POSITIVE AND STABLE PERIODIC TIME-VARYING CONTINUOUS-TIME LINEAR SYSTEMS

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ABSTRACT
The positivity and stability of periodic time-varying continuous-time linear systems are addressed. Necessary and sufficient conditions for the positivity and stability of the system are established. The proof of conditions is based on the Lyapunov transformation of time-varying systems to time-invariant linear systems. Examples of positive and stable linear periodic systems are presented.

Keywords: positive, linear, periodic, continuous-time, time-varying, system, stability, test.

1. INTRODUCTION

A dynamical system is called positive if its trajectory starting from any nonnegative initial state remains forever in the positive orthant for all nonnegative inputs. An overview of state of the art in positive theory is given in the monographs Farina and Rinaldi 2000, Kaczorek 2002. Variety of models having positive behavior can be found in engineering, economics, social sciences, biology and medicine, etc..
The Lyapunov, Bohl and Perron exponents and stability of time-varying discrete-time linear systems have been investigated in Czornik et. all 2012, 2013, 2014. The positivity and stability of fractional time varying discrete-time linear systems have been addressed in Kaczorek 2014, 2015b, 2015c, 2015d and the stability of continuous-time linear systems with delays in Kaczorek 2009. The fractional positive linear systems have been analyzed in Kaczorek 2008, 2011, 2015a. The positive electrical circuits and their reachability have been considered in Kaczorek 2011d, 2011f and the controllability and observability in Kaczorek 2011a. The stability and stabilization of positive fractional linear systems by state-feedbacks have been analyzed in Kaczorek 2010. The normal positive electrical circuits has been introduced in Kaczorek 2014. The positivity and stability of time-varying continuous-time linear systems and electrical systems has been addressed in Kaczorek 2015f.
In this paper the positivity and stability of periodic time-varying continuous-time linear systems will be addressed.
The paper is organized as follows. In section 2 the Lyapunov reduction of the periodic time-varying continuous-time linear systems to time-invariant linear systems is recalled. The positivity of periodic time-variant linear systems is considered in section 3. Necessary and sufficient conditions for the stability of positive periodic time-varying linear systems are established in section 4. Concluding remarks are given in section 5.
The following notation will be used: $\Re$ - the set of real numbers, $\Re^{n\times m}$ - the set of $n\times m$ real matrices, $\Re_+^{n\times m}$ - the set of $n\times m$ matrices with nonnegative entries and $\Re_+^n = \Re_+^{n\times 1}$, M_n - the set of $n\times n$ Metzler matrices (real matrices with nonnegative off-diagonal entries), I_n - the $n\times n$ identity matrix.

2. REDUCTION TO TIME-INVARIANT OF TIME-VARYING LINEAR SYSTEMS BY LYAPUNOV TRANSFORMATION

Consider the time-varying linear system

$$\dot{x} = A(t)x \tag{2.1}$$

where $x = x(t) \in \Re^{n\times n}$ and $A(t) \in \Re^{n\times n}$ is a piecewise continuous-time time-varying and bounded matrix.
Let $x = X(t) \in \Re^{n\times n}$ be a matrix satisfying the equation

$$\dot{X} = A(t)X \tag{2.2}$$

and the initial condition $X(t_0) = X_0 = I_n$.

Definition 2.1. The matrix $L(t) \in \Re^{n\times n}$ satisfying the conditions:

1) there exists continuous-time $\dot{L}(t) = \dfrac{dL(t)}{dt}$ in the interval $[t_0, +\infty)$,
2) the norms of $L(t)$ and $\dot{L}(t)$ are bounded in the interval $[t_0, +\infty)$,
3) $|\det L(t)| > c > 0$ for some constant c,

is called the Lyapunov transformation matrix.
It is well-known Gantmacher 1959 that the inverse matrix $L^{-1}(t)$ exists and it is also the Lyapunov matrix.

Let

$$X = L(t)Z \tag{2.3}$$

where $Z = Z(t) \in \Re^{n\times n}$.
Using (2.2) and (2.3) we obtain

$$\dot{X} = A(t)X = \dot{L}(t)Z + L(t)\dot{Z} = A(t)L(t)Z \tag{2.4}$$

and

$$\dot{Z} = B(t)Z \tag{2.5}$$

where

$$B(t) = L^{-1}(t)[A(t)L(t) - \dot{L}(t)]. \tag{2.6}$$

The time-varying system (2.1) can be reduced to time-invariant system

$$\dot{Z} = BZ \tag{2.7}$$

by the use of the Lyapunov transformation matrix if there exists matrix $L(t)$ satisfying the equality

$$B = L^{-1}(t)[A(t)L(t) - \dot{L}(t)]. \tag{2.8}$$

For given matrices $A(t)$ and B the matrix $L(t)$ can be found from the equation

$$\dot{L}(t) = A(t)L(t) - L(t)B \tag{2.9}$$

The equation (2.9) follows from (2.8).
The solution of (2.7) has the form

$$Z = Z(t) = e^{Bt} \tag{2.10}$$

for $Z(t_0) = I_n$.
Substituting (2.10) into (2.3) we obtain

$$X = X(t) = L(t)e^{Bt}. \tag{2.11}$$

It is well-known (Gantmacher 1959, Kaczorek 1998) that if the periodic matrix $A(t)$ satisfy the condition

$$A(t+T) = A(t) \tag{2.12}$$

where T is the period of the matrix, then the time-varying system (2.2) can be reduced by the Lyapunov transformation to the time-invariant system (2.5).
In this case from (2.2) and (2.11) we have

$$\dot{X}(t+T) = A(t+T)X(t+T) = A(t)X(t+T) \tag{2.13}$$

and the matrices $x(T)$ and $x(t+T)$ are related by

$$X(t+T) = X(t)V \tag{2.14}$$

where $V \in \Re^{n\times n}$ is a time-invariant nonsingular (constant) matrix since from (2.14) we have

$$\begin{aligned}\dot{V} &= X^{-1}(t)[\dot{X}(t+T) - \dot{X}(t)V] \\ &= X^{-1}(t)[A(t)X(t+T) - A(t)X(t)V] \\ &= X^{-1}(t)A(t)[X(t+T) - X(t)V] = 0.\end{aligned} \tag{2.15}$$

From (2.11) we have

$$L(t) = X(t)e^{-Bt}. \tag{2.16}$$

The matrix $L(t)$ satisfies the condition

$$L(t+T) = L(t) \tag{2.17}$$

since by (2.17) and (2.13)

$$\begin{aligned}L(t+T) &= X(t+T)e^{-B(t+T)} = X(t)Ve^{-BT}e^{-Bt} \\ &= X(t)e^{-Bt} = L(t)\end{aligned}. \tag{2.18}$$

From (2.14) and (2.11) it follows that

$$V = e^{BT} \tag{2.19}$$

and

$$B = \frac{1}{T}\ln V. \tag{2.20}$$

From (2.19) it follows that the eigenvalues $v_1,...,v_n$ of the matrix V and eigenvalues $\lambda_1,...,\lambda_n$ of the matrix B are related by

$$v_k = e^{\lambda_k T} \text{ for } k = 1,2,...,n. \tag{2.21}$$

From (2.21) we have $|v_k| < 1$ for $k = 1,2,...,n$ if and only if $\mathrm{Re}\,\lambda_k < 0$ for $k = 1,2,...,n$.
Therefore, the following Lemma has been proved.
Lemma 2.1. The time-variant system with the matrix $A(T)$ satisfying the condition (2.12) is asymptotically stable if and only if

$$|v_k| < 1 \text{ for } k = 1,2,...,n. \tag{2.22}$$

3. POSITIVE TIME-VARYING LINEAR SYSTEMS

Consider the time-varying linear system

$$\dot{x}(t) = A(t)x(t) + B(t)u(t) \tag{3.1a}$$

$$y(t) = C(t)x(t) + D(t)u(t) \tag{3.1b}$$

where $x(t) \in \Re^n$, $u(t) \in \Re^m$, $y(t) \in \Re^p$ are the state, input and output vectors and $A(t) \in \Re^{n\times n}$,

$B(t) \in \Re^{n \times m}$, $C(t) \in \Re^{p \times n}$, $D(t) \in \Re^{p \times m}$ are real matrices with entries depending continuously on time and $\det A(t) \neq 0$ for $t \in [0, +\infty)$.

Definition 3.1. The system (3.1) is called positive if $x(t) \in \Re_+^n$, $y(t) \in \Re_+^p$, $t \in [0, +\infty)$ for any initial conditions $x_0 \in \Re_+^n$ and all inputs $u(t) \in \Re_+^m$, $t \in [0, +\infty)$.

Theorem 3.1. Let $A(t) \in \Re^{n \times n}$, $t \in [0, +\infty)$. Then

$$\Omega_{t_0}^t(A) = I_n + \int_{t_0}^t A(\tau) d\tau + \int_{t_0}^t A(\tau) \int_{t_0}^{\tau} A(\tau_1) d\tau_1(\tau) d\tau + \ldots \in \Re_+^{n \times n} \quad \text{for } t \geq t_0 \tag{3.2}$$

if and only if $A(t) \in M_n$, $t \in [0, +\infty)$.

Proof is given in Gantmacher 1959, Kaczorek 1998.

Corollary 3.1. If the matrix $A(t) \in \Re^{n \times n}$ satisfies the condition

$$A(t_1)A(t_2) = A(t_2)A(t_1) \text{ for } t_1, t_2 \in [t_0, t] \in [0, +\infty) \tag{3.3}$$

then

$$\Omega_{t_0}^t(A) = e^{\int_{t_0}^t A(\tau) d\tau}. \tag{3.4}$$

Proof is given in Gantmacher 1959.

Corollary 3.2. Let $A \in \Re^{n \times n}$ be a matrix with constant entries independent of time *t*. If $A(t) = A$ then

$$\Omega_{t_0}^t(A) = e^{A(t - t_0)}. \tag{3.5}$$

Proof is given in Gantmacher 1959.

Lemma 3.2. The solution of the equation (3.1a) with given initial condition $x_0 = x(t_0) \in \Re^n$ and input $u(t) \in \Re^m$ has the form

$$x = \Omega_{t_0}^t(A)x(t_0) + \int_{t_0}^t K(t, \tau) B(\tau) u(\tau) d\tau \tag{3.6a}$$

where

$$K(t, \tau) = \Omega_{t_0}^t(A)[\Omega_{t_0}^{\tau}(A)]^{-1}. \tag{3.6b}$$

Proof is given in Gantmacher 1959.

Theorem 3.3. The time-varying linear system (3.1) is positive if and only if

$$A(t) \in M_n,\ B(t) \in \Re_+^{n \times m},$$
$$C(t) \in \Re_+^{p \times n},\ D(t) \in \Re_+^{p \times m},\ t \in [0, +\infty). \tag{3.7}$$

Proof is given in Kaczorek 1998.

Definition 3.2. The matrix $L(t) \in \Re_+^{n \times n}$ is called monomial if only one entry in each row and in each column is positive for $t \in [0, +\infty)$ and the remaining entries are zero.

Theorem 3.4. The time-varying linear system (2.1) with periodic matrix $A(t)$ satisfying the condition (2.12) is positive if and only if the matrix $L(t) \in \Re_+^{n \times n}$ is monomial and $B \in M_n$.

Proof. It is well-known that $Z = e^{Bt} \in \Re_+^{n \times n}$ for $t \in [0, +\infty)$ if and only if $B \in M_n$. From (2.3) it follows that $X(t) \in \Re_+^{n \times n}$ for $t \in [0, +\infty)$ if and only if $L(t) \in \Re_+^{n \times n}$ is monomial matrix. □

Theorem 3.5. The positive time-variant linear system (3.1) with matrices satisfying the condition

$$A(t+T) = A(t),\ B(t+T) = B(t),$$
$$C(t+T) = C(t),\ D(t+T) = D(t) \tag{3.8}$$

is positive if and only if:

1) the matrix $L(t) \in \Re_+^{n \times n}$ is monomial and $B \in M_n$,
2) $B(t) \in \Re_+^{n \times m}$, $C(t) \in \Re_+^{p \times n}$, $D(t) \in \Re_+^{p \times m}$ for $t \in [0, +\infty)$. (3.9)

Proof of the first condition follows immediately from Theorem 3.3. Proof of (3.5) is standard and similar to the one given in Gantmacher 2011f. □

Example 3.1. Consider the system (2.1) with the matrices

$$A(t) = \begin{bmatrix} -\dfrac{4 + 2\sin t - \cos t}{2 + \sin t} & 0 \\ \dfrac{2 - \cos t}{2 + \sin t} & \dfrac{-2 + \sin t + \cos t}{2 - \cos t} \end{bmatrix}, \tag{3.10}$$

It is easy to see that the matrix (3.10) is periodic and its period is $T = 2\pi$.

In this case the Lyapunov transformation matrix has the form

$$L(t) = \begin{bmatrix} 0 & 2 + \sin t \\ 2 - \cos t & 0 \end{bmatrix}, \tag{3.11}$$

and its period is also $T = 2\pi$.

Using (2.5), (2.6) and (2.7) we obtain

$$
\begin{aligned}
B &= L^{-1}(t)[A(t)L(t)-\dot{L}(t)] \\
&= \begin{bmatrix} 0 & \dfrac{1}{2-\cos t} \\ \dfrac{1}{2+\sin t} & 0 \end{bmatrix} \\
&\times \left\{ \begin{bmatrix} -\dfrac{4+2\sin t-\cos t}{2+\sin t} & 0 \\ \dfrac{2-\cos t}{2+\sin t} & \dfrac{-2+\sin t+\cos t}{2-\cos t} \end{bmatrix} \times \begin{bmatrix} 0 & 2+\sin t \\ 2-\cos t & 0 \end{bmatrix} - \begin{bmatrix} 0 & \cos t \\ \sin t & 0 \end{bmatrix} \right\} \qquad (3.12) \\
&= \begin{bmatrix} -1 & 1 \\ 0 & -2 \end{bmatrix},
\end{aligned}
$$

The equation (2.7) has the form

$$\dot{Z} = \begin{bmatrix} -1 & 1 \\ 0 & -2 \end{bmatrix} Z \qquad (3.13)$$

and its solution

$$Z = Z(t) = e^{Bt} = \begin{bmatrix} e^{-t} & e^{-t}-e^{-2t} \\ 0 & e^{-2t} \end{bmatrix}. \qquad (3.14)$$

The matrix of solution of (2.2) with (3.10) is given by

$$
\begin{aligned}
X &= X(t) = L(t)Z(t) \\
&= \begin{bmatrix} 0 & 2+\sin t \\ 2-\cos t & 0 \end{bmatrix} \begin{bmatrix} e^{-t} & e^{-t}-e^{-2t} \\ 0 & e^{-2t} \end{bmatrix} \qquad (3.15) \\
&= \begin{bmatrix} 0 & e^{-2t}(2+\sin t) \\ e^{-t}(2-\cos t) & (2-\cos t)(e^{-t}-e^{-2t}) \end{bmatrix}.
\end{aligned}
$$

It is easy to check that every column of the matrix (3.15) is a solution of the equation (2.2) with (3.10).

Example 3.2. Compute the solution of the equation (3.1a) with the matrix $A(t)$ given by (3.10) and

$$B(t) = \begin{bmatrix} 2+\sin t \\ 2-\cos t \end{bmatrix} \qquad (3.16)$$

for given initial condition $x(0) = x_0$ and an input $u(t)$.

The matrices (3.10) and (3.16) are periodic with the same period $T = 2\pi$.

Using (3.6b), (3.11) and (3.14) we obtain

$$
\begin{aligned}
K(t,\tau) &= X(t)[X(\tau)]^{-1} = L(t)Z(t)[L(\tau)Z(\tau)]^{-1} \\
&= L(t)Z(t)[Z(\tau)]^{-1}[L(\tau)]^{-1} \qquad (3.17) \\
&= \begin{bmatrix} 0 & 2+\sin t \\ 2-\cos t & 0 \end{bmatrix} \begin{bmatrix} e^{-t} & e^{-t}-e^{-2t} \\ 0 & e^{-2t} \end{bmatrix} \\
&\quad \times \begin{bmatrix} e^{-\tau} & e^{-\tau}-e^{-2\tau} \\ 0 & e^{-2\tau} \end{bmatrix}^{-1} \begin{bmatrix} 0 & 2+\sin\tau \\ 2-\cos\tau & 0 \end{bmatrix}^{-1} \\
&= \begin{bmatrix} 0 & e^{-2t}(2+\sin t) \\ e^{-t}(2-\cos t) & (e^{-t}-e^{-2t})(2-\cos t) \end{bmatrix} \begin{bmatrix} \dfrac{e^{\tau}-e^{2\tau}}{2+\sin\tau} & \dfrac{e^{\tau}}{2-\cos\tau} \\ \dfrac{e^{2\tau}}{2+\sin\tau} & 0 \end{bmatrix} \\
&= \begin{bmatrix} \dfrac{e^{2(\tau-t)}(2+\sin t)}{2+\sin\tau} & 0 \\ \dfrac{(e^{\tau-t}-e^{2(\tau-t)})(\cos t-2)}{2+\sin\tau} & \dfrac{e^{\tau-t}(\cos t-2)}{\cos\tau-2} \end{bmatrix}
\end{aligned}
\qquad (3.17)
$$

Taking into account that in this case

$$\Phi_{t_0=0}^{t}(A) = X(t) = \begin{bmatrix} 0 & e^{-2t}(2+\sin t) \\ e^{-t}(2-\cos t) & (2-\cos t)(e^{-t}-e^{-2t}) \end{bmatrix} \qquad (3.18)$$

and using (3.6a) and (3.17) we obtain the desired solution of the equation (3.1a) with (3.10) and (3.16) in the form

$$
\begin{aligned}
X(t) &= X(t)x(0) + \int_0^t K(t,\tau)B(\tau)u(\tau)d\tau \\
&= \begin{bmatrix} 0 & e^{-2t}(2+\sin t) \\ e^{-t}(2-\cos t) & (2-\cos t)(e^{-t}-e^{-2t}) \end{bmatrix} x_0 \\
&+ \int_0^t \begin{bmatrix} \dfrac{e^{2(\tau-t)}(8\sin t-\cos 2t+9}{2(2+\sin\tau)} \\ \dfrac{e^{\tau-t}(\cos t-2)^2}{\cos\tau-2} - \dfrac{(e^{\tau-t}-e^{2(\tau-t)})(\cos t-2)(2+\sin t)}{2+\sin\tau} \end{bmatrix} u(\tau)d\tau.
\end{aligned}
\qquad (3.19)
$$

4. STABILITY OF POSITIVE PERIODIC SYSTEMS

Consider the time-varying linear system (2.1) with periodic matrix $A(t)$ satisfying the condition (2.12).

Definition 4.1. The positive periodic time-varying system (2.1) is called asymptotically stable if

$$\lim_{t\to\infty} \|X(t)\| = 0 \ \text{ for all } \ X_0 \in \Re_+^n. \qquad (4.1)$$

It is assumed that there exists the Lyapunov transformation matrix $L(t)$ satisfying the conditions of Definition 2.1 that reduces the system (2.1) to (2.7).

Theorem 4.1. The positive periodic time-varying system (2.1) is asymptotically stable if and only if the coefficients a_k of the characteristic polynomial of the matrix B of the reduced system (2.7)

$$\det[I_n s - B] = s^n + a_{n-1}s^{n-1} + \ldots + a_1 s + a_0, \qquad (4.2)$$

are positive, i.e. $a_k > 0$ for $k = 0,1,\ldots,n-1$.

Proof. It is well-known Kaczorek 2014 that the positive linear time-invariant system is asymptotically stable if and only if the coefficients of the characteristic polynomial of its Metzler matrix $B \in M_n$ are positive. From (2.3) it follows that for Lyapunov transformation matrix $L(t)$ we have

$$\lim_{t \to \infty} \|X(t)\| = 0 \text{ if and only if } \lim_{t \to \infty} \|Z(t)\| = 0 \,. \quad (4.3)$$

This competes the proof. □

Example 4.1. (Continuation of Example 3.1). From the matrix (3.10) it follows that the matrix B is given by (3.12) and its characteristic polynomial

$$\det[I_2 s - B] = \begin{vmatrix} s+1 & -1 \\ 0 & s+2 \end{vmatrix} = s^3 + 3s + 2 \,, \quad (4.4)$$

has positive coefficients: $a_0 = 2, \;\; a_1 = 3$.

Therefore, the positive system (2.7) with the matrix (3.12) is asymptotically stable. The Lyapunov transformation matrix is given by (3.11) and it is easy to verify that (4.3) holds.

Remark 4.1. Note that for checking the asymptotic stability of the positive periodic time-varying linear systems all tests given in Kaczorek 2014 can be applied.

5. CONCLUDING REMARKS

The positivity and stability of the periodic time-varying linear systems have been addressed. Necessary and sufficient conditions for the positivity and asymptotic stability of the system have been established. Using the Lyapunov transformation of the periodic time-varying linear systems to time-invariant linear systems. The considerations have been illustrated by examples of periodic time-varying linear systems. The consideration can be easily extended to discrete-time periodic linear systems.

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Tadeusz Kaczorek (born 1932, Poland) received his M.Sc., Ph.D. and D.Sc. degrees from the Faculty of Electrical Engineering at Warsaw University of Technology in 1956, 1962 and 1964, respectively. He began his professional career in 1954 at Warsaw University of Technology and continued to work there till 2003. On his retirement he has worked at the Faculty of Electrical Engineering of Bialystok University of Technology till now. From 1968 to 1969 he was the Dean of the Faculty of Electrical Engineering and from 1970 to 1973 he was the Prorector of Warsaw University of Technology. Since 1971 he has been a professor and since 1974 a full professor at Warsaw University of Technology. In 1986 he was elected a corresponding member and in 1996 a full member of the Polish Academy of Sciences. From 1988 to 1991 he was the director of the Research Centre of the Polish Academy of Sciences in Rome. In June 1999 he was elected a full member of the Academy of Engineering in Poland. In May 2004 he was elected honorary member of the Hungarian Academy of Sciences. He was awarded a degree honoris causa by the 11 Universitys in Poland.

His research interests cover the theory of systems and the automatic control systems theory, especially singular multidimensional systems, positive multidimensional systems, fractional systems and singular positive 1D and 2D systems. He has initiated research in the field of singular 2D, positive 2D linear systems and positive fractional 1D and 2D systems. He has published 25 books (8 in English) and over 1100 scientific papers.