MULTI-OBJECTIVE GENETIC LOCAL SEARCH ALGORITHM FOR SUPPLY CHAIN SIMULATION OPTIMISATION

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ABSTRACT

This paper presents a hybrid simulation optimisation algorithm that integrates a multi-objective genetic algorithm and response surface-based metamodelling techniques. The optimisation problem involves a search in a high dimensional space with different ranges for decision variable scales, multiple stochastic objective functions and problem specific constraints. A case study demonstrates the application of a hybrid simulation optimisation algorithm to optimal cyclic planning for a generic supply chain network.

Keywords: simulation optimisation, genetic algorithm, response surface-based metamodelling, supply chain cyclic planning

1. INTRODUCTION

Pareto-based evolutionary algorithms are powerful algorithms for solving complex multi-objective problems. They present the class of direct search methods that apply the concepts of Pareto-optimality and dominance relation. They evolve multiple parallel solutions that allow generating a set of non-dominated solutions and preserving a diverse set of the candidate solutions. In addition, they are able to perform a search in a high dimensional space with different ranges of decision variables and could incorporate constraint handling techniques, such as rejection of unfeasible solutions, penalty function, etc. The obvious difference between various types of evolutionary algorithms is in encoding mechanism of candidate solutions, i.e. strings over a finite alphabet in genetic algorithms (GAs); real-valued vectors in evolution strategies; finite state automata in evolutionary programming and trees in genetic evolutionary programming. In particular, GA's encoding mechanism allows easy implementation of specific constraints in supply chain planning problems.

The need for hybridization of Pareto-based search algorithms with other methods and techniques is widely discussed in literature (Hiroyasu et al 1999). In this paper, hybridisation of Pareto-based genetic algorithms with response surface-based metamodelling techniques applied to simulation optimisation is investigated.

Response surface methodology (RSM) is a collection of statistical and mathematical techniques for optimisation of stochastic functions. RSM is developed in order to analyse experimental data and to build empirical models based on observations of the stochastic function. In case of a computer simulation model, simulation output dependence on its input variables could be interpreted by a response surface function. The main advantage of RSM methodology (Merkuryeva 2005) is its applicability for a small number of observations in conditions of costly and time-consuming simulation experiments. RSM-based optimization is based on local approximation of the simulation response surface by a regression type metamodel in a small region of independent factors. Exploration and optimisation of the resulting response surface function approximations provide effective tools for iterative optimisation of the simulation response function applicable for a small number of simulation experiments.

For the last years, there has been an increasing attention placed on the performance, design, and analysis of multi-echelon supply chains (Merkuryev et al 2008). Optimisation of multi-echelon cyclic plans in supply chains that is performed in the case study refers to the class of multi-objective optimisation problems, which are usually characterised by a large search space of decision variables, conflicting and stochastic objectives etc. While there is no a single optimal solution for a number of conflicting objectives, the development of an algorithm, which gives a large number of alternative solutions lying near the Pareto-optimal front and tackles the variations of a response generated from the uncertainties in the environment variables, is of great practical value.

The case study is aimed to find the optimal parameters of a multi-echelon cyclic plan for each of supply chain nodes in order to minimise the supply chain total cost and maximise end-customers fill rate taking into account cyclic planning constraints and assumptions of stochastic demand and backordering.

A network simulation model (Merkuryeva and Napalkova 2009) is built as a process-oriented model with a one-directional flow of goods. It is represented by two types of nodes: stock points and processes. The stock points correspond to stock keeping units, while the processes denote transformation activities (assembly, transportation and packaging operations).

2. OPTIMISATION PROBLEM

The stochastic optimisation problem is formulated as follows:

$$\text{Min } E[\mathbf{f}(\mathbf{x})] = E[f_1(\mathbf{x}),\ldots, f_M(\mathbf{x})], \quad (1)$$
$$\text{subject to: } \mathbf{g}(\mathbf{x}) = E[\mathbf{r}(\mathbf{x})] \le 0 \text{ and } \mathbf{h}(\mathbf{x}) \le 0,$$

where $E[\cdot]$ is a mathematical expectation; $\mathbf{x} = (x_1,\ldots x_K) \in X$, $\mathbf{f} = (f_1,\ldots,f_M) \in Z$; K is the number of decision variables; M is the number of objective functions; X is the decision space; Z is the objective space; $\mathbf{x}$ is a vector of decision variables; $\mathbf{f}$ is a vector of objective functions; $\mathbf{g}$ is a vector of stochastic constraints; $\mathbf{h}$ is a vector of deterministic constraints on the decision variables; $\mathbf{r}$ is a random vector that represents several responses of the simulation model for a given $\mathbf{x}$. Decision variables could be of different types (i.e. discrete, continuous) and have metrics with different ranges of possible values. Proceeding from (1), the solution of multi-objective simulation-based optimisation problem is interpreted as a vector of decision variables $\mathbf{x}$ that satisfies all feasible constraints and provides the best trade-off between multiple objectives.

To describe the objective vector function, one could use traditional methods of aggregating multiple objectives into a single one, or optimising the most important objective while treating others as constraints. The main strength of these techniques is their computational efficiency and simple implementation. The weakness is the difficulty to determine a priori information about the objectives, such as weight coefficients that reflect a relative importance of each criterion or their ranks. Moreover, this approach can produce only one optimal solution during a single experiment, which may not be the best trade-off. However, in multi-objective optimisation problems, each objective function could have individual optimal solution and none of them can be considered to be better than any other with respect to all objective functions. Therefore, in order to find trade-off solutions this paper applies the principles of Pareto optimality. The dominance relation can be formulated in the following way:

A trade-off solution $\mathbf{x}^* \in X$ *is said to dominate a solution* $\mathbf{x} \in X$ *iff* $\forall i \in \{1,\ldots,M\}: f_i(\mathbf{x}^*) \le f_i(\mathbf{x})$ *and* $\exists j \in \{1,\ldots,M\}: f_j(\mathbf{x}^*) < f_j(\mathbf{x})$.

As it follows from the definition, a solution $\mathbf{x}^*$ is Pareto-optimal if it is not worse than any other solutions for all criteria and is better for at least one criterion. For simplification, it is assumed here that all objective functions are minimised.

In the case study, two objective functions that define the quality of a multi-echelon cyclic plan are introduced. The first one is aimed at minimising the average total cost of the supply chain, which includes the sum of production, inventory and reordering costs. The second objective function is aimed to maximise end-customer service requirements specified by the average order fill rate. The parameters of a multi-echelon cyclic plan identify decision variables. They are: replenishment of cycles Cy_i and order-up-to-levels S_i defined at each stock point i on the network. These variables determine the reorder period and quantity to be ordered or produced for each mature product and are interpreted as discrete and continuous type variables, correspondingly. A large number of decision variables in practice make conducting simulation experiments difficult. Specific constraints are introduced that define cycles by the power-of-two policy, in which cycles are integers and multiples of two.

3. ANALYSIS OF THE PROBLEM SOLVING METHODS

In order to solve the aforedescribed simulation-based optimisation problem (1), an appropriate method should be applied. Based on analysing the properties of the objective functions and decision search space, the following requirements are imposed on the problem solving method: (i) it should converge to the approximate Pareto-optimal front while keeping its diversity, (ii) it should be able to guide the search toward a near-optimal direction using only the numeric values of the multiple stochastic objective functions and constraints, (iii) it should incorporate some techniques for generating statistically significant candidate solutions and (iv) it should manage a search process in such a way that the total number of simulation experiments and, in consequence, the total computational time would be decreased.

The first and second requirements restrict the choice to the class of *direct search methods* that apply the concepts of Pareto-optimality and dominance relation. Pareto-based evolutionary algorithms (EAs) refer to the most efficient representatives of this class. The tremendous advantage of EAs over others is that they evolve multiple parallel solutions instead of a single one that allows generating a set of non-dominated solutions at each iteration. On the other hand, EAs are able to preserve a diverse set of non-dominated solutions using specific mechanisms. In addition, EAs are able to perform a search in a high dimensional space with different ranges for decision variables. Moreover, EAs have proved to be independent on strong problem structure, such as, for example, convexity and discontinuity of the objective function. Also, they allow one to incorporate different constraint handling techniques, such as rejection of unfeasible solutions, penalty function, etc.

Although Pareto-based EAs are powerful algorithms for solving complex multi-objective problems, they are unable to fulfil all of the above-formulated problem requirements. This fact clearly illustrates the need for hybridisation of Pareto-based EAs with others methods and techniques.

Typically, the hybridisation is performed following some predefined scheme. In literature, it is possible to outline three hybridisation schemes, such as *parallel hybridisation*, *sequential hybridisation* and *built-in hybridisation*. Parallel hybridisation requires that the

search space is investigated independently by multiple optimisation methods. For instance, a population can be divided into sub-populations called islands, which are associated with particular objective functions or certain ranges of the Pareto-optimal front. Another example includes dividing the control of genetic operators between computer processors. The parallel hybridisation scheme is implemented in a divided range multi-objective genetic algorithm (DRMOGA) (Hiroyasu et al 1999), parallel strength Pareto multi-objective evolutionary algorithm (PSPMEA) and parallel multi-objective evolutionary algorithm with a hypergraph represented population structure (pMOHypEA).

In using sequential hybridisation, separate methods are sequentially combined based on predefined rules. According to the most widespread implementation of this scheme, Pareto-based EAs are combined with local search based methods. The reason is that EAs have overall global perspective, while the local search based methods have good convergence properties to a local optimal solution and can be used to extensively explore the search space around EA solutions. On this way, the simple multi-objective genetic local search (S-MOGLS) algorithm probabilistically applies the local search to candidate solutions found by the fast elitist non-dominated sorting genetic algorithm (NSGA-II) (Deb et al 2000). In using the local search, multiple objective functions are aggregated based on randomly generated weight coefficients. Another example of implementing the sequential hybridisation scheme is to apply the local search based method after running Pareto-based EA. The weight coefficients used are computed for each EA solution based on its location in the Pareto-optimal front.

Built-in hybridisation concerns with introducing some features into mechanisms of Pareto-based EAs. Recent developments in this domain include the application of fuzzy logic to (i) the dynamical adjustment of the crossover and mutation rates in the NSGA-II algorithm (Deb et al 2000), (ii) selection of more preferable solutions from the Pareto-optimal set based on their degrees of fuzzy optimality and (iii) incorporation of fuzzy ranking scheme, in which dominance degrees are measured by using membership functions.

The aforementioned hybridisation schemes have been mainly tested on deterministic analytical models. However, the simple combination of Pareto-based EAs with the simulation model may not provide efficient results because of time consuming simulation experiments and the simulation noise, which influence the objective function estimates and performance of EA operators. Thus, the ongoing section is dedicated to a hybridised approach to multi-objective simulation-based optimisation, which can be mentioned as a very promising and at the same time poorly investigated field of research.

4. OPTIMISATION ALGORITHM

The proposed simulation optimisation algorithm (Figure 1) is based on integration of the multi-objective genetic algorithm (GA) and RSM-based linear search algorithm (Merkuryeva and Napalkova 2008). While a GA is well suited to solve combinatorial problems and is used to guide the search towards the Pareto-optimal front, the RSM-based linear search is appropriate to improve GA solutions based on the local search.

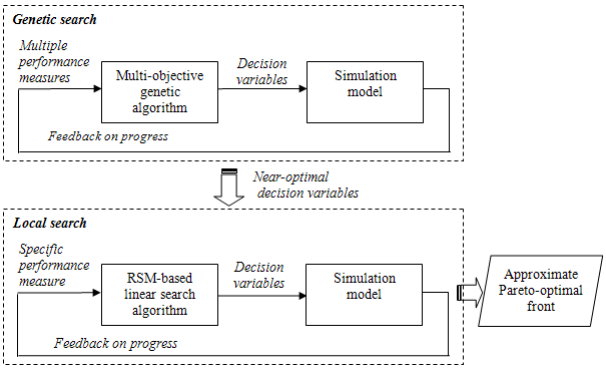

Figure 1: General scheme of the hybrid simulation optimisation algorithm

The multi-objective genetic algorithm starts with generating an initial population of decision variables values (Napalkova and Merkuryeva 2008). In order to smoothly cover the investigated search space, uniform distribution is applied. Decision variables such as cycles are encoded using a modified binary encoding procedure, which satisfy power-of-two synchronisation policy in supply chains. Afterwards, fitness values are defined based on multiple objectives, here by the average total cost and average fill rate that are obtained through simulation experiments. To estimate fitness values of chromosomes, a ranking-based fitness assignment is applied. It concerns the use of a dominance depth that is connected with dividing a population into several fronts in order to represent a front of a certain solution. In order to obtain solutions uniformly distributed over the Pareto-optimal front, the diversity preserving mechanism based on a crowding distance metric is implemented. The crowding distance is an estimate of the density of solutions surrounding the current solution. The larger a crowding distance value becomes, the less crowded an area around the solution is. As a result, every chromosome in the population has the following two attributes: (1) domination depth and (2) crowding distance.

Then, the penalty function is applied to decrease the survival probability of solutions, which provide the average fill rate lower than the pre-defined threshold. In order to choose chromosomes from the current population for breeding purposes, the algorithm applies a crowded two-tournament selection. The main idea of this selection strategy is that a crowded comparison operator is used to compare pairs of chromosomes. From two candidate solutions the one with the lower domination depth is preferable. If both solutions have

the same depth, then the solution with larger crowding distance is selected. The crowded comparison operator ($\geq$) is defined as follows:

$$a \geq b \text{ if } (r_a < r_b) \text{ or } ((r_a = r_b) \text{ and } (\delta_a > \delta_b)), \qquad (2)$$

where r_a and r_b are domination depths, δ_a and δ_b are crowding distances for chromosomes a and b.

After applying the crossover and mutation operators, the new population is replaced by the union of the best parents and offspring to avoid the loss of non-dominated solutions during the evolution process. Domination depths of chromosomes in the combined population are updated. The first N solutions are gathered for the next generation, where N is a population size. This elitist strategy is often called (μ + λ) – selection, where μ and λ assign parents and offspring, respectively. The multi-objective GA is automatically terminated, when the number of generations with stagnant non-dominated set is equal to the predefined value (usually set to 3).

In the local search, the RSM-based linear search iterative algorithm is used to improve decisions solutions of the genetic algorithm by adjusting specific decision variables, e.g. order-up-to levels in supply chains. The algorithm is based on local approximation of the simulation response surface by a regression type meta-model in a small region of independent factors; it integrates linear search techniques for optimising stock points' order-up-to levels. Finally, the approximate Pareto-optimal front initially generated by the GA is updated including solutions found by the response surface-based linear search algorithm.

5. CASE STUDY

The case study is aimed to find an optimal cyclic plan of a chemical product, i.e. liquid based raisin, in order to minimise production, ordering and inventory holding costs, and maximise end-customers fill rate. As a test bed, the chemical manufacturing supply chain is used. The main operations that occur in the supply chain network are the following. In the plant CH, the raw material is converted to the liquid based raisin. It is then either sourced to direct customers or shipped to the plant DE, where other components are added to make different products. From that plant, the end-products are shipped to different types of customers.

The Service Model-based simulation model of the above-described supply chain network is automatically generated in optimisation environment developed in (Merkuryeva and Napalkova 2007). The end-customer demand is normally distributed and cycles are defined according to the power-of-two policy. Cycles are represented in weeks as follows: 7, 14, 28, 56, where 56 days is the maximal cycle that corresponds to one full turn of a "planning wheel". In this business case, specific policies such as nested or inverted-nested ones are not analysed. Order-up-to levels are calculated using analytical formulas, where the cycle service level is set to 95%. Initial stocks are equal to order-up-to levels plus average demand multiplied by cycle delays. Stock point 1 has infinite on hand stock and is not controlled by any policy. Backorders are delivered in full.

Simulation run length is equal to 224 periods. This allows modelling of four full turns of the planning wheel, i.e. 4*56 periods. Number of simulation replications is equal to 5. The GA is executed with the following parameters: the population size is 40; crossover and mutation probabilities are 0.5 and 0.1, correspondingly; a tournament size is equal to 2. The GA works with 66 decision variables (i.e. cycles and order-up-to levels assigned to network stock points). Initial values of order-up-to levels are calculated analytically. When the number of generations with a stagnant non-domination set is equal to 3, the GA is terminated. Figure 2 shows solutions received from the final population.

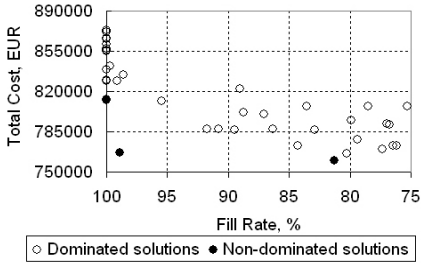

Figure 2: Final GA population

Figures 3 and 4 illustrate the execution of the GA. The average total cost and fill rate of parent chromosomes are plotted against the generation step. The GA makes quick progress at the beginning of the evolutionary process that is typical for genetic algorithms. Then, there are phases when it hits the local optimum before mutations further improve its performance. Finally, the GA finds three non-dominated solutions with the following performance average measures: 1) *total cost* = €787,431, *fill rate* = 100.00%; 2) *total cost* = €766,669, *fill rate* = 98.88%; and 3) *total cost* = €752,300, *fill rate* = 93.76%.

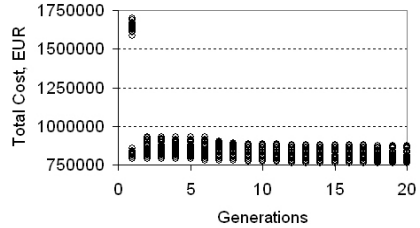

Figure 3: The GA convergence subject to total cost

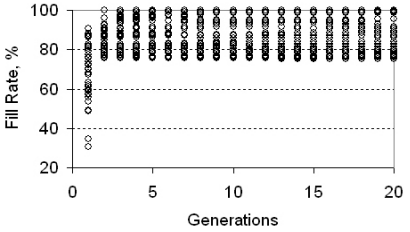

Figure 4: The GA convergence subject to fill rate

The response surface-based linear search algorithm is used to adjust order-up-to levels of three non-dominated solutions received with the GA while fixing stock point cycles. Finally, the average total cost and average fill rate of the second solution are equal to €756,178 and 98.88%, respectively. The updated Pareto-optimal front is given in Figure 5. There are three non-dominated solutions found by the GA, where the second solution is improved by the RSM-based linear search algorithm.

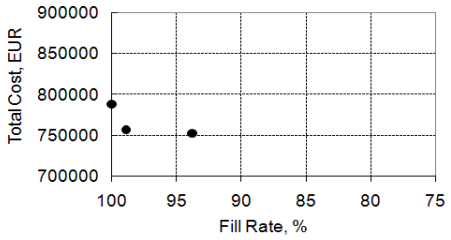

Figure 5: The approximate Pareto-optimal front

CONCLUSIONS

The paper has presented the hybrid simulation optimisation algorithm that integrates the multi-objective genetic algorithm and response surface-based linear search algorithm. Although genetic algorithms are widely applied at solving different real world multi-objective problems, they are often unable to ensure both the convergence to the Pareto-optimal front and its diversity. In this paper, genetic algorithm allows covering a broad region of the search space at each generation, while RSM-based linear search algorithm provides careful investigation of small portions of the search space and improves the current solution by moving to a better "neighbour" solution. The results of the case study have demonstrated performance efficiency of the proposed hybrid algorithm.

REFERENCES

Campbell, G.M. and Mabert, V.A., 1991. Cyclical Schedules for Capacitated Lot Sizing with Dynamic Demands. *Management Science*, 409 – 427.

Deb, K., Agrawal, S., Meyarivan, T., 2000. A Fast Elitist Non-Dominated Sorting Genetic Algorithm for Multi-Objective Optimisation: NSGA-II. *Proceedings of the Parallel Problem Solving from Nature VI Conference*, pp. 849 - 858.

Hiroyasu, T., Miki, M. and Watanabe, S. (1999). Divided range genetic algorithms in multiobjective optimization problems. *Proceedings of International Workshop on Emergent Synthesis (IWES'99)*, pp.57-66.

Merkuryev, Y., Merkuryeva, G., Desmet, B., Jacquet-Lagrèze, E., 2007. Integrating Analytical and Simulation Techniques in Multi-Echelon Cyclic Planning. *Proceedings of the First Asia International Conference on Modelling and Simulation (AMS 2007)*, March 27 – 30, Phuket, Thailand, pp.460 – 464.

Merkuryeva, G., 2005. Response surface-based simulation metamodelling methods with applications to optimisation problems. Chapter 15. In: Dolgui, A., Soldek, J. and Zaikin, O., eds. *Supply chain optimisation Product / Process Design, Facility Location and Flow control*. Springer, 205–215.

Merkuryeva, G., Merkuryev, J. and Napalkova, L., 2007. Simulation-based environment for multi-echelon cyclic planning and optimisation. *Proceedings of the 19th European Modeling and Simulation Symposium*, pp.318-325. October 4-6, Bergeggi, Italy.

Merkuryeva, G. and Napalkova, L., 2009. Supply Chain Cyclic Planning and Optimisation. In: Merkuryev Y., Merkuryeva G., Pierra M.A., Guash A., eds. *Simulation-Based studies in Logistics: Education and Applied Research*. UK: Springer-Verlag, 89.-111.

Merkuryeva, G. and Napalkova, L., 2009. Two-phase simulation optimization algorithm with applications to multi-echelon cyclic planning. *International Journal of Simulation and Process Modelling* (in press).

Merkuryeva, G. and Napalkova, L., 2007. Development of simulation-based environment for multi-echelon cyclic planning and optimisation. *Proceedings of the 6th EUROSIM Congress on Modelling and Simulation*, paper ID 452.

Napalkova, L. and Merkuryeva, G., 2008. Theoretical Framework of Multi-Objective Simulation-Based Genetic Algorithm for Supply Chain Cyclic Planning and Optimisation. *Proceedings of the 10th International Conference of Computer Modelling and Simulation*, pp.467-474. April 1-3, Cambridge, UK.

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