# CONSIDERATION OF SOME PERFORMANCE OF CONTAINERS' FLOW AT YARD

**Maja Škurić[a], Branislav Dragović[b], Nenad Dj. Zrnić[c], Romeo Meštrović[d]**

[a], [b], [d] Maritime Faculty Kotor, University of Montenegro
[c] Faculty of Mechanical Engineering, University of Belgrade

[a] mskuric@ac.me, [b] branod@ac.me, [c] nzrnic@mas.bg.ac.rs, [d] romeo@ac.me

## ABSTRACT

Performance of containers' flow at yard represents very important point of container terminals. The organization of container transport and stacking policies leads to less congestion and lower costs. Otherwise, containers wait in queue before they are serviced. This study presents an analytical approach for obtaining the average number of containers in queue. We proposed two models (constant and geometric) of bulk arrival multi-server queuing system. Traffic intensity and utilization factor are very important parameters that consist of data for arrival and service rate and number of servers (yard cranes). In this paper, we assume that there are three yard cranes that operate at container yard. A given numerical example for two models will improve the best values for performance of containers' flow at container yard.

Keywords: container yard, average number of containers in queue, bulk arrival, numerical example

## 1. INTRODUCTION

For determining the optimal capacity of a container yard, a maximum attention should be paid to the stacking policies defined by yard cranes. This is due to the accommodative capacity of the yard, expressed by the number of yard cranes, determining the required capacity of container yard as a whole. Applying queuing models, container yard can be treated as: a system with the infinite waiting capacity and determined number of yard cranes, a single or multi-server system (depending on the number of yard cranes), a system in which servicing is most often carried out according to the FCFS rule (first come, first served), but it is possible that there are certain containers which have priority in servicing and a system where containers must be serviced at once (Škurić, Dragović, and Meštrović 2011).

In this paper we calculate the average number of containers in queue at container yard. We assume that the containers are arriving in group at yard and follows constant or/and geometric distributions. Their service time follows the exponential distribution. In accordance to the extended Kendall's queuing notation, these two models may be denoted as ( $M^{X=const}/M/c(\infty)$ ) for constant and ( $M^{X=(1-a)^{k-1}a}/M/c(\infty)$ ) geometric distributions of container group arrivals. The number of assumed yard cranes is three. The level of traffic intensity and values of some other parameters are also stated. The objective is to describe these models for defining the strategies at yard and calculate the average number of containers in queue.

This paper is organized as follows. Literature review is given in Section 2, while in Section 3 the analytical formulations are provided. Related numerical results' analysis for obtaining the average number of containers in queue with corresponding graphical results is shown in Section 4. Final conclusions are given in Section 5.

## 2. LITERATURE REVIEW

Generally speaking, authors used queuing models (single or multiple) to describe the arrival and service processes of customers (ships) in ports. They are used to analyze complex dynamic and stochastic situations (see e.g. Dragović, Park, Zrnić, and Meštrović 2012). The models contain analytic formulations and numerical solutions for the performance evaluation of port systems. Various models from simple queues to complex queuing network models have been suggested to analyze: movement of ships in port, ship traffic modelling, mechanism of congestion occurrence, composition and congestion costs, evaluation method for optimal number of berths, optimum allocation and size of ports, optimal berth and crane combination in ports, average cost per ships served, the ship turn-around time at the port and so on. Regarding multiple queuing system, the authors presented in Table 1 have investigated bulk arrivals of the customers.

Their considered problems are based on the following statements: a comparison of analytical and simulation planning models, the analysis of a queue with bulk arrivals and bulk-dedicated servers, an analytical methodology of bulk queuing system that determines the capacity of berths within seaports and river ports, port storage locations as queuing systems with bulk arrivals and a single service, the optimal number of servers with bulk arrivals by minimizing the total costs of system, the anchorage-ship-berth link at the port utilizing queuing theory with bulk arrivals, a multi-server queue with bulk arrivals and finite-buffer space and queuing approaches at container yard with


Proceedings of the Int. Conf. on Harbor Maritime and Multimodal Logistics M&S, 2013 119
ISBN 978-88-97999-24-9; Bruzzone, Gronalt, Merkuryev, Piera Eds.

detailed analytical expressions and real case study (see e.g. Škurić, Dragović, and Meštrović 2011).

The importance of bulk queuing models for container terminal problem is explained in Kozan (1997). The analysis of a queue with bulk arrivals and bulk-dedicated servers is specified in Gullu (2004). In this paper, it is considered the $M/G/\infty$ queuing system with bulk arrivals whose jobs belong to a batch have to be processed by the same server. Similarly to this study, bulk arrivals are presented by pushed and pulled convoys of barges in Radmilović (1992). The queuing system describes that barges in convoy have a constant or geometric probability distribution. On the other hand, in Radmilović, Čolić, and Hrle (1996), the authors deal with the port storage locations with bulk arrivals and a single service. Finally, in Radmilović, Dragović, and Meštrović (2005) the aim was to minimize the total cost of system by determining the optimal number of servers. The processes of anchorage-ship-berth link at the port described by the non-stationary multi-server queuing system are presented in Dragović, Zrnić, and Radmilović (2006) and in Zrnić, Dragović, and Radmilović (1999). Likewise, partial and total bulk rejections and the distributions of the numbers of customers in the system for multi-server queue are explained in Laxmi and Gupta (2000). Again, the related literature overview is given in Table 1.

Table 1: Related Literature Overview

| References | Considered problem | Results |
|---|---|---|
| (Radmilović 1992) | Analytical methodology of bulk queuing system. | Determined the optimum number and capacity of berths within seaports and river ports. |
| (Radmilović, Čolić, and Hrle 1996) | Bulk arrivals and a single service. | Port storage locations as queuing systems are solved. |
| (Kozan 1997) | A comparison of analytical and simulation planning models of container terminals. | The advantages of simulation are shown because it is able to capture all details and the complexity of a real system. |
| (Zrnić, Dragović, and Radmilović 1999; Dragović, Zrnić, and Radmilović 2006) | Analyzed the anchorage-ship-berth link utilizing queuing theory with bulk arrivals. | Determined cost ratio and total system cost. |
| (Laxmi and Gupta 2000) | A multi-server queue with bulk arrivals and finite-buffer space. | Partial and total bulk rejections and the distributions of the numbers of customers in the system are obtained. |
| (Gullu 2004) | The analysis of a queue with bulk arrivals and bulk-dedicated servers. | Considered the $M/G/\infty$ queuing system with bulk arrivals whose jobs belong to a batch have to be processed by the same server. |
| (Radmilović, Dragović, and Meštrović 2005) | Optimal number of servers in with bulk arrivals. | Minimized the total costs of system. |
| (Škurić, Dragović, and Meštrović 2011) | A multi-server queue with bulk arrivals. | Obtained average number of containers in queue and related cost ratio. |
| (Dragović, Park, Zrnić, and Meštrović 2012) | Discuss dynamic system performance evaluation in the river port utilizing queuing models with batch arrivals. | The results have revealed that analytical modelling is a very effective method to examine the impact of introducing priority, for certain class of ships, on the anchorage-ship-berth link performance. |

## 3. ANALYTICAL FORMULATIONS

In this Section, we present the methodology that contains analytical formulations of parameters for calculating the average number of containers in queue. First, we start with obtaining the explicit formulae for steady-state probability that $n$ containers are at the yard. After providing the probabilities of constant and geometric distribution of $X$ in case when there is specified number of yard cranes, we give formulae for related numbers of containers in queue that corresponds to the mentioned probabilities. Finally, numerical example is used for sensitivity analysis of average number of containers in queue in relation to three parameters (number of containers in group, number of yard cranes and traffic intensity).

Traffic intensity of containers at yard is in dependence of their arrival and service rate, denoted as $\theta = \lambda/\mu$ where $\lambda$ is the average arrival rate of containers in group and $\mu$ represents the average service rate of containers. These are serviced by yard cranes ($c$) which represent the number of servers. The average number of containers in group is given as $\overline{a}$ while the utilization factor for bulk queuing system is defined as $\rho = (\lambda\overline{a})/(c\mu)$. Notice that $\rho = (\theta \cdot \overline{a})/c$.

We consider a bulk arrival multi-server queue $M^X/M/c$ where the bulk size $X$ is a constant or geometrically distributed random variable. The yard cranes have independent, exponentially distributed service times. The containers that arrive for service in groups $X$ and the mean of $X$ is equal to $E(X) = \overline{a} = 1/a$ and the variance of $X$ is equal to $\text{var}\, X = \sigma_a^2 = 1/a^2$. The case when $X$ is a constant that is $P(X=b)=1$ for some fixed $b \in \{1,2,3,...\}$, then $E(X)=\bar{a}=b$ and $\sigma^2(X)=0$. The inter-arrival times, the bulk sizes and service times are mutually independent. It is known that a probability that containers are present in queuing system is (Chaudhry and Templeton 1983)

$$-\sum_{n=0}^{c-1} nP_n = c(Q_0 - \rho) \quad (1)$$

where $Q_0$ is a probability that average number of containers that are present in a queuing system, $L_c$, are busy. The above formula immediately yields

$$\sum_{n=0}^{c-1}(c-n)P_n = c\left(\left(Q_0 + \sum_{n=0}^{c-1} P_n\right) - \rho\right) \quad (2)$$

which in view of the fact that $Q_0 = \sum_{n=c}^{\infty} P_n$ becomes

$$\sum_{n=0}^{c-1}(c-n)P_n = c(1-\rho) \quad (3)$$

where $\rho = \lambda\bar{a}/(c\mu)$ is the utilization factor. Following Chaudhry and Tampleton (1983),

$$L_c = L_q - c - \sum_{n=0}^{c}(n-c)P_n \quad (4)$$

where $P_n$ is steady-state probability that $n$ containers are at the yard, i.e., that $n$ containers are just being serviced or are waiting in a queue to be serviced. On the other hand, it is suitable to determine the probabilities $P_0$ and $P_n$ using Kabak's recurrence formulae (Dragović, Zrnić, and Radmilović 2006; Kabak 1970; Škurić, Dragović, and Meštrović 2011). These probabilities follow recurrence relations:

$$P_n = y(n)\sum_{k=0}^{n-1} P_k A_{n-k}, \qquad n = 1, 2, ...$$

(5)

with

$$y(n) = \lambda / \mu(n),\ \mu(n) = \mu \min\{n, c\} \quad (6)$$

and

$$A_{n-k} = 1 - \sum_{i=0}^{n-k-1} a_i \quad (A_1 = 1) \quad (7)$$

where $a_i$ is a probability that a group of $i$ containers arrives in the bulk queuing system, $P(X=i)=a_i$, $i \ge 1$. Substituting (6) and (7) into (5), the probability $P_0$ is obtained as

$$P_0 = 1 - \rho - \frac{\sum_{n=1}^{c}(c-n)P_n}{c}. \quad (8)$$

Furthermore, the average number of containers present in queuing system with $c$ yard cranes is

$$L_c = \sum_{n=0}^{\infty} nP_n \quad (9)$$

and it also holds for the bulk arrivals queuing system. Following Chaudhry and Templeton (1983),

$$L_c = \frac{\theta}{2}\cdot\frac{\sigma_a^2 + \bar{a}^2 + \bar{a}}{c - \theta\cdot\bar{a}} + \frac{\sum_{n=0}^{c-1} n(c-n)P_n}{c - \theta\cdot\bar{a}}. \quad (10)$$

In the case when there are three yard cranes i.e. $c=3$, it is necessary to substitute $\bar{a}=1/a$, $\sigma_a^2 = 1/a^2$ into (10) and (6), respectively. Also, for taking the values for geometric distribution of $X$ with $P(X=k)=a_k=(1-a)^{k-1}a$, $k=1, 2, 3 ...$, where $0<a<1$, and putting $A_k = 1 - \sum_{i=1}^{k-1} a_i = 1 - a\sum_{i=1}^{k-1}(1-a)^{i-1} = (1-a)^{k-1}$, $k=1, 2, ...$ in (7), we obtain the formulae for $P_k$ related to geometric distribution of $X$. The corresponding formulae for average number of containers in queue, $P_k^{const}$ and $P_k^{g}$ related to the constant and geometric distribution of batch size $X$, respectively, are as follows (Škurić, Dragović, and Meštrović 2011):

- The probabilities of constant distribution of $X$ in case when $c=3$ yield

$$P_0^{const} = \begin{cases} \dfrac{2(1-\rho)}{2+4\rho+3\rho^2} & if\ b=1 \\ \dfrac{2b^2(1-\rho)}{2b^2+5b\rho+3\rho^2} & if\ b>1 \end{cases},$$

$$P_1^{const} = \begin{cases} \dfrac{6\rho(1-\rho)}{b(2+4\rho+3\rho^2)} & if\ b=1 \\ \dfrac{6\rho b(1-\rho)}{2b^2+5b\rho+3\rho^2} & if\ b>1 \end{cases},$$

$$P_2^{const} = \begin{cases} \dfrac{9\rho^2(1-\rho)}{b^2(2+4\rho+3\rho^2)} & if\ b=1 \\ \dfrac{9\rho^2(1-\rho)}{2b^2+5b\rho+3\rho^2} & if\ b>1 \end{cases} \text{ and}$$

$$P_3^{const} = \begin{cases} \dfrac{9\rho^3(1-\rho)}{b^3(2+4\rho+3\rho^2)} & if\ b=1 \\ \dfrac{9\rho^3(1-\rho)}{2b^3+5b^2\rho+3b\rho^2} & if\ b>1 \end{cases}. \quad (11)$$

- The probabilities of geometric distribution of $X$ in case when $c=3$ are

$$P_0^{g} = \frac{2(1-\rho)}{2+a(5-a)\rho+3a^2\rho^2},$$


Proceedings of the Int. Conf. on Harbor Maritime and Multimodal Logistics M&S, 2013 121
ISBN 978-88-97999-24-9; Bruzzone, Gronalt, Merkuryev, Piera Eds.

$$P_1^g = \frac{6a\rho(1-\rho)}{2+a(5-a)\rho+3a^2\rho^2},$$

$$P_2^g = \frac{3a\rho(1-\rho)(1-a+3a\rho)}{2+a(5-a)\rho+3a^2\rho^2} \text{ and}$$

$$P_3^g = \rho a\left(\left(1-a^2\right)+\frac{9\rho a(1-a)}{2}+\frac{9\rho^2 a^2}{2}\right)P_0 . \qquad (12)$$

- Related numbers of containers in queue that corresponds to the probabilities given by (11) and (12) are

$$L_3^{const} = \frac{14\rho+10\rho^2-15\rho^3}{(2+4\rho+3\rho^2)(1-\rho)} \; if\ b=1,$$

$$L_3^{const} = \frac{18\rho(b+\rho)}{2b^2+5b\rho+3\rho^2}+\frac{\rho(b+1)}{2(1-\rho)} \; if\ b>1 \qquad (13)$$

and

$$L_3^g = \frac{6a\rho(3+3a\rho-a)}{2+a(5-a)\rho+3a^2\rho^2}+\frac{\rho(2+a)}{2a(1-\rho)} . \qquad (14)$$

## 4. NUMERICAL RESULTS' ANALYSIS

The numerical example is in relation to container terminal in port of Bar, Montenegro. The container terminal throughput from 2008 to 2012 is given in Figure 1 (PBR 2012). The biggest throughput of 43708 TEU is reached in 2008. We observe that the container yard is consisted of three yard cranes that serve for container stacking. Tractor trailer system and fork lifters are used for terminal transport of containers from berth to yard and vice versa.

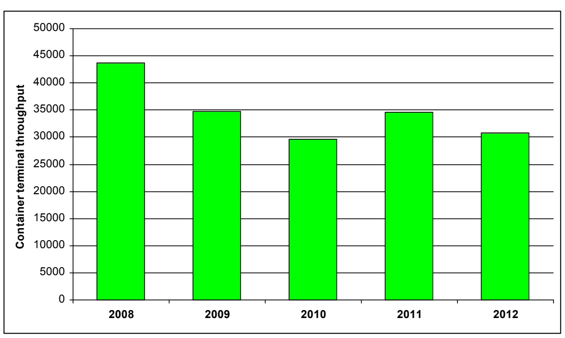


Figure 1: Container terminal throughput in port of Bar

At time, container terminal in port of Bar is consisted of one berth and one quay crane for servicing the container ships. The maximum carrying capacity of container ships is 4000 TEU. There are also the rail and road vehicles for inland connection (Škurić, Dragović, and Meštrović 2011). The input data for the analytical models are based on the actual containers' arrivals at the terminal of port of Bar where we assumed that the containers' arrivals fit constant or geometric distribution.

Using formulae (13) and (14), in Figures 2 and 3 we compare the values for the average number of containers in queue which are in function of traffic intensity for $\overline{a}=b=2$ and $\overline{a}=b=4$ with constant and geometric distribution. The graphs are obtained in *Mathematica* 8. Considering Figure 2, the input data such as average number of containers in group $\overline{a}=b=2$ is specified as well as the different values of traffic intensity are from $\theta=0.1$ to $\theta=1.3$ while the number of yard cranes is 3.

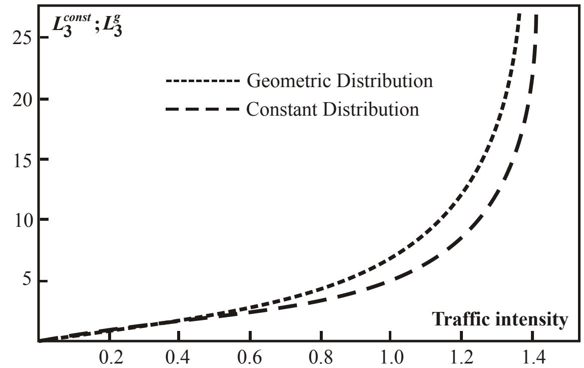


Figure 2: Average Number of Containers in Queue for Constant and Geometric Distributions of $X$ with $\overline{a}=b=2$, $c=3$ and $\theta\in(0,1.3)$

From Figure 2, we can notice that the results for average number of containers in queue are in function of three different parameters (average number of containers in group, traffic intensity and number of yard cranes) and may serve to see related parameters for comparing results for constant and geometric distributions. The increase of average number of containers in queue causes higher traffic intensity of containers at yard. Therefore, for the same traffic intensity, the average number of containers in queue for geometric arrivals of containers implies higher values then those for constant distribution.

The results for average number of containers in queue in the case of $\overline{a}=b=4$ are given in Figure 3 with the same number of yard cranes as in the first case and traffic intensity values are from $\theta=0.1$ to $\theta=1.3$. Obviously in Figure 3, the values for geometric distributions are more dynamic and are increasing faster then those for constant distributions. It means that in case that the group arrivals of containers have its behaviour by constant distribution; it implies that the average number of containers in queue is lower in comparison to the geometric distributed containers' arrivals with the same value of traffic intensity.

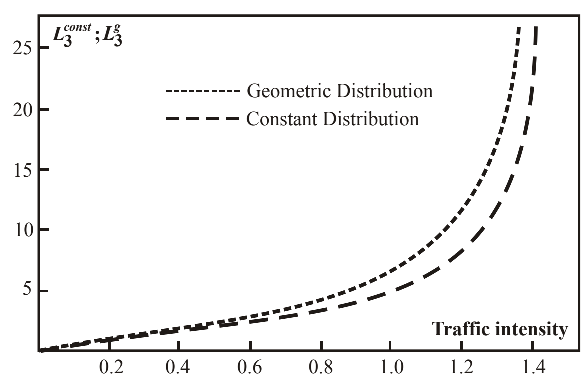


Figure 3: Average Number of Containers in Queue for Constant and Geometric Distributions of $X$ with $\overline{a}=b=4$, $c=3$ and $\theta\in(0,1.3)$



ISBN 978-88-97999-18-8; Bruzzone, Gronalt, Merkuryev, Piera Eds.


Proceedings of the Int. Conf. on Harbor Maritime and Multimodal Logistics M&S, 2013 122


ISBN 978-88-97999-24-9; Bruzzone, Gronalt, Merkuryev, Piera Eds.

## 5. CONCLUSION

The numerical results' analysis for different values of parameters for presenting the performances of containers' flow at container yard in port of Bar leads to the following conclusions:

- The dynamical arrivals of containers in accordance to constant distribution showed better results in the view of average number of containers in queue which may lead to less congestion in comparison to the geometric distributed containers' arrivals.
- As a matter of fact that Figure 1 implies that there would not be huge fluctuations in container terminal throughput in coming years, this suggests that the level of traffic intensity will not be drastically changed and that assumed values in numerical example represent the real situation in port.
- The values of average number of containers in queue directly impact on specific cost ratio of total annual cost for queuing system to the annual container cost and total system costs.
- The obtained results suggest that the operational strategy at container yard can be improved by reducing the average number of containers in queue. This can be evaluated through the employment of another yard crane or to observe other group arrivals of containers.

On the other hand, this analysis also has some limitations. There are a lot of parameters that did not taken into account, but no matter to that, we suppose that it represents a convenient approach for implementing some other modelling techniques and in some further investigations simulation model employment would be able to capture the complexity of a real system such as container yard.

## REFERENCES


Chaudhry, M.L. and Templeton, J.G.C., 1983. *A first course in bulk queues*. 1st ed. New York: John Wiley and Sons, Inc.

Dragović, B., Zrnić, Dj. and Radmilović, Z., 2006. *Ports & container terminals modeling*. 1st ed. University of Belgrade: Faculty of Transport and Traffic Engineering.

Dragović, B., Park, N.K., Zrnić, Dj. N. and Meštrović, R., 2012, Mathematical models of multi-server queuing system for dynamic performance evaluation in port. *Mathematical Problems in Engineering*, Volume 2012 (2012), Article ID 710834, 19 pages, doi:10.1155/2012/710834.

Gullu, R., 2004. Analysis of an $M/G/\infty$ queue with batch arrivals and batch-dedicated servers. *Operations Research Letters*, 32, 431-438.

Kabak, I.W., 1970. Blocking and delays in $M^{(X)}/M/c$ bulk arrival queuing systems. *Management Sciences*, 17 (1), 112-115.

Kozan, E., 1997. Comparison of analytical and simulation planning models of seaport container terminals. *Transportation Planning and Technology*, 20 (3), 235-248.

Laxmi, P. V. and Gupta, U.C., 2000. Analysis of finite-buffer multi-server queues with group arrivals: $GI^X/M/c/N$. *Queueing Systems*, 36, 125-140.

Port of Bar Reports (PBR), 2012.

Radmilović, Z., 1992. Ship-berth link as bulk queuing system in ports. *Journal of Waterway*, *Port*, *Coastal and Ocean Engineering*, *ASCE*, 118 (5), 1-30.

Radmilović, Z., Čolić, V. and Hrle, Z., 1996. Some aspects of storage and bulk queueing systems in transport operations. *Transportation Planning and Technology*, 20, 67-81.

Radmilović, Z., Dragović, B. and Meštrović, R., 2005. Optimal number and capacity of servers in $M^{X=\bar{a}}/M/c(\infty)$ queuing systems. *International Journal of Information and Management Sciences*, 16 (3), 1-22.

Zrnić, Dj., Dragović, B. and Radmilović, Z., 1999. Anchorage-ship-berth link as multiple server queuing system. *Journal of Waterway*, *Port*, *Coastal and Ocean Engineering*, *ASCE*, 125 (5), 1-27.

Škurić, M., Dragović, B. and Meštrović, R., 2011. Some results of queuing approaches at container yard. *Int. J. Decision Sciences, Risk and Management,* 3 (3/4), 260-273.


## AUTHORS BIOGRAPHY

Maja Škuric is a PhD student at Maritime Faculty, University of Montenegro. After graduations, she started to work as a Teaching Assistant at the same faculty. She received her Master of Science degree at Faculty of Transport and Traffic Engineering, University of Belgrade. Up to now she published more then 30 conference papers and few journal papers. Her main research interests are related to modelling and optimizing processes at container terminals and lastly she included the cruise ship traffic modelling in her interests. These topics are mainly concerned with the investigations of queuing systems, planning transportation systems and operations research in transport.

Branislav Dragović is a Professor at the Maritime Faculty, University of Montenegro. He received his BSc and MSc at Maritime Transport and Traffic. Further, he received his PhD from Faculty of Mechanical Engineering, University of Belgrade with specialization on planning and design of transport systems. He was a Visiting Professor of the Korea Maritime University from 2006 to 2007. He was invited as a Lecturer in the country and abroad many times. He has published over 130 scientific papers including ones in journals such as Journal of Waterway, Port, Coastal and Ocean



Proceedings of the Int. Conf. on Harbor Maritime and Multimodal Logistics M&S, 2013 123
ISBN 978-88-97999-24-9; Bruzzone, Gronalt, Merkuryev, Piera Eds.

Engineering; Maritime Economics and Logistics; Maritime Policy and Management; Mathematical Problems in Engineering among others. He has written four books, three national and two international monographs. He was quoted many times. He is a member of IAME and WCTRS.

Nenad Zrnić is Associate Professor at the Faculty of Mechanical Engineering of the University of Belgrade. He received engineering degree Dipl.-Ing. (equivalent to MSc) from the UB, FME in 1992, MagTechSc (equivalent to CSc) in 1996 at the same institution. He received his DrSc from the Faculty of Mechanical Engineering, University of Belgrade in 2005 with in material handling equipment in ports and terminals. He is a reviewer in the journals Mechanics Research Communications (SCI), Mechatronics (SCI), Natural Hazards and Earth System Sciences (SCI), among others. He has published over 120 scientific papers in the country and abroad, among them 15 in world's leading journal (SCI). He presented 7 invited plenary lectures at international conferences. He wrote 1 national monograph, 1 international monograph and 1 book. He was quoted several times.

Romeo Meštrović is a Professor of Mathematics at University of Montenegro. He received his BSc from the University of Zagreb and his MSc and PhD from the University of Montenegro, in the area of mathematics. His topics are complex analysis, combinatorial number theory, operation researches and queuing theory in transport systems. He has published more than 20 papers in academic journals such as Journal of Mathematical Analysis and Applications, Journal of Inequalities and Applications, Ars Combinatoria, American Mathematical Monthly etc.