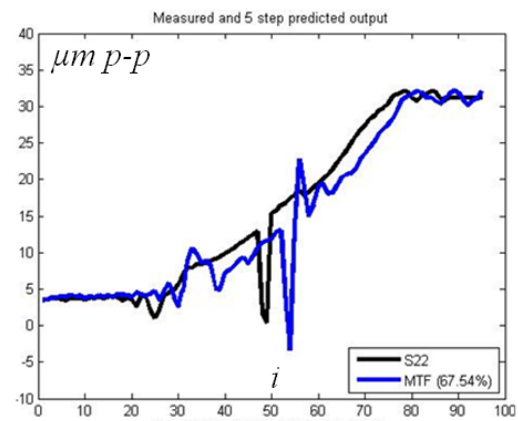


Figure 17: Validation of MTF: measured (S22), predicted and simulated model output (MTF) of the rotor 22.

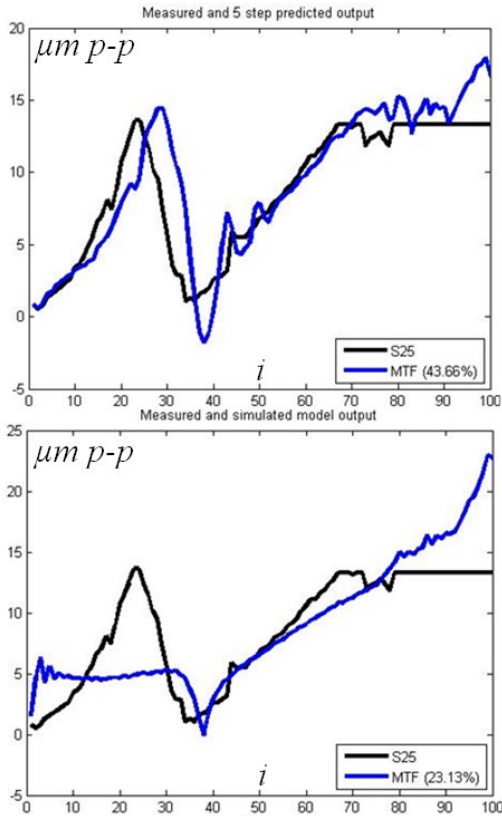


Figure 18: Validation of MTF: measured (S25), predicted and simulated model output (MTF) of the rotor 25.

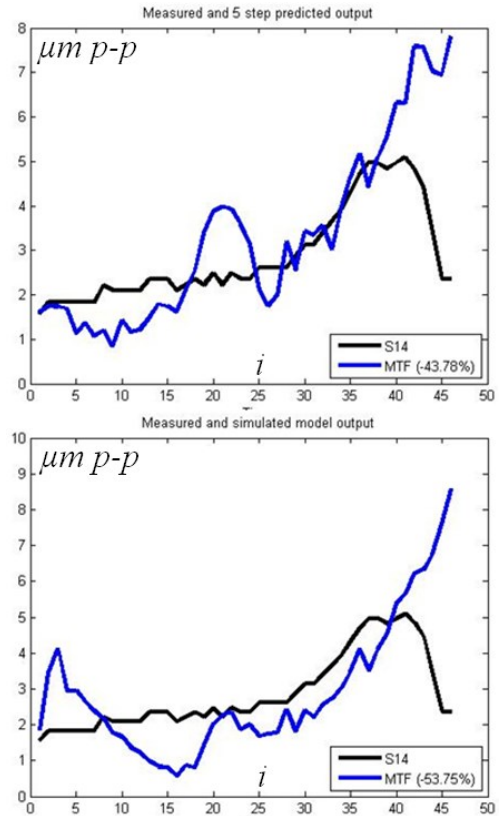


Figure 20: Validation of MTF: measured (S14), predicted and simulated model output (MTF) of the rotor 14.

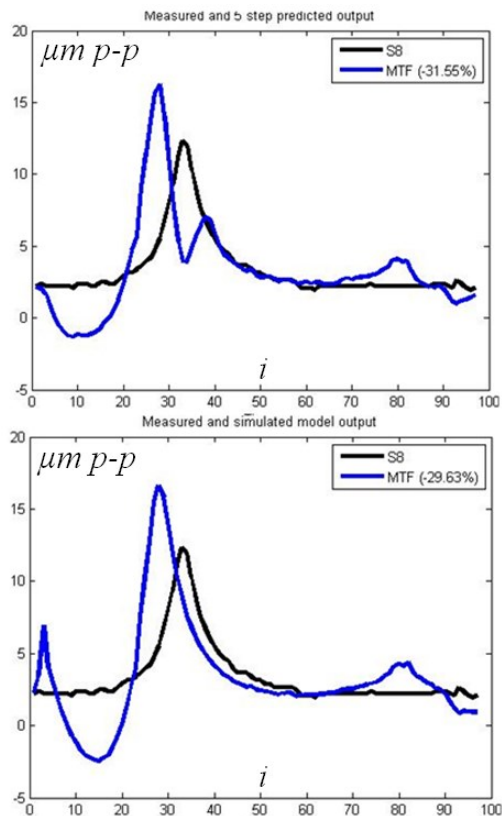


Figure 19: Validation of MTF: measured (S8), predicted and simulated model output (MTF) of the rotor 8.

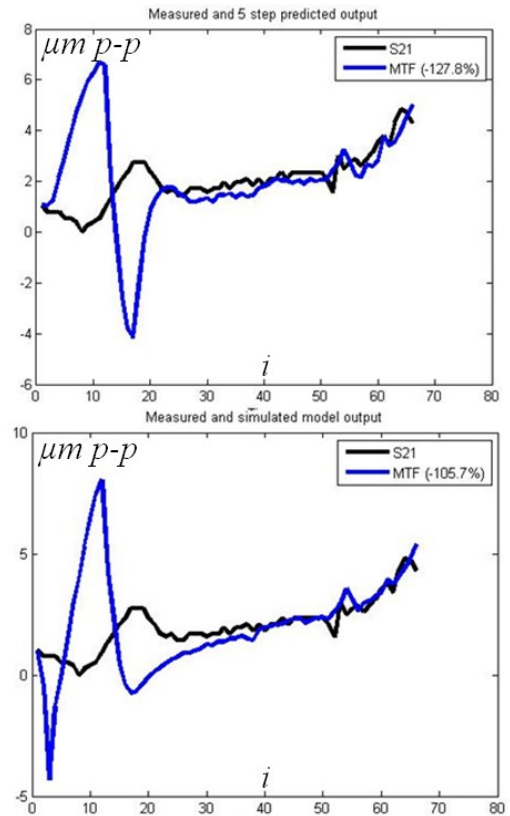


Figure 21: Validation of MTF: measured (S21), predicted and simulated model output (MTF) of the rotor 21.

Table 3: Negative predicted and simulated Best Fit values

Predicted %	Simulated %	Rotor number	Figure
-31.55	-29.63	8	19
43.78	53.75	14	20
-39.06	-32.8	16	
-155.8	-255.4	18	
-89.17	-146.1	19	
-80.46	-298.7	20	
-127.8	-105.7	21	21
-3.2	-17.96	23	
-7.87	-20.31	24	

The predicted MTF model of rotors 12, 13, 15, 17, 22, 23 and 25 confirms that the MTF reproduces the experimental response of rotors as seen for rotors number 2, 5, 6, 7. The predicted graphs of rotors 8, 14, 16, 18, 19, 20, 21 and 24 overestimate the maximum limit of vibration in the first and second critical speed, respect to the experimental graphs.

The prediction of the response of rotor 21 (Figure 21), using MTF, generates a trend similar to the real signal S21, but a peak precedes the first critical speed of the signal S21.

However, the maximum value of the predicted response is lower than the allowable value of 25 micrometers peak-to-peak and the predicted signal with the MTF represents a good safety factor. As can be seen from Figures, also with no bigger best fit values, all simulated or predicted models follow the real trends of the rotor. In particular, the rotor number 21 of the Figure 21 has a negative best fit value in prediction and simulation, but by graphs analysis it is possible to evidence the similarity between S21 and MTF after the 20 i (3000 rpm) of the rotor.

In conclusion, the MTF can be used also as a first step for studying a non-linear rotor behavior underlining non-linear rotor behavior zones respect to the linear one.

A comparison between the experimental graph of the rotor 9 (Figure 10) used for determination of the MTF and the graph of the rotor 8 (Figure 19), used for the validation of the MTF shows that the real signal S9 is different respect to the real signal S8 even if two rotors (8, and 9) are structurally and functionally the same.

Thus, the non-linearity of the system is not only correlated to the structural and manufacturing characteristics of rotors, but also to other factors that cannot be described using classical approaches and as described in (Muscolo 2008).

The validation of the MTF was also confirmed in the prediction and simulation of the trends of other 3 remaining probes. It was conducted the validation of the MTF also considering the probes B, C and D (relating

to the other six lines of the graphs of Figure 1 in addition to the probe A).

The analysis of validation of the formula MTF for the three probes B, C, D, gave the following results:

The probe B, for all 25 rotors has the same trends of the probe A, at least in the proximity of the first critical speed. In proximity to the second critical speed seems that the probe B reproduces, in some cases, responses similar to rotors of the probe C.

The trends of the probe C are predicted with the same goodness which predicted with the probe A.

For most of the rotors the probe D has the same trends in prediction of the probe C. For some rotors near of the second critical speed the probe D follows the probe A, when the probe B follows the probe C.

The transfer function MTF is representative for the probe that has the largest number of resonances (probe A).

5. CONCLUSIONS

In this paper the authors used a black-box approach based on system identification to find a transfer function, called MTF, between the rotor responses on a high speed balancing machine (HSB) and in turbo machinery (SP).

MTF has been presented in past works having considered the limitations of the classical (physical and statistical) approaches, the high complexity of the systems, and the unavailability of the necessary data.

In this paper some graphs on prediction and simulation using MTF were presented and discussed. MTF was determined by a regression analysis of the responses in HSB and SP of 10 rotors; subsequently it was tested and validated on other 15 rotors.

The first tests have been carried out in the labs of GE Oil & Gas Company. Only the rotors of compressors were considered because they have more problems in balancing (maybe related to impellers) compared to rotors of steam and gas turbines.

The first research started because some of these rotors presented a stable response in HSB and an unacceptable response in SP. MTF should allow to predict the vibration amplitude of the rotor in SP, already during the balancing steps in HSB.

The proposed formula is the first attempt to find a relation between the two systems (HSB and SP) and must be considered preliminary. The linear system identification models, studied in (Muscolo 2008; Muscolo, Casadio, and Forte 2012), are actually a first step of this research.

The best fit negative value obtained for some rotors indicate that the estimation algorithm failed to converge using linear identification and that it is necessary to apply non-linear identification methods.

Moreover, the formula was obtained on the basis of the signals of one probe but with some additional work it could be optimized on the responses of all the other probes.

Improvements could be made also differentiating the formula for classes of rotors or for ranges of

operating conditions. The future planned steps are therefore focused on the optimization of the formula using a non-linear system identification approach.

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