PETRI NET REDUCTION RULES THROUGH INCIDENCE MATRIX OPERATIONS

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ABSTRACT
A Petri net (PN) is a powerful tool that has been used to model and analyze discrete event systems. Such systems can be concurrent, asynchronous, distributed, parallel, non-deterministic, and/or stochastic. A problem in PN modelling is related to its graphical representation because it increases for each element of the system. Consequently, incidence matrix of the PN also increases the number of rows and/or columns. To verify properties in PN such as liveness, safeness, and boundedness, computer time is required, even more if we need to verify huge Petri nets. There are six simple reduction rules, which are used to produce a smaller PN preserving the properties of the original PN. In order to apply these reduction rules, we have to find the pattern and then apply the corresponding rule. In this paper, we propose to apply the reduction rules directly in the incidence matrix of the PN modelled, detecting the pattern of each rule on the incidence matrix and applying the corresponding changes on the incidence matrix.

Keywords: Petri nets, reduction method, incidence matrix

1. INTRODUCTION

Petri net (PN) is a powerful tool that gives support to theoretical and practitioners to develop models representing Discrete Event Systems (DES). It has been widely used in several fields to model and analyse flexible manufacturing systems (FMS) and information processing systems trough the application of analysis methods of PN theory, such as the coverability tree, the incidence matrix and state equation, and the reduction rules (Murata 1989).

There are several works where analysis methods of PN are applied in the study of DES.

In (Henry, Layer, and Zaret 2010) a framework that incorporates an application of the coverability analysis is presented. The coverability analysis was coupled with process failure mode analysis in order to quantify the risk induced by potential cyber attacks against network-supported operations.

The work presented in (Cabastino, Giua, and Seatzu 2006) uses the coverability graph to determine a PN system from the knowledge of its coverability graph. The authors faced the following problem: given an automaton that represents the coverability graph of a PN, determine a PN system whose coverability graph is isomorph to the automaton.

In (Latorre-Biel, and Jimenez-Macias 2011), four incidence matrix-based operations are applied to perform transformations in PN models in order to validate and verify them as models of discrete event systems. Properties of the initial PN model are preserved with these matrix operations.

A process to convert A and B contacts in the ladder diagrams into a PN model is described in (Lee, and Lee 2000). The authors construct the incidence matrix for each contact in order to obtain their corresponding state equation and perform their analysis.

In (Verbeek, et. al. 2010) the reduction method is applied to PNs with reset and inhibitor arcs. This PN extension is used to model cancellation and blocking. In (Xi-zuo, Gui-ying, and Sun-ho 2006) the reduction method of PN is applied to verify the correctness of workflow models.

Reduction rules and deadlock detection methods are proposed in (Lu, and Zhang 2010). This proposal is based on Object Oriented Petri net models and the authors take advantage of object oriented concepts to develop their methods. In (Uzam 2004) the PN reduction approach is used to set a policy for deadlock prevention in FMS.

We can notice the importance of the use of analysis methods in PN theory and applications. Nevertheless, the structures of the PN models obtained from DES have several places and transitions, which produces huge coverability trees and very big matrices. Hence, the importance to apply reduction rules in order to have smaller petri net models and perform faster analysis to DES. Therefore, the use of reduction rules and its application on the incidence matrix of the PN, instead of its graphical representation, is proposed in this work.

The remainder of the paper is organized as follows. Section 2 gives fundamental concepts of PNs and

introduces the incidence matrix and reduction rules method. Section 3 describes the proposal of using incidence matrix operations to apply reduction rules. Section 4 presents two illustrative examples. Finally, section 6 shows conclusions of the work and further research.

2. PETRI NET FUNDAMENTALS

A PN is a graphical and mathematical tool that has been used to model concurrent, asynchronous, distributed, parallel, non-deterministic, and/or stochastic systems.

The graph of a PN is directed, with weights in their arcs, and bipartite, whose nodes are of two types: *places* and *transitions*. Graphically, places are depicted as circles and transition as boxes or bars. PN arcs connect places to transitions or transition to places; it is not permissible to connect nodes of the same type. The state of the system is denoted in PN by the use of *tokens*, which are assigned to place nodes.

A formal definition of a PN is presented in table 1 (Murata 1989).

Table 1: Formal definition of a PN

A Petri net is a 5-tuple, $PN = \{P, T, F, W, M_0\}$ where:
$P = \{p_1, p_2, \ldots, p_m\}$ is a finite set of places,
$T = \{t_1, t_2, \ldots, t_n\}$ is a finite set of transitions,
$F \subseteq \{P \times T\} \cup \{T \times P\}$ is a set of arcs,
$W = F \rightarrow \{1, 2, 3, \ldots\}$ is a weight function,
$M_0 = P \rightarrow \{0, 1, 2, 3, \ldots\}$ is the initial marking,
$P \cap T = \varnothing$ and $P \cup T \neq \varnothing$.

The token movement through the PN represents the dynamical behaviour of the system. In order to change the token position, the following transition firing rule is used (Murata 1989):

1. A transition $t \in T$ is enabled if every input place $p \in P$ of t has *w(p,t)* tokens or more. *w(p,t)* is the weight of the arc from *p* to *t*.
2. An enabled transition *t* will fire if the event represented by *t* takes place.
3. When an enabled transition *t* fires, *w(p,t)* tokens are removed from every input place *p* of *t* and *w(t,p)* tokens are added to every output place *p* of *t*. *w(t,p)* is the weight of the arc from *t* to *p*.

2.1. Analysis methods

PN theory considers three groups of analysis methods: a) the coverability tree method, b) the matrix-equation approach, and 3) the reduction method. For the intention of this paper, the matrix equation approach and reduction methods are presented.

2.1.1. Incidence matrix and state equation

A *PN* with *n* transitions and *m* places can be expressed mathematically as a $n \times m$ matrix of integers $A = [a_{ij}]$. The values for each element of the matrix are given by: $a_{ij} = a_{ij}^{+} - a_{ij}^{-}$, where a_{ij}^{+} is the weight of the arc from t_i to p_j, and a_{ij}^{-} is the weight of the arc from p_j to t_i.

The state equation is used to determine the marking of a *PN* after a transition firing, and it can be written as follows:

$$M_k = M_{k-1} + A^T u_k, \; k = 1,2, \ldots \qquad (1)$$

where u_k is a $n \times 1$ column vector of n - 1 zeros and one nonzero entries, which represents the transition t_j that will fire. The nonzero entry is located in the position j of u_k. A^T is the transpose of incidence matrix. M_{k-1} is the marking before the firing of t_j. And Mk is the reached marking after the firing of t_j denoted in u_k.

2.1.2. Reduction rules

In order to work with smaller PN models and analyse them in an easier way, six simple reduction rules have been proposed (Murata 1989; Zhou and Venkatesh 1999). These rules guaranty the preservation of system properties in the system modelled, such properties are safeness, liveness and boundedness.

1. Fusion of Series Places (FSP).
2. Fusion of Series Transitions (FST).
3. Fusion of Parallels Places (FPP).
4. Fusion of Parallels Transitions (FPT).
5. Elimination of Self-loop Places (ESP)
6. Elimination of Self-loop Transitions (EST).

Figure 1 shows the transformations in the PN through the application of reduction rules.

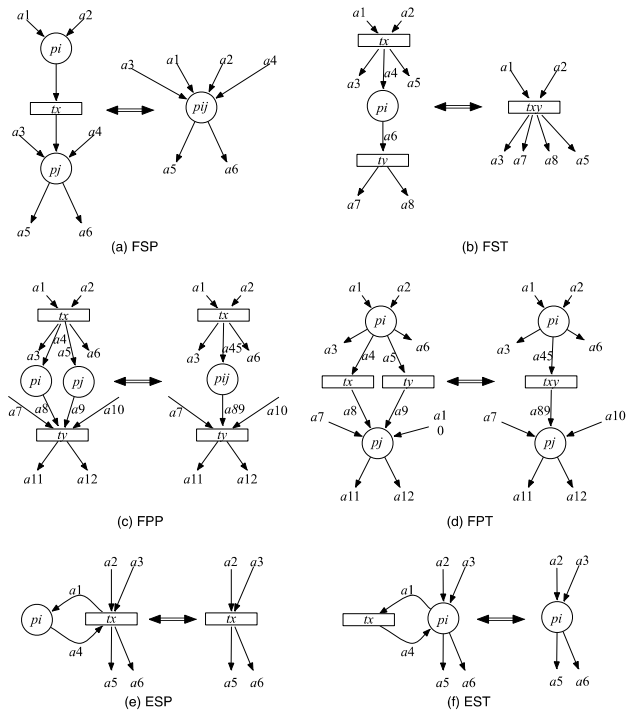

Figure 1: Reduction rules applied to PN models.

3. REDUCTION RULES ON INCIDENCE MATRIX

3.1. Fusion of Series Places (FSP) rule

In the FSP rule, the transition t_x located between the places p_i and p_j is deleted and both places are merged. Then, the result is a unique place p_{ij} with the sum of input and output arcs of places p_i and p_j less the arcs connected to t_x. (Figure 1a).

The incidence matrix for PN of figure 1a is the following.

$$A = \begin{array}{c} \\ t_1 \\ \cdots \\ t_x \\ \cdots \\ t_n \end{array} \begin{array}{c} \begin{array}{ccccccc} p_1 & \cdots & p_i & \cdots & p_j & \cdots & p_m \end{array} \\ \begin{bmatrix} \cdots & \cdots & i_1 & \cdots & j_1 & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & i_n & \cdots & j_n & \cdots & \cdots \end{bmatrix} \end{array} \quad (2)$$

On the incidence matrix we have to do the next steps:

1. Delete the x row from the incidence matrix.
   $A_1 = A[1 \ldots x-1, 1 \ldots m]$
   $A_2 = A[x+1 \ldots n, 1 \ldots m]$
   $A_r = \begin{bmatrix} A_1 \\ A_2 \end{bmatrix}$
2. Sum columns i and j of A_r,
   $A_3 = A_r[1 \ldots n-1, i]$
   $A_4 = A_r[1 \ldots n-1, j]$
   $A_{34} = A_3 + A_4$
3. Replace column in position i by the column vector resulting A_{34}, and remove the column in position j from A_r.
   $A_5 = A_r[1 \ldots n-1, 1 \ldots i-1]$
   $A_6 = A_r[1 \ldots n-1, i+1 \ldots j-1]$
   $A_7 = A_r[1 \ldots n-1, j+1 \ldots m]$
   $A_{fsp} = A[A_5 \quad A_{34} \quad A_6 \quad A_7]$

The column vector A_{34} can be placed either on position i or j. In this case, column in position i is replaced by the column vector resulting A_{34}. At the end, the dimension of incidence matrix A_{fsp} is n-1 × m-1.

3.2. Fusion of Series Transitions (FST) rule

Now, place p_i will be deleted and transitions t_x and t_y will be merged (figure 1b). As result of this rule, we get a single transition t_{xy} whose input and output arcs are the join of input and output arcs of t_x and t_y less those arcs connecting from t_x to p_i and from p_i to t_y.

The incidence matrix of PN depicted in figure 1b is the next.

$$A = \begin{array}{c} \\ t_1 \\ \cdots \\ t_x \\ \cdots \\ t_y \\ \cdots \\ t_n \end{array} \begin{array}{c} \begin{array}{ccccc} p_1 & \cdots & p_i & \cdots & p_m \end{array} \\ \begin{bmatrix} \cdots & \cdots & 0 & \cdots & \cdots \\ \cdots & \cdots & 0 & \cdots & \cdots \\ x_1 & \cdots & 1 & \cdots & x_m \\ \cdots & \cdots & 0 & \cdots & \cdots \\ y_1 & \cdots & -1 & \cdots & y_m \\ \cdots & \cdots & 0 & \cdots & \cdots \\ \cdots & \cdots & 0 & \cdots & \cdots \end{bmatrix} \end{array} \quad (3)$$

For the application of the FST rule on the incidence matrix, we have to perform the following steps.

1. Delete the i–th column from the incidence matrix.
   $A_1 = A[1 \ldots n, 1 \ldots i-1]$
   $A_2 = A[1 \ldots n, i+1 \ldots m]$
   $A_r = [A_1 \quad A_2]$
2. Sum rows x and y of A_r,
   $A_3 = A_r[x, 1 \ldots m-1]$
   $A_4 = A_r[y, 1 \ldots m-1]$
   $A_{34} = A_3 + A_4$
3. Replace row in position x by the row vector resulting A_{34}, and remove row in position y from A_r.
   $A_5 = A_r[1 \ldots x-1, 1 \ldots m-1]$
   $A_6 = A_r[x+1 \ldots y-1, 1 \ldots m-1]$
   $A_7 = A_r[y+1 \ldots n, 1 \ldots m-1]$
   $A_{fst} = A \begin{bmatrix} A_5 \\ A_{43} \\ A_6 \\ A_7 \end{bmatrix}$

A_{fst} is a n-1 × m-1 matrix, after the elimination of p_i and the fusion of t_x and t_y.

3.3. Fusion of Parallel Places (FPP) rule

The aim of this rule is to fuse places with the same input transition, same output transition, and only with one input arc and one output arc.

In order to apply FPP rule on the incidence matrix, parallel places involved are deleted except one of them. The incidence matrix related to PN of figure 1c is the following.

$$A = \begin{array}{c} \\ t_1 \\ \cdots \\ t_x \\ \cdots \\ t_y \\ \cdots \\ t_n \end{array} \begin{array}{c} \begin{array}{ccccccc} p_1 & \cdots & p_i & \cdots & p_j & \cdots & p_m \end{array} \\ \begin{bmatrix} \cdots & \cdots & 0 & \cdots & 0 & \cdots & \cdots \\ \cdots & \cdots & 0 & \cdots & 0 & \cdots & \cdots \\ x_1 & \cdots & 1 & \cdots & 1 & \cdots & x_m \\ \cdots & \cdots & 0 & \cdots & 0 & \cdots & \cdots \\ y_1 & \cdots & -1 & \cdots & -1 & \cdots & y_m \\ \cdots & \cdots & 0 & \cdots & 0 & \cdots & \cdots \\ \cdots & \cdots & 0 & \cdots & 0 & \cdots & \cdots \end{bmatrix} \end{array} \quad (4)$$

In this case, there is only one operation to perform on the incidence matrix.

1. Delete the j–th column from the incidence matrix.
   $A_1 = A[1 \ldots n, 1 \ldots j-1]$
   $A_2 = A[1 \ldots n, j+1 \ldots m]$
   $A_{fpp} = [A_1 \quad A_2]$

The fusion of places p_i and p_j is denoted on the incidence matrix with the elimination of either p_i or p_j. Both places have the same arcs, then $A[1\ldots n, i] = A[1\ldots n, j]$. The dimension of A_{fpp} is n rows by m-1 columns.

3.4. Fusion of Parallel Transitions (FPT) rule

For this rule, the rows of merged transitions are deleted except one of them. The transitions that will be fused have the same input place and output place, so the rows in incidence matrix corresponding to these transitions are similar.

The incidence matrix representing the PN with parallel transitions (figure 1d) is the following.

$$A = \begin{array}{c} \\ t_1 \\ \cdots \\ t_x \\ \cdots \\ t_y \\ \cdots \\ t_n \end{array} \begin{array}{c} \begin{array}{ccccccc} p_1 & \cdots & p_i & \cdots & p_j & \cdots & p_m \end{array} \\ \begin{bmatrix} \cdots & \cdots & i_1 & \cdots & j_1 & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & -1 & 0 & 1 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & i_n & \cdots & j_n & \cdots & \cdots \end{bmatrix} \end{array} \quad (5)$$

To apply the FPT rule on incidence matrix, either t_x or t_y must be deleted, and the other one must be kept.

1. Delete the y–th row from the incidence matrix.
   $A_1 = A[1 \ldots y-1, 1 \ldots m]$
   $A_2 = A[1 \ldots y+1, 1 \ldots m]$
   $A_{fpt} = \begin{bmatrix} A_1 \\ A_2 \end{bmatrix}$

A_{fpt} is a matrix of n-1 rows by m columns.

3.5. Elimination of Self-loop Places (ESP) rule

Self-loop places can be seen in the PN graph, moreover on the incidence matrix this kind of places have only zero entries in their corresponding column. However, isolated places also present only zero entries on the incidence matrix, but in the intention of reduction method isolated places can also be deleted.

Incidence matrix for PN depicted in figure 1e is defined as follows.

$$A = \begin{array}{c} \\ t_1 \\ \cdots \\ t_x \\ \cdots \\ t_n \end{array} \begin{array}{c} \begin{array}{ccccc} p_1 & \cdots & p_i & \cdots & p_m \end{array} \\ \begin{bmatrix} \cdots & \cdots & 0 & \cdots & \cdots \\ \cdots & \cdots & 0 & \cdots & \cdots \\ x_1 & \cdots & 0 & \cdots & x_m \\ \cdots & \cdots & 0 & \cdots & \cdots \\ \cdots & \cdots & 0 & \cdots & \cdots \end{bmatrix} \end{array} \quad (6)$$

The elimination of the self-loop place on the incidence matrix is done through the following step:

1. Delete the i–th column from the incidence matrix.
   $A_1 = A[1 \ldots n, 1 \ldots i-1]$
   $A_2 = A[1 \ldots n, i+1 \ldots m]$
   $A_{esp} = [A_1 \quad A_2]$

After the elimination of i-th column, the A_{esp} matrix has n rows by m-1 columns.

3.6. Elimination of Self-loop Transitions (EST) rule

EST rule indicates that transitions with only an input arc and an output arc to the same place have to be deleted. On the incidence matrix, rows with zero entries in all values denote a self-loop transition or even an isolated one. In both cases the row must be removed from the matrix.

The PN depicted in figure 1f has the following incidence matrix.

$$A = \begin{array}{c} \\ t_1 \\ \cdots \\ t_x \\ \cdots \\ t_n \end{array} \begin{array}{c} \begin{array}{ccccc} p_1 & \cdots & p_i & \cdots & p_m \end{array} \\ \begin{bmatrix} \cdots & \cdots & i_1 & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & i_n & \cdots & \cdots \end{bmatrix} \end{array} \quad (7)$$

t_x row is a zero row vector because p_i is an input an output place of t_x, i.e. $A(t_x,p_i) = 1^+ - 1^- = 0$.

The elimination of t_x on the incidence matrix can be done with the following instruction.

1. Delete the x–th row from the incidence matrix.
   $A_1 = A[1 \ldots x-1, 1 \ldots m]$
   $A_2 = A[x+1, \ldots n, 1 \ldots m]$
   $A_{est} = \begin{bmatrix} A_1 \\ A_2 \end{bmatrix}$

Incidence matrix A_{est} has one row less that A, and the dimension of the matrix now is n-1 rows by m columns.

4. ILLUSTRATIVE EXAMPLES

In order to show the applicability of reduction rules on incidence matrix of PN, two examples taken from the literature are presented.

4.1. Example 1

The PN used in this example was presented in (Murata 1989) and it is shown in figure 2.

The incidence matrix is the following.

$$A = \begin{bmatrix} 1 & -1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & -1 & -1 & 1 \\ 0 & 0 & 1 & -1 \end{bmatrix}$$

Places p_3 and p_4 are serial places. To reduce the PN firstly we apply the steps described for FSP rule in section 3.1.

1. Delete the 4th row from the incidence matrix.
2. Sum column vectors 3 and 4.
3. Replace the 3rd column with the values obtained in the sum, and remove the 4th column.

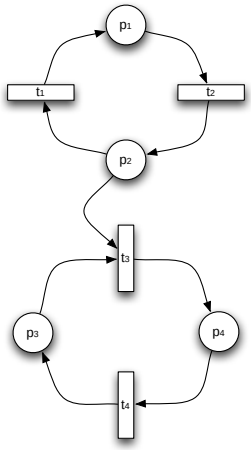

Figure 2: PN model for example 1.

After these operations, the resulting incidence matrix is the following.

$$A_{fsp} = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 0 \end{bmatrix}$$

Now, the rule that can be applied is the FST, because transitions t_1 and t_2 are series transitions.

1. Delete the 1st column from the incidence matrix (place p_1).
2. Sum row vectors 1 and 2.
3. Replace the first row by the result of the sum and delete row 2.

The incidence matrix after these operations is the following.

$$A_{fst} = \begin{bmatrix} 0 & 0 \\ -1 & 0 \end{bmatrix}$$

In this phase, the ESP rule can now be applied to this incidence matrix, because the place of second column of A_{fst} is a self-loop place. The instruction for ESP rule is applied.

1. Delete the second column from the incidence matrix.

The incidence matrix is the following.

$$A_{esp} = \begin{bmatrix} 0 \\ -1 \end{bmatrix}$$

Finally, EST rule can be applied on incidence matrix A_{esp} because the transition of first row represents a self-loop transition.

1. Delete the first row from the incidence matrix A_{esp}.

Applying the EST instruction the last incidence matrix is as follows.

$$A_{esp} = [-1]$$

Figure 3 shows the evolution in the PN when the reduction rules are applied.

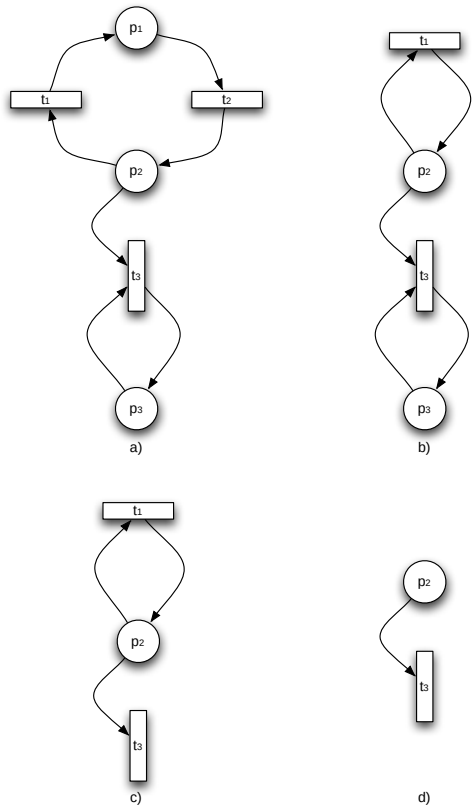

Figure 3. PN changes after the application of some reduction rules.

4.2. Example 2

The second example is a PN model that was reduced applying the reduction method by (Zhou and Venkatesh 1999). Figure 4 shows the initial PN model and its incidence matrix is the following.

$$A = \begin{array}{c} \\ t_1 \\ t_2 \\ t_3 \\ t_4 \\ t_5 \\ t_6 \\ t_7 \\ t_8 \end{array} \begin{array}{c} \begin{array}{cccccccc} p_1 & p_2 & p_3 & p_4 & p_5 & p_6 & p_7 & p_8 \end{array} \\ \begin{bmatrix} -1 & 2 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & -2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 \end{bmatrix} \end{array}$$

FSP rule is applied to fuse places p_2 and p_3, where transition t_2 is between these places.

1. Delete the *2nd* row from the incidence matrix (t_2).

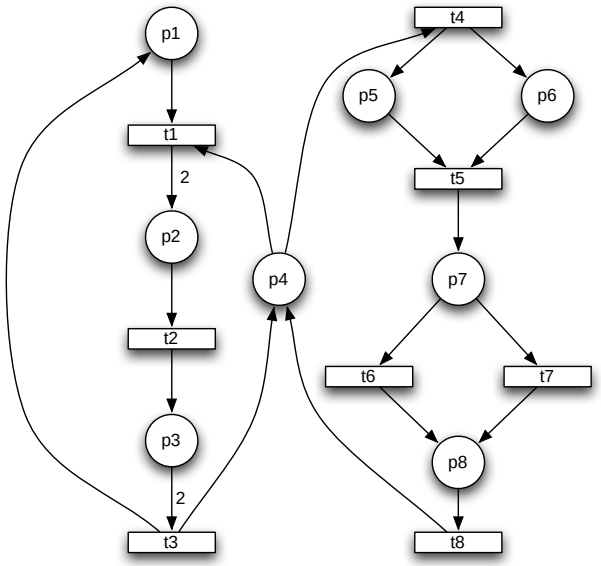

Figure 4. PN model used in example 2.

$$A_{fsp} = \begin{array}{c} \\ t_1 \\ t_3 \\ t_4 \\ t_5 \\ t_6 \\ t_7 \\ t_8 \end{array} \begin{array}{c} \begin{array}{cccccccc} p_1 & p_2 & p_3 & p_4 & p_5 & p_6 & p_7 & p_8 \end{array} \\ \begin{bmatrix} -1 & 2 & 0 & -1 & 0 & 0 & 0 & 0 \\ 1 & 0 & -2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 \end{bmatrix} \end{array}$$

2. Sum columns *2* and *3* of A_{esp},

$$\begin{bmatrix} 2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ -2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ -2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

3. Replace the column in position *2* by the result of the sum, and remove the column in position *3* from A_{esp}.

$$A_{fsp} = \begin{array}{c} \\ t_1 \\ t_3 \\ t_4 \\ t_5 \\ t_6 \\ t_7 \\ t_8 \end{array} \begin{array}{c} \begin{array}{ccccccc} p_1 & p_2 & p_4 & p_5 & p_6 & p_7 & p_8 \end{array} \\ \begin{bmatrix} -1 & 2 & -1 & 0 & 0 & 0 & 0 \\ 1 & -2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & 0 & 0 & 0 & -1 \end{bmatrix} \end{array}$$

Next, rule FPP is applied to parallel places p_5 and p_6.

1. Delete the column corresponding to p_6 from the incidence matrix.

$$A_{fpp} = \begin{array}{c} \\ t_1 \\ t_3 \\ t_4 \\ t_5 \\ t_6 \\ t_7 \\ t_8 \end{array} \begin{array}{c} \begin{array}{cccccc} p_1 & p_2 & p_4 & p_5 & p_7 & p_8 \end{array} \\ \begin{bmatrix} -1 & 2 & -1 & 0 & 0 & 0 \\ 1 & -2 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & 0 & 0 & -1 \end{bmatrix} \end{array}$$

Parallel transitions t_6 and t_7 are reduced with the rule FPT.

1. Delete the t_7 row from the incidence matrix.

$$A_{fpt} = \begin{array}{c} \\ t_1 \\ t_3 \\ t_4 \\ t_5 \\ t_6 \\ t_8 \end{array} \begin{array}{c} \begin{array}{cccccc} p_1 & p_2 & p_4 & p_5 & p_7 & p_8 \end{array} \\ \begin{bmatrix} -1 & 2 & -1 & 0 & 0 & 0 \\ 1 & -2 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 \\ 0 & 0 & 1 & 0 & 0 & -1 \end{bmatrix} \end{array}$$

Serial transitions t_1 and t_3 are fused through FST rule. Place p_2 is the output and input place from t_1 and to t_3, respectively.

1. Delete the p_2 column from the incidence matrix.

$$A_{fst} = \begin{array}{c} \\ t_1 \\ t_3 \\ t_4 \\ t_5 \\ t_6 \\ t_8 \end{array} \begin{array}{c} \begin{array}{ccccc} p_1 & p_4 & p_5 & p_7 & p_8 \end{array} \\ \begin{bmatrix} -1 & -1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & -1 & 1 \\ 0 & 1 & 0 & 0 & -1 \end{bmatrix} \end{array}$$

2. Sum rows t_1 and t_3 from A_{fst},

$$\begin{matrix} [-1 & -1 & 0 & 0 & 0] + \\ [\ 1 & 1 & 0 & 0 & 0] = \\ [\ 0 & 0 & 0 & 0 & 0] \end{matrix}$$

3. Replace the row in t_1 position by the result of the sum, and remove the row of t_3 from A_{fst}.

$$A_{fst} = \begin{array}{c} \\ t_1 \\ t_4 \\ t_5 \\ t_6 \\ t_8 \end{array} \begin{array}{c} \begin{array}{ccccc} p_1 & p_4 & p_5 & p_7 & p_8 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & -1 & 1 \\ 0 & 1 & 0 & 0 & -1 \end{bmatrix} \end{array}$$

Next, FST rule is used to fuse serial transitions t_4 and t_5. Place p_5 is between t_4 and t_5.

1. Delete the p_5 column from the incidence matrix.

$$A_{fst'} = \begin{array}{c} \\ t_1 \\ t_4 \\ t_5 \\ t_6 \\ t_8 \end{array} \begin{array}{c} \begin{array}{cccc} p_1 & p_4 & p_7 & p_8 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 1 & 0 & -1 \end{bmatrix} \end{array}$$

2. Sum rows t_4 and t_5 from $A_{fst'}$,

$$\begin{matrix} [0 & -1 & 0 & 0] + \\ [0 & 0 & 1 & 0] = \\ [0 & -1 & 1 & 0] \end{matrix}$$

3. Replace the row in t_4 position by the result of the sum, and remove the row of t_5 from $A_{fst'}$.

$$A_{fst'} = \begin{array}{c} \\ t_1 \\ t_4 \\ t_6 \\ t_8 \end{array} \begin{array}{c} \begin{array}{cccc} p_1 & p_4 & p_7 & p_8 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 1 & 0 & -1 \end{bmatrix} \end{array}$$

Then, serial transitions t_6 and t_8 are fused through the application of FST rule. Place p_8 is the output place from t_6 and the input place to t_8.

1. Delete the p_8 column from the incidence matrix.

$$A_{fst''} = \begin{array}{c} \\ t_1 \\ t_4 \\ t_6 \\ t_8 \end{array} \begin{array}{c} \begin{array}{ccc} p_1 & p_4 & p_7 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 \\ 0 & -1 & 1 \\ 0 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix} \end{array}$$

2. Sum rows t_6 and t_8 from $A_{fst''}$,

$$\begin{matrix} [0 & 0 & -1] + \\ [0 & 1 & 0] = \\ [0 & 1 & -1] \end{matrix}$$

3. Replace the row in the position of t_6 by the result of the sum, and remove the row of t_8 from $A_{fst''}$.

$$A_{fst''} = \begin{array}{c} \\ t_1 \\ t_4 \\ t_6 \end{array} \begin{array}{c} \begin{array}{ccc} p_1 & p_4 & p_7 \end{array} \\ \begin{bmatrix} 0 & 0 & 0 \\ 0 & -1 & 1 \\ 0 & 1 & -1 \end{bmatrix} \end{array}$$

Serial transitions t_4 and t_6 are merged with the FST rule, where p_7 is the output place from t_4 and the input place to t_6.

1. Delete the p_7 column from the incidence matrix.

$$A_{fst'''} = \begin{array}{c} \\ t_1 \\ t_4 \\ t_6 \end{array} \begin{array}{c} \begin{array}{cc} p_1 & p_4 \end{array} \\ \begin{bmatrix} 0 & 0 \\ 0 & -1 \\ 0 & 1 \end{bmatrix} \end{array}$$

2. Sum rows t_4 and t_6 from $A_{fst'''}$,

$$\begin{matrix} [0 & -1] + \\ [0 & 1] = \\ [0 & 0] \end{matrix}$$

3. Replace the row in the position of t_4 by the result of the sum, and remove the row of t_6 from $A_{fst'''}$.

$$A_{fst'''} = \begin{array}{c} \\ t_1 \\ t_4 \end{array} \begin{array}{c} \begin{array}{cc} p_1 & p_4 \end{array} \\ \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \end{array}$$

Finally, place p1 is removed because it is a self-loop place.

1. Delete the column of p_1 from the incidence matrix $A_{fst'''}$.

$$A_{esp} = \begin{array}{c} \\ t_1 \\ t_4 \end{array} \begin{array}{c} p_4 \\ \begin{bmatrix} 0 \\ 0 \end{bmatrix} \end{array}$$

A_{esp} is the incidence matrix of the PN model obtained in (Zhou and Venkatesh 1999). Figure 5 shows the reduced PN after the application of the reduction rules.

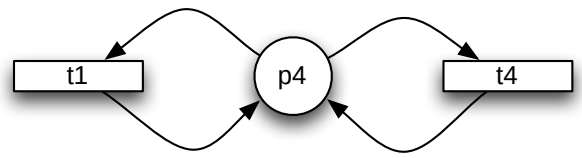

Figure 5. Reduced PN after the application of reduction rules.

5. CONCLUSIONS AND FUTURE WORK

The reduction method in PN is used to generate smaller PNs that preserve structural properties from the initial model.

Other analysis method in PN is the state equation, which uses the concept of incidence matrix. The incidence matrix is the mathematical representation of PNs, and denotes the relationship between places and transitions of the PN.

Reduction rules have been applied on the PN graphical representation; however, this work shows that reduction method can be applied on the incidence matrix, with the same results. This result is important because now the reduction method, based on matrix operations, can be inserted in a computational algorithm.

Two examples were developed to show the feasibility of matrix operations on PN reduction method.

As further research, it is possible to develop an algorithm taking into account the matrix operations proposed, and reduce very big PNs into smaller ones.

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