JOINING SIMULATION AND SITUATION AWARENESS FOR AN ITALIAN PRODUCTION SYSTEM

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ABSTRACT

The paper focuses on simulation results for a real supply network, dealing with tomatoes production, which is widespread in Southern Italy. The dynamics of the system is studied via differential equations, that involve either parts on arcs or queues that consider the exceeding goods. Two different numerical schemes are compared to test the approximation degrees, and then used for simulations. Through a procedure of Situation Awareness, a possible choice of the input flow to the supply network is analyzed. The obtained results prove that using Situation Awareness allows, at least for the real system in consideration, good compromises in order to modulate production queues.

Keywords: supply networks, Situation Awareness, differential equations, tomatoes production

1. INTRODUCTION

Italian economy, as well as the wealth of each Italian region, depends on various phenomena that are also connected to the distribution/production of different goods. As the intention is an increment of prestige at international and national levels, a careful attention is always devoted to the analysis of some marketing strategies that foresee the exporting of cultivated goods. Hence, big efforts aim to guarantee a suitable treatment of products via supply networks, by which producers and consumers are both satisfied. Such a situation is highly studied in Campania, an Italian region where production and distribution of tomatoes represent a serious issue. Indeed, the primary key factor is the production, as its possible delays have consequent negative impacts on the delivery in the foreseen times.

The aim of this paper is to focus on supply networks for tomatoes, also considering environment factors that always establish a hard constraint in terms of input flows to production systems. In particular, supply networks are modelled using a fluid-dynamic model, based on Partial and Ordinary Differential Equations (PDEs, ODEs). Input flows to such networks are chosen by an approach of Situation Awareness (Endsley 1995, Endsley 2015, Wickens 2008), applied to the tomatoes production. The advantages are evident: on one hand, the fluid-dynamic model allows focusing on time-space dynamics of goods; on the other hand, Situation Awareness establishes, considering environment parameters, possible correct inputs to the production systems in order to avoid unsuitable situations, such as remainders of goods to process.

Supply networks and their behaviors have been studied by different mathematical models, see for instance Daganzo 2003, Helbing 2005, Kleijnen 2003, Longo 2008 and Wang 2010. Some approaches are discrete and based on dynamics of individual parts; others are continuous and deal with differential equations (see, for example, Cascone 2008 and Manzo 2012 for applications to road networks). The first work in this direction is by Armbruster, Degond and Ringhofer (Armbruster 2006), who used a limit procedure to obtain a conservation law (Bressan 2000a, Bressan 2000b and Dafermos 1999), which refers to densities of parts. Other papers have been introduced to focus on further phenomena of supply systems (Armbruster 2007, Armbruster 2003). In our case, we consider the model proposed in Göttlich 2005 and Göttlich 2006, where conservation laws for densities of parts and queues for each supplier are analyzed. Considering various discretizations for PDEs (examples are in Canic 2015, De Falco 2016 and Leveque 2002), two different numerical schemes are analyzed for the fluid-dynamic model: an Upwind-Euler approach (precisely, Upwind method for PDEs and Euler scheme for ODEs), with different space meshes and a fixed time grid mesh to overcome problems of not rational ratios for lengths of suppliers (details are in Cutolo 2011); a Differential Quadrature (DQ) approach, firstly introduced in Bellman 1971 and considered as higher order Finite Differences (FDs), see Shu 2000, Shvartsman 2016, Tomasiello 2013. DQ based methods have found many applications in science and engineering (De Rosa 1998, De Rosa 2007, De Rosa 2016, Fantuzzi 2016, Kamarian 2016, Tomasiello 2007, Tornabene 2016, Tornabene 2015a, Tornabene 2015b), because of the improved computational efficiency.

As there is the exigency of controlling the production processes and hence the input flows to the supply networks, Situation Awareness is necessary. As for Endsley's opinion (Endsley 1995), Situation Awareness deals with "the perception of the elements in the environment within a volume of time and space, the comprehension of their meaning, and the projection of their status in the near future". Such a model deals with three different levels, useful to plan decisions on input flows of supply systems: perception, by which elements of the environment are perceived; comprehension, that considers which data from the environment are useful for the goals to achieve; projection, which is the capability of projecting the recognized elements in the future. Great advantages of Situation Awareness are simply obtained focusing on the characteristics of the three levels, useful to take decisions for particular domains. An example is in D'Aniello 2016, where the authors discuss a possible Decision Support System for smart commerce environment.

Finally, simulations are made: first, the Upwind-Euler method and the Differential Quadrature approach are compared, showing that they have similar characteristics in terms of numerical approximations and computational times. Then, starting from the numerical results, a real example of supply system for tomatoes production is considered. In this case, input flows are chosen in two different cases: decisions planned by the leadership of a little business company in Campania region (Italy); decisions obtained by a model of Situation Awareness, considering real environment data. It is proven that Situation Awareness is useful to accelerate the system dynamics, in terms of emptying of queues in some parts of the system.

The outline of the paper is the following. Section 2 presents Situation Awareness within the context of tomatoes production. Section 3 considers the mathematical model for supply networks. Section 4 deals with numerical methods for the proposed model: Upwind-Euler with different space meshes for different suppliers; Differential Quadrature rules. Finally, Section 5 contains the simulation results: first, numerical errors for the described numerical approaches are considered; then, the case study of a supply network for tomatoes is presented. Conclusions end the paper in Section 6.

2. SITUATION AWARENESS FOR TOMATOES PRODUCTION

In this section, we discuss a possible application of Situation Awareness using the Endsley's model (Endsley 1995), that is contextualized to processes for tomatoes production within a real little business company in Campania region (Italy). The overall approach is in Figure 1.

In detail, the *environments* consist of conditions by which high quality tomatoes depend, namely: presence of wind, humidity for arable fields and weather. In such a framework, a *situation* describes a state for a good growth of tomatoes and has three different phases:

- *Perception*: environment data are obtained and kept.
- *Comprehension*: data of the previous step are elaborated. This operation represents a serious issue, as combinations of parameters for tomatoes growth imply possible forecasts on the quality of goods. In this work, the comprehension step is made by analysing time series.
- *Projection*: results of the second phase are used to plan possible future decisions. The effect of this phase is to define a Decision Support System (DSS).

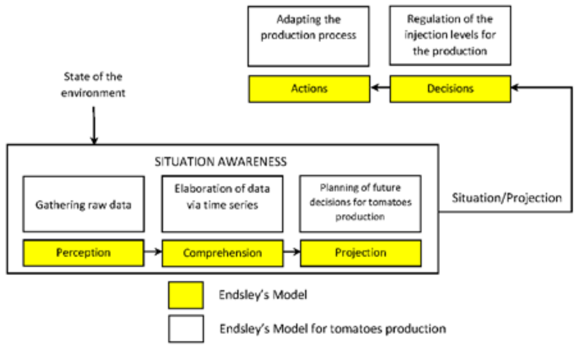

Figure 1: Situation-aware Decision-making process

The DSS, not described in detail here, represents a support for the leadership of the business company. The DSS has rules, based on Fuzzy Logic, that allow to understand possible correct levels of injection to the production networks.

3. A MODEL FOR SUPPLY NETWORKS

In this section, we present an ODE-PDE model for supply networks (see Göttlich 2006), based on the analysis of Armbruster 2006 and described for supply chains in Göttlich 2005. The model considers: a conservation laws formulation for density of parts over the suppliers; time - dependent queues for the transition of parts among suppliers; distribution coefficients which indicate how the outgoing flows from a given node distribute over suppliers which are downstream.

A supply network is a directed graph with a set of arcs J and a set of vertices V.

Each arc $j \in J$ is parameterized by a real interval $\left[a_j, b_j\right]$, represents a supplier (possibly having infinite endpoints) and is considered: incoming if $b_j < +\infty$; outgoing if $a_j > -\infty$. For each outgoing arc $j \in J$, there exists a queue.

Each vertex $v \in V$ is connected to a set of incoming arcs $Inc(v) \subset J$ and a set of outgoing arcs $Out(v) \subset J$.

There are distributions coefficients $(\alpha_{v,j})_{j \in Out(v)}$ such that $\alpha_{v,j} \in]0,1[$ and $\sum_{j \in Out(v)} \alpha_{j,v} = 1 \quad \forall \quad v \in V$.

Notice that the coefficient $\alpha_{v,j}$ indicates the percentage of flux outgoing from v and directed to the supplier j.

For each arc $j \in J$, indicate by: $\mu_j > 0$ the maximum processing capacity; $L_j > 0$ the length; $T_j > 0$ the processing time; $v_j := L_j/T_j$ the processing velocity; $\rho_j(t,x) \in [0, \rho_j^{\max}]$ the density of parts at point x and time t; $f_j(\rho_j(t,x)) := \min\{\mu_j, v_j\rho_j(t,x)\}$ the flux function.

Considering that parts over each arc $j \in J$ are processed with velocity v_j and with a maximal flux μ_j, the phenomenon is defined by the conservation law:

$$\partial_t \rho_j(t,x) + \partial_x f_j(\rho_j(t,x)) = 0, \quad \forall x \in [a_j, b_j], \quad t > 0, \tag{1}$$

$$\rho_j(0,x) = \rho_{j,0}(x) \geq 0, \quad \rho_j(t,a_j) = \frac{f_{j,inc}(t)}{v_j}, \tag{2}$$

where $\rho_{j,0}$ and the inflow, $f_{j,inc}(t)$, have to be assigned.

If, for some vertex $v \in V$, we get that arc $j \in Out(v)$, we have a time dependent queue, $q_j(t)$, that obeys this equation:

$$\frac{d}{dt} q_j(t) = \alpha_{v,j} \sum_{k \in Inc(v)} f_k(\rho_k(b_k,t)) - f_{j,inc}(t). \tag{3}$$

For each arc $j \in J$, we assume that:

$$f_{j,inc}(t) := \begin{cases} \varphi_j(t), \text{ if } a_j = -\infty, \\ min\left\{\alpha_{v,j} \sum_{k \in Inc(v)} f_k(\rho_k(b_k,t)), \mu_j\right\}, \text{ if } q_j(t) = 0, \\ \mu_j, \text{ if } q_j(t) > 0. \end{cases} \tag{4}$$

where $\varphi_j(t)$ is an assigned input profile on the left boundary $\{(a_j,t) : t \geq 0\}$.

The following holds:

Lemma 1. Consider a solution to (1), (2), (3) and (4), given by $\rho_j(t,x)$ and $q_j(t)$. Then, $\rho_j(t,x) \geq 0$, $q_j(t) \geq 0$ for every $j \in J$, $t \geq 0$ and x.

Proof. As $\alpha_{j,v} > 0$, the inflows (4) are positive and the fluxes f_j vanish at 0, the density ρ_j is always positive by the comparison principle of conservation laws. Equations (3) and (4) ensure that $q'(t) > 0$ when $q(t) = 0$, hence the conclusion follows.

Remark 2. Lemma 1 also holds for supply chains, considering $\alpha_{j,v} = 1$.

4. NUMERICAL SCHEMES

We deal with some numerical schemes for the model of Section 3, in order to define approximations of densities and queues. Two possible alternatives are considered:

- UE scheme: Upwind method for the PDE (1) and explicit Euler for the ODE (3) in case of different space meshes, see further details in Cutolo 2011.
- DQ scheme: Differential Quadrature rules (e.g. see Tomasiello 2007, Tomasiello 2013).

4.1. UE Schemes

In this Section we introduce the Upwind-Euler method with different space meshes for different suppliers. This is useful either when L_j have not rational ratios or when computational complexity reductions are needed, see Cutolo 2011.

For each arc $j \in J$, define a numerical grid in $[0, L_j] \times [0, T]$ with points $(x_i, t^n)_j$, $i = 0, ..., N_j$, $n = 0, ..., \eta_j$, where: N_j is the number of segments into which the j-th supplier is divided; η_j is the number of segments into which $[0,T]$ is divided.

Then, we indicate by: ${}^j\rho_i^n$ the value assumed by the approximated density at the point $(x_i, t^n)_j$; q_j^n the value of the approximate queue buffer occupancy at time t^n.

For each supplier $j \in J$, set a fixed time grid mesh Δt and different spaces grid meshes $\Delta x_j = v_j \Delta t$. Then, grid points are $(x_i, t^n)_j = (i\Delta x_j, n\Delta t)$, $i = 0, ..., N_j$, $n = 0, ..., \eta_j$. The Upwind scheme for the parts density of arc j reads as:

$${}^j\rho_i^{n+1} = {}^j\rho_i^n - \frac{\Delta t}{\Delta x_j} v_j \left({}^j\rho_i^n - {}^j\rho_{i-1}^n\right), \; j \in J, \tag{5}$$

$i = 0, ..., N_j$, $n = 0, ..., \eta_j$. Notice that, as $\Delta t = \min\left\{\frac{\Delta x_j}{v_j} : j \in J\right\}$, the CFL condition (see Leveque 2002) is automatically satisfied.

For queues, if $a_j < -\infty$, the explicit Euler method gives:

$$q_j^{n+1} = q_j^n + \Delta t \left[\alpha_{v,j} \sum_{k \in Inc(v)} f_k({}^k\rho_{N_k}^n) - f_{j,inc}^n\right], \tag{6}$$

$n = 0,\ldots,\eta_j$, where:

$$f_{j,inc}^n = \begin{cases} min\left\{\alpha_{v,j} \sum_{k \in Inc(v)} f_k({}^k\rho_{N_k}^n),\ \mu_j\right\},\ q_j^n(t) = 0, \\ \mu_j,\ q_j^n(t) > 0. \end{cases} \qquad (7)$$

As we need ${}^j\rho_i^n \geq 0$, $q_j^n \geq 0$ $\forall$ $n \geq 0$, $j \in J$, $i = 1,\ldots,N_j$, the following numerical correction for $f_{j,inc}^n$ is provided:

$$f_{j,inc}^n = \frac{\Delta t' \mu_j + (\Delta t - \Delta t')\alpha_{v,j} \sum_{k \in Inc(v)} f_k({}^k\rho_{N_k}^n)}{\Delta t}, \qquad (8)$$

where:

$$\Delta t' := \frac{q_j^n}{\mu_j - \alpha_{v,j} \sum_{k \in Inc(v)} f_k({}^k\rho_{N_k}^n)}. \qquad (9)$$

Finally, if $a_j = -\infty$, suitable boundary data are needed, using ghost cells and the expression of inflows given by $\varphi_j(t)$, see equation (4).

4.2. DQ Scheme

The numerical scheme is defined through DQ rules (Bellman 1971), applied to approximate the derivatives of the equations.

In what follows, first we consider a short overview of DQ rules. Then, a suitable scheme for the model of Section 3 is provided.

4.2.1. Differential Quadrature rules: an overview

Consider a continuous function $u(x)$ in the interval $I = [0, L]$, whose a fixed and arbitrary partition is $0 = x_1 < x_2 < \ldots < x_M = L$. The DQ rules allow to approximate the r-th order derivative of $u(x)$ by a weighted sum of the functional values at the grid points $u_j = u(x_j)$ as:

$$\frac{d^r}{dx^r} = \sum_{j=1}^{M} A_{ij}^{(r)} u_j, \quad i = 1,\ldots,M. \qquad (10)$$

The weighting coefficients are computed as follows, see Shu 2000: with regard to the first–order derivative weighting coefficients, we get:

$$A_{ij}^{(1)} = \frac{\prod_{p=1,\,p\neq i}^{M} (x_i - x_p)}{(x_i - x_j) \prod_{p=1,\,p\neq i}^{M} (x_i - x_p)},\ i,\ j = 1,2,\ldots,M,\ j \neq i; \qquad (11)$$

with regard to the higher–order derivative weighting coefficients ($2 \leq r \leq M-1$), we have:

$$A_{ij}^{(r)} = r\left[A_{ii}^{(r-1)} A_{ij}^{(1)} - \frac{A_{ij}^{(r-1)}}{(x_i - x_j)}\right],\ i,\ j = 1,2,\ldots,M,\ j \neq i; \qquad (12)$$

on the other hand, for $i = j$ ($1 \leq r \leq M-1$):

$$A_{ii}^{(r)} = -\sum_{p=1,\,p\neq i}^{M} A_{ip}^{(r)},\ i = 1,2,\ldots,M. \qquad (13)$$

As for the partition, it can be uniform or not. An usual choice for non-uniform partitions is given by the Gauss-Chebyshev-Lobatto (GCL) distribution:

$$x_i = \frac{1}{2}\left[1 - \cos\frac{(i-1)}{(M-1)}\pi\right],\ i = 1,2,\ldots,M. \qquad (14)$$

The approximate solution, which is obtained by applying the DQ rules, is in general written as (Tomasiello 2007):

$$\overline{u}(x) = \mathbf{V}(x)^T \mathbf{d}, \qquad (15)$$

where $\mathbf{d}^T = (u_1,\ldots,u_M)$, while $\mathbf{V}(x)$ is the shape functions vector of which the j-th element is:

$$V_j(x) = \delta_{1j} + \sum_{r=1}^{M-1} A_{1k}^{(r)} \frac{x^r}{r!}, \qquad (16)$$

being δ_{1j} the well–known Kronecker operator. Notice that equation (15) (with equation (16)) expresses an $M-1$ terms Taylor expansion around $x = 0$. So, the residual is $O(x^M)$ (see also Shu 2000).

4.2.2. The discretized equations

Let us consider equation (1). By applying DQ rules to the spatial derivative, we get for the j-th arc:

$$\partial_t \rho_j(t, x_i) + \sum_{k=1}^{M_j} A_{ik}^{(1)} f_j(\rho_j(t, x_k)) = 0,\ i = 1,\ldots,M_j. \qquad (17)$$

By discretizing with respect to the time simply by conventional FDs, as in the UE scheme, we finally obtain:

$$ {}^{j}\rho_i^{n+1} = {}^{j}\rho_i^{n} - \Delta t \sum_{k=1}^{M_j} A_{ik}^{(1)} f_j\left(\rho_j\left(t_n, x_k\right)\right) = 0, \qquad (18) $$

$i = 1,\ldots,M_j$, $n = 0,\ldots,\eta_j$. For the queues, we apply the Euler scheme as described in Section 4.1.

As one can notice, the main difference with the UE scheme is in the fact that in the DQ scheme we have M_j (in general not equally spaced) grid points over the spatial axis instead of N_j intervals with length Δx_j.

5. SIMULATIONS

This section is devoted to the presentation of simulation results for the dynamics of queues and parts on supply networks. In particular, after considering a short case study to analyze the goodness of approximations for different numerical approaches, the attention is focused on a supply network to simulate a typical process for the tomatoes treatment.

5.1.1. Test 1 – Comparison between UE and DQ schemes

In what follows, we consider a supply chain with $N = 6$ suppliers, whose parameters (lengths, processing times and maximal fluxes) are in Table 1.

Table 1: Parameters of the supply chains

Supplier j	L_j	T_j	μ_j
1	1	1	40
2	0.5	1	35
3	1.5	3	30
4	2	4	15
5	0.5	1	10
6	0.5	1	5

For the simulation results, we assume:

- Empty suppliers and queues at $t = 0$.
- Total simulation time $T = 350$.
- Inflow $\varphi(t)$ for the first supplier defined as:

$$ \varphi(t) := \begin{cases} 15, \ 0 \le t \le 20, \\ 5 + \dfrac{t}{2}, \ 20 < t \le 50, \\ 60 - \dfrac{3}{5}t, \ 50 < t \le 100, \\ 0, \ 100 < t \le T. \end{cases} \qquad (19) $$

Using the UE scheme with $\Delta t = 0.0125$ and $\Delta x_j = v_j \Delta t$, $j = 1,...,6$, we have the queue buffer occupancies in Figure 2. Notice that the various queues decrease with slow velocities as $\mu_2 > \mu_3 > \mu_4 > \mu_5 > \mu_6$. Although the processing velocities v_j, $j = 2,...,6$, are the same, q_6 is the highest queue.

The same test is made by the DQ scheme, using $M = 6$ CGL grid points on each arc. Figure 3 shows the behaviour of the queue buffer occupancy $q_4(t)$. Evident similar numerical approximations are obtained.

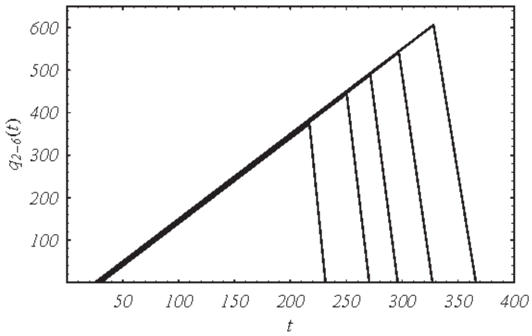

Figure 2: Queues $q_j(t)$, $j = 2,...,5$; $q_2(t)$ is the first on the left; $q_3(t)$ is the second on the left, and so on

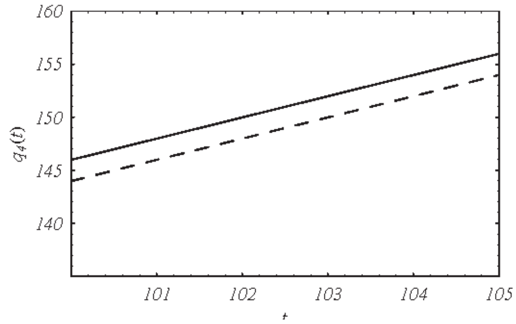

Figure 3: Numerical approximation of $q_4(t)$ via different numerical methods: UE scheme (continuous line); DQ scheme (dashed line)

Further remarks about the convergence errors (see Cutolo 2011 and De Falco 2016) are useful to compare the numerical methods, see Table 2.

Table 2: Errors for UE and DQ schemes

Δt	Errors for UE	Errors for DQ
0.00625	0.01175	0.01554
0.0125	0.02512	0.02812
0.025	0.07822	0.08212

From the obtained results, we get that the considered schemes have similar characteristics as for goodness of approximation.

5.1.2. Test 2 – Simulation of a process for treating tomatoes

We analyze some simulation results of a real supply network, that deals with tomatoes, see Figure 4. The network is used inside a little business company in Campania region (Italy) and, following the interpretation given in Göttlich 2006, each arc is either a conveyor belt or a machine.

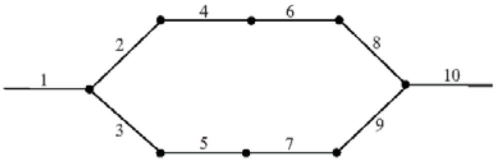

Figure 4: Supply network for tomatoes

In this case, the roles of each arc are described as follows. Arc 1 is a conveyor belt that transports tomatoes. According to a distribution coefficient 0.5, tomatoes are equally distributed to arcs 2 and 3, conveyor belts useful to discriminate goods for the production of peeled and diced tomatoes, respectively. Arcs 4 and 5 are machines with the same function, namely: peeling and skins separations for tomatoes. Arcs 6 and 7 work for a suitable selection of the processed tomatoes. Arcs 8 and 9 consider the closure of containers for tomatoes. Finally, arc 10 is useful for palletizing operations.

The supply network is simulated by the UE scheme with $\Delta t = 0.00625$ and the following parameters for the arcs: $L_i = T_i = 1$, $i = 1,...,10$; $\mu_1 = 500$; $\mu_{10} = 15$; $\mu_j = 34 - 2j$, $j = 2,...,9$; $\rho_i(0,x) = 0$, $i = 1,...,10$; $q_i(0) = 0$, $i = 2,...,10$; total simulation time $\overline{T} = 400$; input profile for arc 1 given by:

$$\varphi(t) = \begin{cases} t, & 0 \le t \le 70, \\ 70, & 70 < t \le 80, \\ 80, & 80 < t \le 150, \\ 0, & t > 150. \end{cases} \quad (20)$$

Function (20) is chosen considering the real cases of production inside the business company in discussion, namely: tomatoes are injected inside the system following, first, a linear increasing profile (hard injection); then, constant ones; finally, a decreasing one (light injection).

In Figures 5–7, queues are depicted. Notice that $q_2(t)$ is smoother than other queues as arc 2 receives directly goods from arc 1. Slopes of queues $q_j(t)$, $j = 3,...,9$, are quite different from the one of $q_{10}(t)$, due to the values of μ_j, $j = 1,...,10$. Moreover, although (20) is zero $\forall \; t > 150$, queues dynamics is very slow. This is confirmed by $q_{10}(t)$ that vanishes at a time instant, which is about 390, much higher than 150.

In Figure 8, we have the density of arc 3 for various instants of time. Unlike the behaviour of queues, arcs become full in a short time, i. e. arc 3 at $t = 40$ already reaches the maximal density (coincident with μ_3 as $v_3 = 1$).

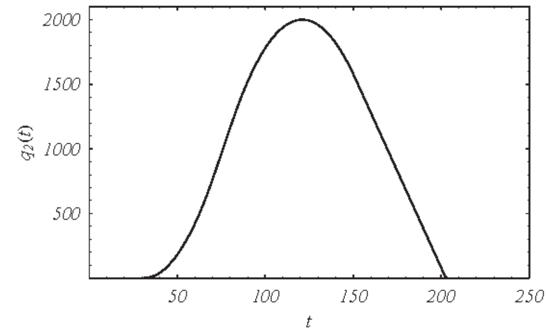

Figure 5: Behaviour of $q_2(t)$

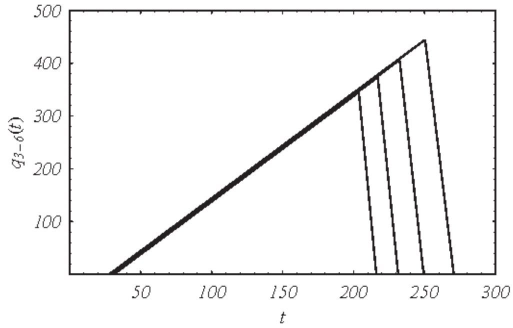

Figure 6: Dynamics of queues $q_j(t)$, $j = 3,...,6$; $q_3(t)$ is the first on the left; $q_4(t)$ is the second on the left, and so on

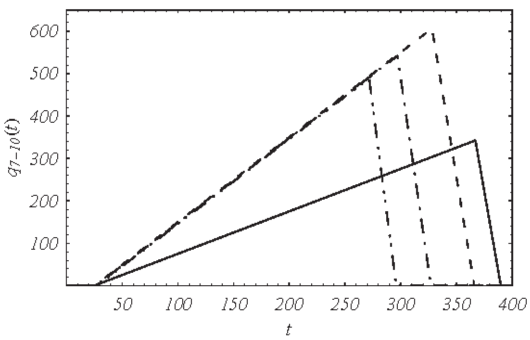

Figure 7: Queues $q_j(t)$, $j = 7,...,10$; $q_7(t)$: dot dot dashed line; $q_8(t)$: dot dashed line; $q_9(t)$: dashed line; $q_{10}(t)$: continuous line

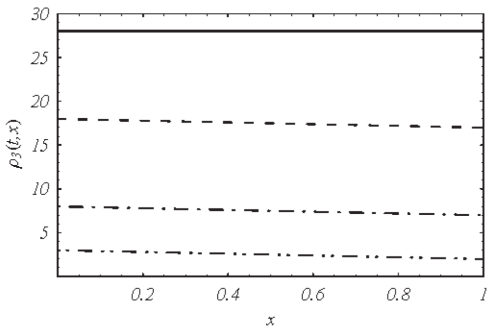

Figure 8: Density $\rho_3(t,x)$ for different time instants; $t = 5$ (dot dot dashed line); $t = 10$ (dot dashed line); $t = 20$ (dashed line); $t = 40$ (continuous line)

Notice that the presence of queues is principally due to $\varphi(t)$, v_j and μ_j, $j = 1,...,10$. In general, as it is not possible to redesign the system in terms of lengths, processing times and maximal fluxes, the levels of $\varphi(t)$ determine the dynamics of the overall supply network. A possible optimization of supply systems modelled by a fluid-dynamic approach is nowadays still under investigation, especially in terms of criteria for a correct choice of $\varphi(t)$ in order to erase queues.

In our case, using an approach based on Situation Awareness, the aim is to establish an alternative choice for $\varphi(t)$. From environment data and using the procedure described in Section 1, we get that $\tilde{\varphi}(t)$, the new choice of $\varphi(t)$, foresees only constant levels, namely:

$$\tilde{\varphi}(t) = \begin{cases} 30, & 0 \le t < 45, \\ 80, & 45 \le t \le 90, \\ 50, & 90 < t \le 150, \\ 10, & t > 150. \end{cases} \tag{21}$$

Decisions for (21) are due to typical Italian weather conditions in months useful for tomatoes, from May to September. Assuming that the time t is expressed in days, a Situation Awareness procedure suggests to have constant levels of injections, that obey the following criteria: a light injection from May 1st to June 15th (about 45 days); a high profile from June 15 to July 30th; a medium injection from August 1st to September 30th; a low profile from September 30th.

Indeed, performances due to $\varphi(t)$ and $\tilde{\varphi}(t)$ are evaluated via the cost functional:

$$J := \frac{1}{10}\sum_{i=1}^{10}\left(\int_0^{\bar{T}} q_i(t)\,dt\right), \tag{22}$$

that measures the average area due to queues inside the supply system. As queues are strictly dependent on the input profiles, different values occur for the choices $\varphi(t)$ and $\tilde{\varphi}(t)$. We get that:

$$J(\varphi(t)) = 149889, \quad J(\tilde{\varphi}(t)) = 112742. \tag{23}$$

As expected, the adoption of a new profile is not suitable to erase queues in the overall system. Indeed, choosing $\tilde{\varphi}(t)$ allows only to decrease the contribution of queues. This last aspect is still under investigation.

6. CONCLUSIONS

Focusing on the model for supply systems proposed in Göttlich 2005 and Göttlich 2006, a real case of production network for tomatoes has been studied.

First, two numerical approaches have been proposed, proving that different schemes produce the same orders of approximation.

Then, using a procedure based on Situation Awareness, the simulations have showed that input profiles are able to modulate production queues, but not to erase them completely.

Further studies, based on Situation Awareness and Fuzzy Logic for the comprehension and the projection phases, are going to be developed in order to obtain robust optimization criteria for the performances of supply networks.

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