Ambulance location using linear programming: The case of the National Autonomous University of Mexico (UNAM)

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ABSTRACT

Every year the Universidad Nacional Autónoma de Mexico (UNAM) holds four entry exams for higher education and high school, for which the candidates do a written exam in any one of 25 schools in the Metropolitan Area of the Valley of Mexico (ZMVM) that the UNAM uses to hold its entry exams. A medical services module manned by a doctor and nurse with an equipped first-aid kit is installed in each school. Moreover, five properly equipped ambulances are available and distributed around the Metropolitan Area to reinforce this medical attention as regards the treatment and transfer of patients. At the present time the five ambulances are located empirically based on the experience of the Emergency Medical Technicians (TUMs) so that the distance between the current location of the ambulance and the modules or schools is what they believe to be the shortest. This study proposes a three-step methodology for optimizing the location of five ambulances based on the Location Set Covering Problem (LSCP) model and the *p*-median model. The solution compares these two models, allowing us to shorten the response time of the Pre-Hospital Care (PHC) service by locating the five ambulances better, considering a variety of speeds and coverage times. This enables us to lower the rate of patient mortality and morbidity.

Keywords: Coverage models, p-median, Pre-Hospital Care, Ambulance location, Heuristic algorithms.

1 INTRODUCTION

The UNAM, in conjunction with eight other institutions, forms part of the admissions processing organization known as the High School and Higher Education Entry Examination Board for the Metropolitan Area of the Valley of Mexico (COMIPEMS), that gives people who are interested in continuing their high school or university studies the opportunity to enter a public school in the open or regular system. The UNAM uses 25 colleges throughout the Metropolitan Area for every entrance exam. The UNAM Medical Services Bureau (DGSM) is in charge of the pre-hospital care service (PHC), which consists of installing a medical service module in every college and deploying five ambulances to provide emergency services. The module consists of a doctor, a nurse and a first-aid kit. The ambulances are properly equipped and capable of providing service to the 25 schools where the entrance exam is being applied. The ambulances are staffed by two emergency medical technicians.

Pre-hospital care is a service that is based on timely medical attention and the immediate transfer of patients to a hospital. It is offered to the sick or injured who are not inside an institution where they could be given the medical service they require, its primary purpose is to take care of and transfer the and sick or injured person to the respective hospital and to treat them in the shortest time possible, thus contributing to lowering the rate of mortality and lessening any sequels that patients could have as a result of organ failure.

2 DESCRIPTION OF THE PROBLEM

The demand for places has grown year by year. Just in the last entrance exam there were 120 thousand candidates (Olivares, 2014) representing a large number of applicants for very few places, as the UNAM can only offer seven thousand places (Olivares, 2014). Thus candidates have to have very good exam results to get a place in this institution. This means that examinees and their relatives are faced with a stressful situation that can give rise to medical emergencies. According to the PHC Coordinator of the Medical Services Bureau, the main ones are: crises in diabetics, road accidents, sunstroke, anxiety attacks, hypoglycemia, convulsions and heart problems. The first three happen more often to the relatives while the rest are problems suffered by the candidates. Based on information provided by the UNAM Entrance Examination Registration and Application Office (SRAEA), we were able to obtain an approximate number of the incidents that occurred in each school during the 2009 admission process. Unfortunately the SRAEA refused to give us any further information, so the study only focuses on that year. However, the information has proven very useful by giving us a parameter of approximate demand in each school.

The Emergency Medical Technicians experimentally establish five regions in the Metropolitan Area, locating one ambulance in each region with the corresponding schools to be served, thus shortening the response time for any incident. The Emergency Medical Technicians makes sure that there is at least one public hospital, belonging to the Institute of Social Services for State Employees (ISSSTE), the Mexican Institute of Social Security (IMSS) or the Government of the Federal District, in each region. There are two shifts for each entrance exam: the first shift from 7.00 a.m. to 2.00 p.m. and the second from 2.00 p.m. to 9.00 p.m.

2.1 Objective of the study

This study proposes the minimization of the response time of the pre-hospital care (PHC) service by locating the ambulances based on the model of the location set covering problem (LSCP) and the p-median model.

3 LOCATION MODELS

Location problems arise from the need to find the most convenient place to locate facilities such as: distribution centers, production plants, garbage dumps, fire, police and ambulance stations, among many others. In general terms, the problem can, according to (Daskin 1995), be expressed as: Given the location of each user, demand and costs (time, distance, etc.) of transport in the region in question, the number of services, the geographical location and capacity of each must be determined in order to optimize the costs of transport, operation, etc.

Location models have been studied since the 1970s. The best known basic problems are cited by Current (2002). Daskin (1995) classifies location models as: continuous, network and discrete models. The models based on coverage and the models based on the p-median are to be found within the discrete models. In this project we are focusing on the LSCM and *p*-median models

3.1 Review of the literature on ambulance location models

The optimization of the PHC system or Emergency Medical Service (EMS) is a subject that has been studied since the middle of the 1960s. It is a very attractive area for applied mathematics and operational research. Some of the reasons for the attraction are that it is a very important issue for society. On the one hand, owing to the high costs involved in maintaining the equipment and having the highly qualified personnel that is required, it is vitally important to guarantee that the best use possible is made of the available resources (Restrepo, 2008). On the other hand, these are deep, interesting problems from a mathematical point of view, requiring one to keep up to date with their inherent subtleties and complexities while finding approaches that can be implemented in practice, given the constraints on the available data and computer resources.

There are hundreds of articles that approach the PHC from the perspective of developing models to support significant decisions, such as (Restrepo, 2008):

1. Location, skills and bases of personnel;
2. Crew schedules;
3. The number and type of vehicles to be located at each base;
4. Choosing the type of vehicle that will attend to the emergency; and
5. The redistribution of vehicles as a function of the state of the system.

According to (Henderson and Mason, 2004) these decisions can be classified as operational, strategic and tactical decisions. This set of decisions directly influences the time the system takes to arrive at the place where the patient is to be found (Sasaki, 2010).

There are two basic documents, Brotones et al. (2003) and Parra (2011) that show the state of the art in PHC, where the models are classified as deterministic, stochastic and dynamic. The first location models were explained by Hakimi (1964) and Toregas et al. (1971) with the proposal of the p-median model and the Location Set Covering Model (LSCM) respectively. Church and Re Velle (1974) improve the LSCMP (Location Set Covering Models P Median) problem by restricting coverage of the population and maximizing the number of available resources to make better use of them. This model is known as a Maximal Covering Location Problem (MCLP). In this project, the p-median models and LSCP (Location Set Covering Problem) model are used to optimize the location of the PHC system's ambulances during the UNAM's entrance exams.

4. LOCATION METHODOLOGY

In this study we propose a three-phase methodology for improving the current location of ambulances. Phase 1 is for gathering information (distances between schools, number of incidents per school and speeds that the ambulances can achieve). Phase 2 is when an initial solution is obtained for the p-median model using the myopic heuristic algorithm (Daskin, 1995), while the schools are also assigned to the ambulance location that is found. In other words, five regions are determined with an ambulance located in each one and the set of schools assigned that are to be served by each ambulance. At the same time a set of initial solutions is obtained by solving the LSCP model and they are established as coverage parameters: ambulance speeds 20, 40, 60 and 90 km/h, and response times 5, 10, 15 y 20 min. Therefore a total of 17 scenarios are generated, 16 for the

LSCP model and one for the p-median model. In each scenario the 20 schools are assigned to the five ambulances located in schools. In phase 3, the 17 initial solutions are improved by neighborhood search algorithms and this solution can, in some cases, be improved by applying a second improving algorithm called an exchange algorithm (Daskin, 1995). It is worth mentioning that the models employed (p-median and LSCP) to optimize ambulance location in the specific case of the UNAM satisfy the particular constraints detected in the University's admissions procedure. A set of scenarios were generated to represent the real situation. For example, the speed variations represent situations where the city streets are not congested so the ambulances can travel at high speed (90Km/h) or else situations where the ambulances cannot travel at high speeds (20 Km/h). The purpose of the solutions to the scenarios is to give the emergency medical technicians a broader vision of the location of the ambulances under different circumstances. This is why we use simulation in order to be able to assess the various scenarios.

4.1 First step: information gathering

The information was provided by the Medical Services Bureau of the UNAM through interviews with the coordinator of the PHC service department, who knows the type and number of incidents that normally occur at each entrance exam. It is important to point out that the emergency medical technicians do not keep a record of the incidents, so, in parallel, we went to the SRAEA of the UNAM where we were given the number of students per school who presented the exam, as well as the number of recorded incidents. This case study had the limitation that we were only given information for the year 2009. Finally we used Google maps to calculate the distances between the 25 schools.

4.2 Second step of the methodology: initial location of ambulances

In this step 17 initial solutions are obtained, the first is obtained by solving the *p*-median model using the myopic algorithm and a set of 16 initial solutions is obtained by solving the LSCP model. We programmed, in this study, the myopic algorithm in Excel and the LSCP model in the LINGO optimizer (Lindo Systems Inc.). That is to say that in each solution or scenario we obtained the location of the five ambulances as well as the set of schools to be served by each one of them.

4.2.1 Initial location based on the *p*-median model

The *p*-median model considers the distance between the schools and the demand in each one of them, i.e., the number of incidents in each school, as well as the five available ambulances. Bearing in mind the above, the model in this case study is posed as follows: Equation (1) minimizes the weighted distance between 25 schools, where h_i is the demand in the module or school i and d_{ij} is the distance between the schools or modules i y j. Equation (2) assigns each school to only one ambulance. Equation (3) assigns school i to point j only if there is an ambulance in that module or school. Equation (4) determines that the number of ambulances to be located is five. Equation (5) indicates that the binary variable x_{ij} is 1 if school i is assigned to ambulance j, 0 if not. Equation (6) indicates that the binary variable w_j is 1 if the ambulance is located at j, 0 if not.

$$Min\ Z = \sum_{i=1}^{25} \sum_{j=1}^{25} \boldsymbol{h}_i d_{ij} x_{ij} \qquad (1)$$

$$\sum_{j=1}^{25} x_{ij} = 1 \quad i = 1,2,..,25 \tag{2}$$

$$x_{ij} \leq w_j \quad i = 1,2,..,25 \; j = 1,2,...,25 \tag{3}$$

$$\sum_{j=1}^{25} w_j = 5 \tag{4}$$

$$x_{ij} = (0,1) \; i = 1,2,...,25 \; j = 1,2,...,25 \tag{5}$$

$$w_j = (0,1) \; j = 1,2,...,25 \tag{6}$$

Myopic algorithm (Daskin, 1995) is applied to solve the p-median model (Equations 1 to 5). This algorithm is designed to find a potential location point for each ambulance in each iteration. This is achieved by minimizing the weighted distance (Equation 1) in each iteration, the prior location is kept fixed and it stops when there are five located ambulances.

4.2.2 Initial location of ambulances based on the covering model (LSCP)

For the application of the LSCP model variations are carried out in desired response times and speeds of the ambulances considering that time [h] is equal to distance [km] divided by the speed [km/h], $t = d / v$. This provides a set of scenarios that determine locations based on distances and speeds to cover patient demand. In this case, it was necessary to add the constraint (equation 9) on the number of ambulances there are to obtain results that fit the available resources.

The LSCP model is applied to this case study for determining the location of the five ambulances. Equation (7) expresses the objective function which is to minimize the number of ambulances required to cover all the points of demand. Equation (8) contemplates those schools or modules that fulfill the various time scenarios established for each one of the proposed speeds. Equation (9) determines that there only are five ambulances. Equation (10) indicates that the binary variable y_i is 1 if the ambulance is located at i, 0 if not.

$$\min z = \sum_{i=1}^{25} y_i \tag{7}$$

$$\sum_{i=1}^{25} y_i \geq 1 \tag{8}$$

$$\sum_{i=1}^{25} y_i = 5 \tag{9}$$

$$y_i = 0,1 \quad i = 1,...,25 \tag{10}$$

4.2.2.1 Getting scenarios that involve variations in speeds and service times

For the solution of the LSCP model, we consider the distance between schools and several speeds that the ambulance can reach. This is achieved owing to the fact that $v = d / t$. Assuming that the ambulances can maintain the following speeds 20, 40, 60 and 90 km/h, we get four tables (with size of 25*25 elements), one per speed. The tables contain the times between schools in minutes. The coverage constraints are generated when considering a desired response time. Response time refers to the time in which we want the ambulance to reach the point where the patient is to be found, which can range from 5, 10, 15 to 20 minutes. Thus 16 total scenarios are generated. For example, if we consider an ambulance speed of 20 Km/h and a desired response time of 5 minutes, a set of constraints (equations 8) will be generated that comply with the desired 5-minute response time between each pair of schools, which is known as a 5-minute radius of coverage. The model for each and every one of the 16 scenarios is developed and we get the location of the 5 ambulances with the respective schools to be served by each one of them.

4.3 Improvement of initial location

In this step the 17 initial scenarios are improved (the one proposed by the p-median model and the 16 proposed for the solution of the LSCP model) by applying the neighborhood search algorithm and, once the solution is improved, the exchange algorithm is applied so that we can get an even better solution. Every scenario consists of the location of five ambulances and the assignment of the schools they need to serve. If the solution is based on the minimum distance, this is obtained by using the *p*-median model, whereas if the solution is based on a desired response time, the solution is obtained by using the LSCP model.

4.3.1 Improvement of initial solution using the neighborhood search algorithm

The neighborhood search algorithm, which is responsible for finding the closest modules to each ambulance, is applied to each and every one of the 17 scenarios obtained. For further reading about the neighborhood search model, we recommend Daskin's book (1995).

4.3.2 Improvement of initial solution using the exchange algorithm

This method is based on swapping around every one of the regions, in this case, the 5 regions. In other words, it considers each one of the schools as a possible ambulance location point by analyzing where the value of the weighted distance is smaller.

5. Analysis of results

After applying the proposed methodology we get the following results. Table 1 gives the 16 possible initial scenarios found by solving the LSCP model. It is important to point out that three scenarios were found not to be feasible. In other words, for an ambulance speed of 20 and 40 km/h and a desired response time (radius of coverage) of between 5 and 10 min, we find pairs of schools that comply with both parameters at the same time.

Table 1: Scenarios proposed for the coverage model

	5 min.	**10 min.**	**15 min.**	**20 min.**
20 km/hr.				
40 km/hr.				
60 km/hr.				
90 km/hr.				

Table 2 shows the results obtained after applying the methodology. We observe that the current situation is drastically improved going from an average ambulance location distance of 3570.27 meters to a distance of 1238.98 meters. This solution was achieved by solving the LSCP model considering a speed of 40 Km/h and of 20-minute radius of coverage. The first ambulance must be located in the UVM school and serves three other schools, the second ambulance must be located at the CSB school and serves three other schools, the third ambulance must be located in the CMA school and serves two other schools, the fourth ambulance must be located in the UL school and serves three other schools and finally the fifth ambulance must be located in the ISEC school and serves 9 other schools. Furthermore, the solution we got, based on the p-median model, also represents a good solution, achieving an average distance between the location of the ambulance and the assigned schools of 1451.27 meters. In Table 2 scenarios in grey are used for the simulation analysis.

Table 2: Summary of the results obtained in each case

Case	Ambulance location (number of modules to be served)	Average distance (in meters)
Current situation	CM (5), CUM (4), ULV (6), UVMC (6)	3570.27
P-median	PSAPE (6), UL (2), CUM (3), CH (5), ULV (4)	1451.27
20 km/hr. - 15 min.	PSAPE (6), UL (3), CUM (6), CH (5), CCP (0)	1548.71
20 km/hr. - 20 min.	CMA (2), UVM (5), ULV (7), CH (6), CCP (0)	1740.01
40 km/hr. - 10 min.	CMA (2), UVM (5), ULV (6), CH (7), CCP (0)	1956.42
40 km/hr. - 15 min.	PSAPE (4), USJ (0), IN (0), UL (3), ISEC (13)	1398.72
40 km/hr. - 20 min.	UVM (3), CSB (3), CMA(2), UL (3), ISEC (9)	1238.98
60 km/hr. - 5 min.	PSAPE (6), ILM (2), ISEC (7), CH (5), CCP (0)	1557.94
60 km/hr. - 10 min.	PSAPE (5), CCP (0), IN (0), UL (2), ISEC (13)	1872.05
60 km/hr. - 15 min.	UVM (2), IN (0), CMA (2), UL (3), ISEC (13)	1924.11
60 km/hr. - 20 min.	PSAPE (5), CCP (0), IN (0), UL (2), ISEC (13)	1872.05
90 km/hr. - 5 min.	IN (0), PSAPE (5), UL (2), CH (7), ULV (6)	1685.13
90 km/hr. - 10 min.	UVM (4), IN (0), CMA (3), UVMC (2), ISEC (11)	2310.26
90 km/hr. - 15 min.	CMA (2), PSAPE (0), UVM (5), ISEC (13), CCP (0)	2275.90
90 km/hr. - 20 min.	CMA (2), PSAPE (0), UVM (5), ISEC (13), CCP (0)	2275.90

5.1 Validation using simulation

To validate the results obtained using the location models, simulation is used considering the more favorable scenarios, that are the ones with less distance, as it can see in next table 3

Table 3 Analyzed scenarios with simulation

Case	Ambulance location (number of modules to be served)	Average distance (in meters)
20 km/hr. - 15 min.	PSAPE (6), UL (3), CUM (6), CH (5), CCP (0)	1548.71
40 km/hr. - 15 min.	PSAPE (4), USJ (0), IN (0), UL (3), ISEC (13)	1398.72
40 km/hr. - 20 min.	UVM (3), CSB (3), CMA(2), UL (3), ISEC (9)	1238.98
60 km/hr. - 5 min.	PSAPE (6), ILM (2), ISEC (7), CH (5), CCP (0)	1557.94

The model programmed in Simio is a representation of the proposal for locating ambulances during the UNAM entrance exam. This exam is sat in different centers, which constitute the demand nodes, the sites of the incidents that are served by the various ambulances that the UNAM puts into service when this exam is being applied. For that reason they are treated as source nodes where each node has a probability distribution that was adjusted based on some historical records of the services provided by the ambulances.

The general structure of the model contemplates 5 areas, in accordance with the p-median location model, each area has an ambulance to cover it and said ambulance has an assigned speed of 20 km / hr. For the transfer of a patient, we consider that when the ambulance is inactive, it returns to the base node. Base nodes are the nodes where the ambulances are located in the location model. We also, in addition to this node, consider a sink node, while the arcs that join the nodes are two-way.

After analyzing the data, as location, type of service, etc. obtained by the pre hospital care reports that record the events attended during the day of evaluation; These data are adjusted to a Poisson distribution, which describes the time between arrivals at a server, in this case it describes the time between occurrences of events that require pre-hospital care provided by ambulance. Once the probability distribution is defined, scenarios are constructed as shown in Figure 1

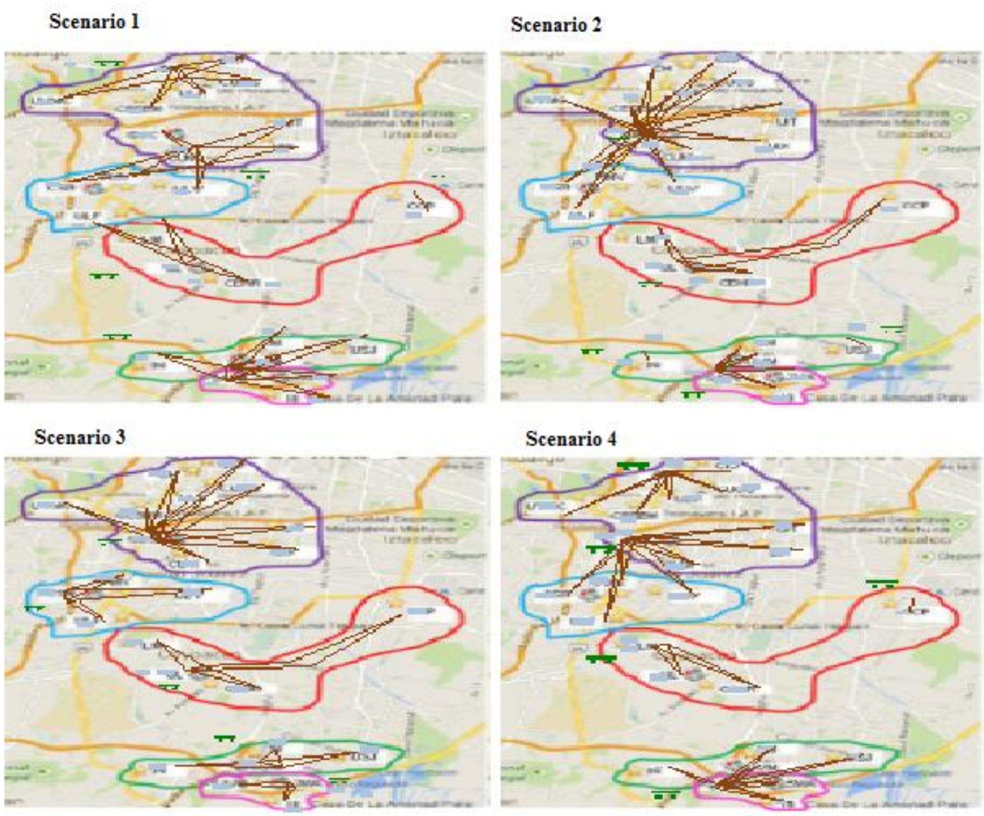

Figure 1. SIMIO scenarios

The model's scale tries to be as close as possible to a ratio 1:1 in order to contemplate the distances of the arcs that join the nodes. An experiment consisting of 100 replicates with a confidence level of 95% was done for the four scenarios. Results obtained are shown in Figures 2 and 3.

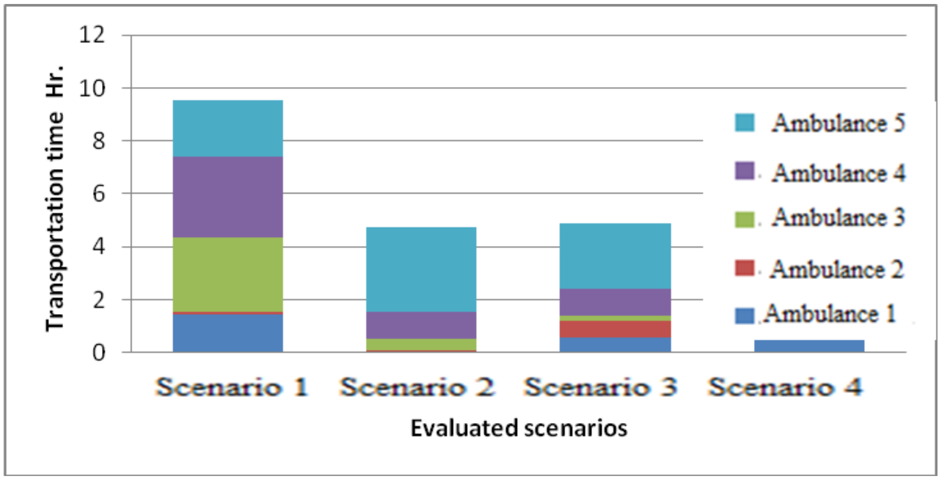

Figure 2 Distribution time per ambulance and scenario

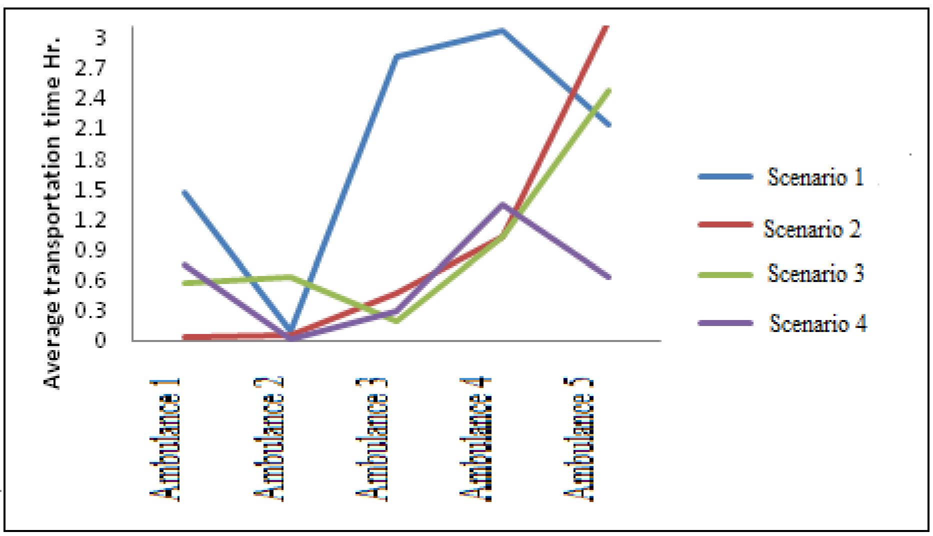

Figure 3 Average time vs. scenarios and ambulances

Based on figures 2 and 3 it can be seen that scenario 4 is better and in section 6 these results will be part of our conclusions.

6. Conclusions

We conclude that the linear programming models based on *p*-medians and covering (LSCP) for ambulance location during the UNAM entrance examinations are an excellent tool, as they considerably shorten the average distances between the location of the ambulance and the schools to be served. A set of scenarios is given that the Emergency Medical Technicians can use for better decision-making in the reduction of the patient response time and a considerable reduction in the probability of patient mortality and morbidity.

Thanks to Simio we can easily expand the detail of the model, with some observations that arise after the model is built such as:

- Defining the routes there and back that can be marked out on a sketch to represent the roads taken by the ambulances.
- Improving the sketch
- Considering the possible sink nodes

And finally it was possible to validate the scenarios built with optimization and to conclude that as it was said in the previous section, from Figures 2 and 3 is noticeable that scenario 4 shows a shorter transportation time, however, one of the conditions for this scenario is a restriction of 5 min as radio coverage, and given the conditions in Mexico City, where the test is applied, it is almost impossible for this reason is better to choose scenario 3, considering that in Figure 3 the line the average time per ambulance remains under scenario 2.

References

1. L. Brotcorne, G. Laporte, F. Semet. Ambulance location and relocation models. *European Journal of Operational Research*, 147, 451-463. 2003.
2. R.L. Church y C.S. Re Velle. The maximal covering location problem. *Papers of the regional science association*, 32, 101 – 18. 1974.
3. J. R. Current and H. A. Eiselt. Location Analysis. *Computers & Operations Research* 29, v-vii (Invited Editorial). 2002.
4. M. S. Daskin. *Network and Discrete Location. Models, Algorithms, and Applications*. Ed. John Wiley & Sons, Inc., New York, 1995. ISBN: 0-471-01897-X.
5. Google Earth:http://www.google.es/intl/es/earth/index.html
6. S. Hakimi.Optimum Location of switching centers in a communication network and some related graphs theoretic problems. *Operation Research* 13, 462-475.1965.
7. S.G. Henderson y A.J. Mason. Ambulance Service Planning Simulation and Data Visualisation. Operations Research and Health Care International Series in *Operations Research & Management Science*. 70, 70-102, 2004.
8. LINGO: www.lingo.com
9. E. Olivares..Buscan 120 mil aspirantes uno de 7 mil espacios en la UNAM. Periódico la Jornada, México D.F., 10/Marzo/2014. http://www.jornada.unam.mx/2014/03/10/sociedad/036n1soc
10. O.J. Parra. Revisión del estado del arte en modelos de localización y relocalización de vehículos para atención de emergencias. *Revista Elementos* 1, 2011.
11. M. Restrepo. Computational methods for static allocation and real-time redeployment of ambulances. Dissertation Faculty of the Graduate School of Cornell University. 2008.
12. S. Sasaki., A. Comber, H. Suzuki, C. Brunsdon. Using genetic algorithms to optimise current and future health planning - the example of ambulance locations. *International Journal of Health Geographics*, 9 (4), 1-10. (2010)
13. C.R. Toregas, R. Swain, C.S. Re Velle, L. Bergman. The location of emergency service facilities. *Operations Research,* 19, pp. 1363 – 1373.1971.