ANALYSIS OF INFORMATION PARTIAL ENCRYPTION OPTIONS FOR EXCHANGING PETRI NETS SYSTEMS

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ABSTRACT

The aim of this work is to create a framework of definitions and notations to hide part of a Petri net, facing a possible delivery, maintaining the privacy of the critical, secret, or complex parts of the system.
From these definitions and notations we work with the incidence matrices and we analyze the implications of hiding. In this work only the structure of the network is processed. The study of markings and properties of networks with hidden pieces will we analyzed in further works.

1. INTRODUCTION

Petri nets are widespread for modeling many classes of systems, such as manufacturing, logistics, processes and services [3] [5], and in general discrete concurrent systems [4]. However, all these nets are described in a comprehensive manner and must have the information of the entire net to determine their evolution. It would be interesting to take a Petri net and hide a part of it. This can be useful, for example, when distributing a process with some secret [6], or simply to be a part of complex net that is not interested to be handle globally for any reason [5].
In advanced work, we studied the possibilities of Petri nets reduction [10], grouping in one place or transition a subnet, so that what happens on this subnet is encapsulated in a single point of execution. However, we want to go further by defining parts of the net that are hidden, not clustered, and even the implications within the network properties. The aim of this work is the creation of the theoretical basis for a further study of Petri nets in which certain parts are hidden.

So we setup a generic framework of definitions and notations that allows us to deepen in the study of the characteristics and properties of Petri nets. We will expand the vision of Petri nets, providing them with greater functionality in an interesting way for practical applications.

The first part of this paper studies the state the art in this field. We are going to deepen in the basic Petri nets definitions and properties [7] related with hidden information. All this will be necessary to create the framework that allows us to study occultation in PN.

For this paper we will always deal with ordinary networks and pure, unless otherwise expressly.

2. PETRI SUBNETS. DEFINITIONS AND PROPERTIES

Let be P and T the non-empty finite sets of places and transitions, respectively. Let $|P| = n$ (the number of places network) and $|T| = m$ (number of transitions).
Let be α and β pre and post incidence matrices respectively. Let $R = \langle P, T, \alpha, \beta \rangle$ be a Petri net and let C the incidence matrix of R

Definition 1 (Subnet [8]). A subnet of $R = \langle P, T, \alpha, \beta \rangle$ is a net $\overline{R} = \langle \overline{P}, \overline{T}, \overline{\alpha}, \overline{\beta} \rangle$ such that $\overline{P} \subseteq P$ and $\overline{T} \subseteq T$, $\overline{\alpha}$ and $\overline{\beta}$ are restrictions of α and β over $\overline{P} \times \overline{T}$.

In other words, a subnet is a subset of places and transitions with their arcs, joined together.

Let's look at the implications of the latter definition since it is one of the most important with regard to this work.

A subnet corresponds [6], from the matrix point of view, with the resulting submatrix obtained by keeping only the rows corresponding to transitions and places columns for the selected subnet.
Example 1. We take the Petri net which has the following incidence matrix:

$$C = \begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \\ p_6 \end{array} \begin{array}{c} \begin{array}{cccccc} t_1 & t_2 & t_3 & t_4 & t_5 & t_6 \end{array} \\ \left(\begin{array}{cccccc} -1 & 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 & -1 & 1 \\ 0 & 0 & 0 & 0 & 1 & -1 \end{array} \right) \end{array}$$

If we stay with places p_1, p_3, and p_5 P_4 and transitions t_1, t_2, and t_3 we have the subnet defined by this incidence matrix.

$$C' = \begin{array}{c} \\ p_1 \\ p_3 \\ p_4 \\ p_5 \end{array} \begin{array}{c} \begin{array}{ccc} t_1 & t_2 & t_3 \end{array} \\ \begin{pmatrix} -1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & -1 & -1 \\ 0 & 1 & 0 \end{pmatrix} \end{array}$$

In [6] is shown that the set of all possible permutations of rows and/or columns of a matrix of incidence corresponding to a network, either the previous or subsequent actual Incidence call, make an equivalence relation. In other words, given an incidence matrix can be rearranged both rows and columns and this rearrangement end is perfectly describing the original network.

In this way, we can study the incidence matrices reordering rows and columns as preferred one at any time, without loss of generality.

From all these definitions and proofs we can draw several trivial conclusions:

1. A subnet, like generic network does not have to be square.
2. If a row or column of the incidence matrix is all zeros, no mean that that place or that transition is isolated. this only occur with pure networks.
3. It does not matter the number of places and / or transitions are chosen for the subnet, if they are not empty sets.

3. SPLITTING A NETWORK INTO SUBNETS

Let $R = \langle P, T, \alpha, \beta \rangle$ a Petri net where $|P| = n$ and $|T| = m$. So $P = \{p_1, p_2 \ldots p_n\}$ and $T = \{t_1, t_2 \ldots t_m\}$. Select two subsets $P' \subseteq P$ and $T' \subseteq T$ so that $|P'| = r \leq n$ and $|T'| = s \leq m$. With these premises divide into two subnets the original one.

We have seen that we can identify a subnetwork simply removing rows and columns (places/transitions) of an incidence matrix. Taking advantage of the equivalence relation defined in [6], we reorder the incidence matrix so that they are in the top places and transitions of the subnet defined. Rename also the places and transitions (without loss of generality, and for convenience) so that the incidence matrix is as follows:

$$C = \begin{array}{c} \\ p_1 \\ \vdots \\ p_r \\ p_{r+1} \\ \vdots \\ p_n \end{array} \begin{array}{c} \begin{array}{ccc|ccc} t_1 & \cdots & t_s & t_{s+1} & \cdots & t_m \end{array} \\ \left(\begin{array}{ccc|ccc} a_{11} & \cdots & a_{1s} & a_{1(s+1)} & \cdots & a_{1m} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ a_{r1} & \cdots & a_{rs} & a_{r(s+1)} & \cdots & a_{rm} \\ \hline a_{(r+1)1} & \cdots & a_{(r+1)s} & a_{(r+1)(s+1)} & \cdots & a_{(r+1)m} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots \\ a_{n1} & \cdots & a_{ns} & a_{n(s+1)} & \cdots & a_{nr} \end{array} \right) \end{array}$$

We now have the network divided into two disjoint and complementary subnets. They are disjoint because there is no place and no common transition, and complementary because the union of the two we gives the complete network. At this point note that the incidence matrix is divided into four blocks $C = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$. The interpretation is as follows:

- A subnet made up of places $p_1 \ldots p_r$ and transitions $t_1 \ldots t_s$
- D subnet is complementary to A, made up of the places $p_{r+1} \ldots p_n$ and transitions $t_{s+1} \ldots t_m$.
- B is the matrix that defines the interaction of the places of A with D transitions
- C is the matrix that defines the interaction of D places with A transitions

This can be generalized to multiple disjoint and complementary subnets without further to re-apply the same process to any of the subnets already defined. Thus, generically we can divide a network into *i* subnetworks, so we'll have a matrix of this style:

$$\left(\begin{array}{ccc|ccc|c|ccc} a_{11} & \cdots & a_{1s} & a_{1(s+1)} & \cdots & a_{1t} & & a_{1u} & \cdots & a_{1m} \\ \vdots & SN_1 & \vdots & \vdots & \ddots & \vdots & \cdots & \vdots & \ddots & \vdots \\ a_{p1} & \cdots & a_{ps} & a_{p(s+1)} & \cdots & a_{pt} & & a_{pu} & \cdots & a_{pm} \\ \hline a_{(p+1)1} & \cdots & a_{(p+1)s} & a_{(p+1)(s+1)} & \cdots & a_{(p+1)t} & & a_{(p+1)u} & \cdots & a_{(p+1)m} \\ \vdots & \ddots & \vdots & \vdots & SN_2 & \vdots & \cdots & \vdots & \ddots & \vdots \\ a_{q1} & \cdots & a_{qs} & a_{q(s+1)} & \cdots & a_{qt} & & a_{qu} & \cdots & a_{qm} \\ \hline & \vdots & & & \vdots & & \ddots & & \vdots & \\ \hline a_{r1} & \cdots & a_{rs} & a_{r(s+1)} & \cdots & a_{rt} & & a_{ru} & \cdots & a_{rm} \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \cdots & \vdots & SN_i & \vdots \\ a_{n1} & \cdots & a_{ns} & a_{n(s+1)} & \cdots & a_{nt} & & a_{nu} & \cdots & a_{nm} \end{array} \right)$$

In this situation, if we select two subnets SN_j and SN_k , we locate the zones of influence of each with respect to the other:

$$\left(\begin{array}{c|c|c|c|c} \ddots & \cdots & \cdots & \cdots & \cdots \\ \hline \vdots & SN_j & \cdots & IM_1 & \cdots \\ \hline \vdots & \cdots & \ddots & \cdots & \cdots \\ \hline \vdots & IM_2 & \cdots & SN_k & \cdots \\ \hline \vdots & \vdots & \cdots & \vdots & \ddots \end{array} \right)$$

Thus, the submatrix IM_1 represents the arcs that connect places of the submatrix SN_j with SN_t*ransitionsk* and the matrix IM_2 represents the arcs that connect places of SN_k to Sn_t*ransitionsj*.
Arcs that are in one way or another indicates the sign of the corresponding element of A or B.

Definition 2 (Partition of a network into subnets). We say that a set $P = \{R_1 R_2 \ldots R_k\}$ is a partition into subnets of R if the following holds:

- $R_1 \cup R_2 \cup \ldots \cup R_k = R$
- $\forall i, j | 1 \leq i, j \leq k \Rightarrow R_i \cap R_j = \emptyset$

ie, the binding of the total network subnets and subnets are pairwise disjoint.

$$
\begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \\ p_6 \\ p_7 \\ p_8 \end{array}
\begin{array}{c} \begin{array}{cccccc} t_1 & t_2 & t_3 & t_4 & t_5 & t_6 \end{array} \\
\left(\begin{array}{cccccc}
-1 & 0 & 1 & 0 & 0 & 0 \\
-1 & 0 & 0 & 1 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 \\
1 & -1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & -1 & 1 \\
0 & 0 & 0 & 0 & 1 & -1
\end{array}\right) \end{array}
\cong
\begin{array}{c} \\ p_8 \\ p_1 \\ p_3 \\ p_6 \\ p_4 \\ p_5 \\ p_2 \\ p_7 \end{array}
\begin{array}{c} \begin{array}{cccccc} t_1 & t_6 & t_3 & t_5 & t_4 & t_2 \end{array} \\
\left(\begin{array}{cccccc}
0 & -1 & 0 & 1 & 0 & 0 \\
-1 & 0 & 1 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -1 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & -1 \\
0 & 0 & 0 & 0 & 0 & 1 \\
-1 & 0 & 0 & 0 & 1 & 0 \\
0 & 1 & 0 & -1 & -1 & 0
\end{array}\right) \end{array}
$$

Figure 1 – Two equivalent incidence matrices to describe the same a Petri net.

4. DESCRIPTION OF THE PARTS OF A MATRIX ONCE DEFINED THE SUBNETS

As can be reordered places and transitions smoothly, we study a network N divided into 2 subnets, for simplicity and without loss of generality.

For consistency with [6] we will follow this notation:

$$
\left(\begin{array}{c|c} H & HP \\ \hline HT & V \end{array}\right)
$$

where

- H (Hidden Subnet) is the subnet you want to hide.
- V (Visible Subnet) is the subnet that is visible.
- HT (Hidden Transitions Submatrix) are the relationships between places of V and H transitions
- HP (Hidden Places Submatrix) are the relations between transitions of V and H sites

Note. Following this notation can be convenient because it is clear what is each of the submatrices. However, elsewhere in the document be referenced as $R1$ and $R2$ for be more clarifying or being something generic and independent networks concealment. However, using $R1$ and $R2$ the notation of subnets of influence with respect to the other is more diffuse.

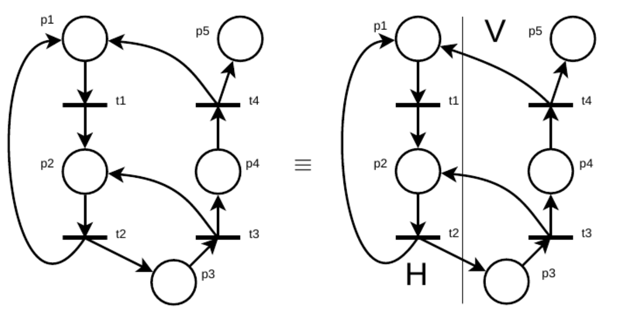


Figure 2 – Selecting subnet to hide

Example 2. Consider the Petri net of the figure 2 with the next incidence matrix:

$$
\begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \end{array}
\begin{array}{c} \begin{array}{cccc} t_1 & t_2 & t_3 & t_4 \end{array} \\
\left(\begin{array}{cccc}
-1 & 1 & 0 & 1 \\
1 & -1 & 1 & 0 \\
0 & 1 & -1 & 0 \\
0 & 0 & 1 & -1 \\
0 & 0 & 0 & 1
\end{array}\right) \end{array}
$$

The subnet we want to hide is formed by sites 1and 2 and 1 and 2 transitions. Graphically, separate places and transitions to hide (H) from the rest of the network (V)

The incidence matrix is already sorted by the places and transitions to the top of it. Here's the four parts described above.

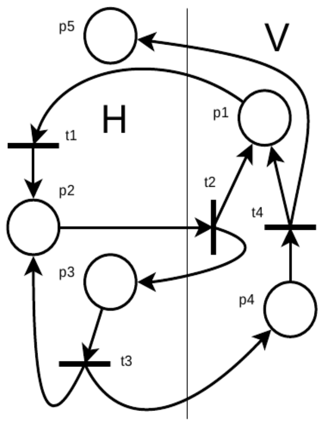
$$
\begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \end{array}
\begin{array}{c} \begin{array}{cc|cc} t_1 & t_2 & t_3 & t_4 \end{array} \\
\left(\begin{array}{cc|cc}
-1 & 1 & 0 & 1 \\
1 & -1 & 1 & 0 \\ \hline
0 & 1 & -1 & 0 \\
0 & 0 & 1 & -1 \\
0 & 0 & 0 & 1
\end{array}\right) \end{array}
$$

In this matrix we can see the four described parts:

- $H = \begin{array}{c} \\ p_1 \\ p_2 \end{array} \begin{array}{c} \begin{array}{cc} t_1 & t_2 \end{array} \\ \left(\begin{array}{cc} -1 & 1 \\ 1 & -1 \end{array}\right) \end{array}$ is the subnet we want to hide.
- $V = \begin{array}{c} \\ p_3 \\ p_4 \\ p_5 \end{array} \begin{array}{c} \begin{array}{cc} t_3 & t_4 \end{array} \\ \left(\begin{array}{cc} -1 & 0 \\ 1 & -1 \\ 0 & 1 \end{array}\right) \end{array}$ is the subnet that is visible.
- $HP = \begin{array}{c} \\ p_1 \\ p_2 \end{array} \begin{array}{c} \begin{array}{cc} t_3 & t_4 \end{array} \\ \left(\begin{array}{cc} 0 & 1 \\ 1 & 0 \end{array}\right) \end{array}$ are the relationships between transitions of V and H places.

- $HT = \begin{pmatrix} & t_1 & t_2 \\ p_3 & 0 & 1 \\ p_4 & 0 & 0 \\ p_5 & 0 & 0 \end{pmatrix}$ are the relationships between places of V and H transitions.

Example 3. In the previous example we have seen a fairly simple option selection subnet and we have chosen the locations 1 and 2 and the transitions 1 and 2. However, we can choose any other subset of places and transitions. In this example we will select locations 2, 3 and 5 and the transitions 1 and 3. Thus, in the graph of the previous example move the locations and transitions to hide on one side and the rest on the other.



Although more confusing, can be seen that the graph is the same as the incidence matrix is the same (not just part of the equivalence class, it is exactly the same). Now, in this matrix move places 2, 3 and 5, and 1 and 3 transitions at the beginning of the matrix:

	t_1	t_3	t_2	t_4
p_2	1	1	−1	0
p_3	0	−1	1	0
p_5	0	0	0	1
p_1	−1	0	1	1
p_4	0	1	0	−1

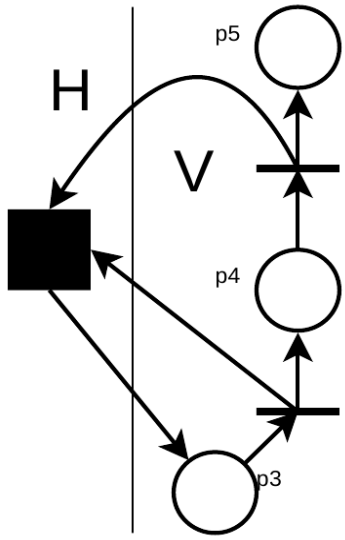
Interpreting each of the chunks of the matrix is similar to the previous example.

5. HIDING THE SUBNET

Once you select the subnet to hide we proceed to the occultation as such [6]. Graphically, it seems simple. Just replace the subnet to hide by a black box and modify some arcs according to the following rules:

1. The arcs originating in a place or transition within the black box, and target a place or transition out of it will have the black box as the source.
2. The arcs originating in a place or transition out of the black box, and target a place or transition within it, are replaced by the black box as a destination.

Example 4. We consider the Petri net of the Figure 2. The result of hiding the part of the graph H is the following:



In the associated incidence matrix also replace the subnetwork H by a black box:

	t_1	t_2	t_3	t_4
p_1	■	■	0	1
p_2	■	■	1	0
p_3	0	1	−1	0
p_4	0	0	1	−1
p_5	0	0	0	1

However, in this matrix notation is given information should also be hidden: it gives us information about the number of places and transitions of the hidden subnet, besides indicating hidden places and transitions with which it interacts. To solve this problem we proceed as follows. We can group all rows for the screened subnet into one. In each row position examine all elements of the original rows corresponding to that position, and will put:

- If all these elements are zero, in the grouped row will be a zero.
- If one and only one of those elements is nonzero, will put that item.
- If there are several non-zero elements, we will post a list of these items separated by commas, creating a d-dimensional element (in *d* dimensions).

In the same way we have done with the rows, proceed with columns. Thus, if the hidden subnet has *i* columns and *j* rows, we will get a matrix like this:

$$\begin{pmatrix} \blacksquare & a_{1(i+1)} & \cdots & a_{1m} \\ a_{(j+1)1} & & & \\ \vdots & & V & \\ a_{n1} & & & \end{pmatrix}$$

Where $\forall p, \forall q | i + 1 \le p \le m \wedge j + 1 \le q \le n$

$$a_{1p} = \begin{cases} 0 & \text{if } \forall r | 1 \le r \le j, c_{rp} = 0 \\ c_{rp} & \text{if } \exists! r, 1 \le r \le j | c_{rp} \neq 0 \\ (c_{r1p}, c_{r2p}, \ldots) & \text{if } \exists! r_1 \neq r_2 \neq \ldots, 1 \le r_1, r_2, \ldots \le j \\ & | c_{r1p}, c_{r2p}, \ldots \neq 0 \end{cases}$$

$$a_{q1} = \begin{cases} 0 & \text{if } \forall s|1 \leq s \leq i, c_{qs} = 0 \\ c_{qs} & \text{if } \exists! s, 1 \leq s \leq i | c_{qs} \neq 0 \\ (c_{qs1}, c_{qs2}, \ldots) & \text{if } \exists! s_1 \neq s_2 \neq \ldots, 1 \leq s_1, s_2, \ldots \leq s \\ & |c_{qs1}, c_{qs2}, \ldots \neq 0 \end{cases}$$

So we hide the number of places and transitions of the hidden subnet and their relationships. Yes, some information is given about the hidden network. Really if this resulting matrix some node that is d-dimensional, at least in the hidden network must exist d nodes of this type.

Example 5. We consider the Petri net defined by the following incidence matrix, separated into H,V ,HT and HP .

$$\left(\begin{array}{ccc|ccc} -1 & 0 & 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & -1 & 0 & 0 \\ 1 & -1 & 0 & 1 & 0 & 0 \\ \hline 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & -1 & 1 \\ 0 & 0 & 0 & 0 & 1 & -1 \end{array}\right)$$

After applying the above steps for the group, we would have the following:

$$\left(\begin{array}{c|ccc} \blacksquare & (1,-1,1) & 0 & 1 \\ \hline 1 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & -1 & 1 \\ 0 & 0 & 1 & -1 \end{array}\right)$$

Here we see that the information about the number of hidden places and transitions is minimized. So we know that at least there is a hidden transition and at least three hidden places (there is a transition of dimension 3). However, we do not know the exact number of either.

6. HIDING VS. REDUCTION

Both Silva works [8] [9] as in the article by Xia [10] discusses possible Petri nets reductions for grouping and simplifying, under certain circumstances, places and / or transitions. These reductions can be structural (only dependent on the structure and initial marking of the net) or depending on the interpretation of the Petri net.

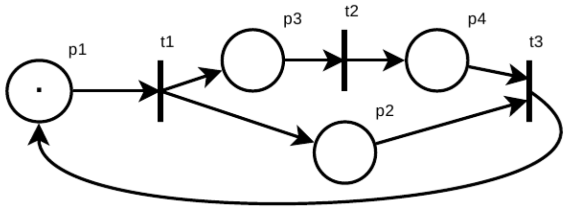
Should be clear that these reductions are not the same thing we are describing. We do not try to simplify the network together elements to have more or fewer places or transitions or to make it easier. What we want is to hide part of the network, regardless of how simple or complicated it is.

Here we have an example of what a reduction is.

Example 6 (Reduction of an implicit place [8]). In a marked Petri net, an implicit place is one that meets the following:

1. its marking can be calculated from other points marking
2. never is the only place that prevents the enabling of its output transitions

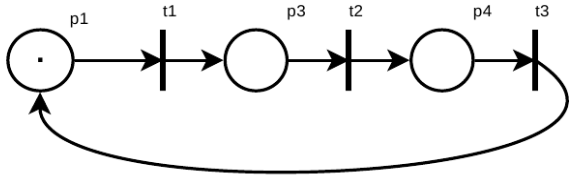
If we consider the following Petri net



we can notice that p_2 is an implicit place because its marking can be calculated as a function of p_3 y p_4:

$$M(p_2) = M(p_3) + M(p_4)$$

Moreover, by this same formula, it is clear that $M(p_2) \geq M(P_4)$ (marking cannot be negative) so the only place that can prevent enabling of T_3 is P_4 . Thus eliminating p_2 does not alter the behavior of the network, which would be as follows:



In this network elements have been removed, no hidden. This example helps us to see the difference between hiding and a reduction.

7. CLASSIFICATION BY TYPE OF SUBNET HIDING

We have seen how to hide part of a network. We have also studied how to make relations between the visible and hidden parts of the network providing minimal information about the network structure.

Then we see occultation special cases with special features. Suppose we take a pure network and want to hide part of it. Depending on how they are each of the four pieces of matrix (H, V, HP and HT) we can see some special cases.

7.1. Disjointed subnets

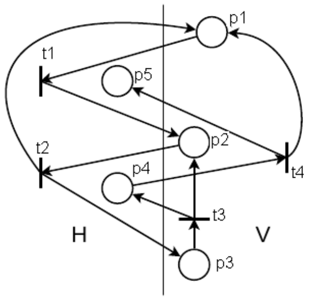
Suppose that in the incidence matrix divided into the four pieces explained, are H or V be a null matrix. In this case the interpretation is that there arcs between places and transitions of the subnet, which would simply places and / or no transitions related to each other but with the additional subnetwork. Subnet talk then disjointed.

Definition 3 (Disjointed subnet). Pure subnet said disjointed if there is no arc between places and transitions of that subnet, ie if its incidence matrix is zero.

Example 7. Consider the Petri net of figure 2. The incidence matrix is:

$$\begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \end{array} \begin{array}{c} \begin{array}{cccc} t_1 & t_2 & t_3 & t_4 \end{array} \\ \left(\begin{array}{cccc} -1 & 1 & 0 & 1 \\ 1 & -1 & 1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{array}\right) \end{array}$$

We assume that we select as subnet formed by 4th and 5th places and transitions 1 and 2. Then the graph and the incidence matrix are thus:



$$C = \begin{array}{c} \\ p_4 \\ p_5 \\ p_1 \\ p_2 \\ p_3 \end{array} \begin{array}{c} \begin{array}{cc|cc} t_1 & t_2 & t_3 & t_4 \end{array} \\ \left(\begin{array}{cc|cc} 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \\ \hline -1 & 1 & 0 & 1 \\ 1 & -1 & 1 & 0 \\ 0 & 1 & -1 & 0 \end{array}\right) \end{array}$$

Here we can see that although really p_4, p_5, t_1 and t_2 are not isolated, there is no arc that connects them together. In the incidence matrix, the corresponding submatrix is the zero matrix. Therefore, whether or not there are elements isolated in the net, total subnet formed by p_4, p_5 , t_1 and t_2 is a disjointed net.

7.2. Macroplace

Suppose now that the incidence matrix divided into the four pieces explained, HT appears to be the zero matrix. Then we conclude that the subnet H is only related by arcs with places of subnet V. All arcs entering H come from transitions of V and all arcs coming out from H go to transitions of V. Stated another way, the subnet V behaves like a spot, but may contain places and transitions.

Definition 4 (Macroplace). A macroplace is a subnet H or V that meets the following:

1. arcs entering any node of the subnet from an external node come from a transition.
2. arcs leaving any node on the subnet to an external node go to a transition.

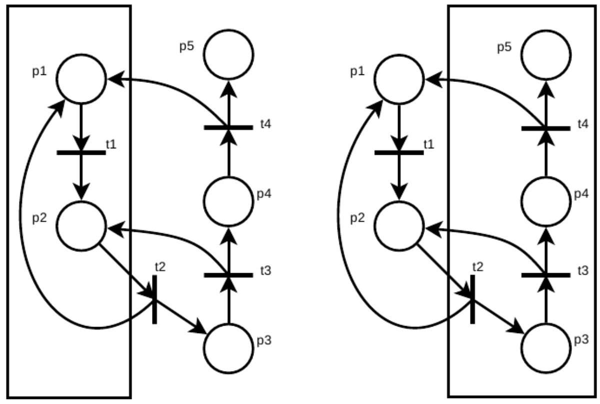
Note that this is not really a place, and that the subnet has not marked as such. The marking is on the places within the subnet and depend on the arches of arrival.

7.3. Macrotransition

Another option that can happen is that in the incidence matrix, HP appears to be the zero matrix. Then we conclude that the subnet H is only related by arcs with transitions of subnet V. All arcs entering H come from places of V and all arcs coming out from H go to places of V. Stated another way, the subnet V behaves like a transition, but may contain places andtransitions.

Definition 5 (Macrotransition). A macrotransition is a subnet H or V that meets with the following:

1. arcs entering any node of the subnet from an external node come from a place.
2. arcs leaving any node on the subnet to an external node go to a place.



a) Macroplace b) Macrotransition

Figure 3 – Macroplace y macrotransition

Like macroplaces, macrotransitions are not transitions as such, it is not necessary that all entries are marked to fire the macrotransition, and not all output places are marked after entering it. Everything depends on the inner workings of the macrotransition.

7.4. Sinkhole subnet and Source subnet

Another thing that can happen is that the hidden subnet reach only arcs. We then find that you can not leave the subnet. We speak then of a sinkhole subnet.

Definition 6 (Sinkhole subnet). It is said that a subnet is a sinkhole subnet if no arc has its origin in an internal node (place or transition) of the subnet.

It is easy to see that a subnet is sinkhole if and only if all elements of HP are greater or equal to zero and all elements of HT are less than or equal to zero.

H is sinkhole $\Leftrightarrow \forall a_{ij} \in HP,\ a_{ij} \geq 0 \wedge \forall a_{pq} \in HT,\ a_{pq} \leq 0$

If instead of this what happens is no arc gets into the subnet, we have a source subnet. In a source subnet we can not enter.

Definition 7 (Source subnet). It is said that a subnet is a source subnet if no arc has its destination in an internal node (place or transition) of the subnet.

It is easy to see that a subnet is a source if and only if all elements of HT are greater than or equal to zero and all elements of HP are less than or equal to zero.

H is source $\Leftrightarrow \forall a_{ij} \in HT,\ a_{ij} \geq 0 \wedge \forall a_{pq} \in HP,\ a_{pq} \leq 0$

8. FRONT-END INTERACTION WITH THE SUBNET. INPUT AND OUTPUT FUNCTIONS

Once you have defined all this environment, we will try to go a little further. Let’s assume that we want to export a subnet we have hidden in another network, like a black box. Our intention is to connect this hidden network to another network, and can thus be reused subnets. For example, let’s assume that we have a process modeling with Petri net modeling and in this there is a subnet we want to hide, but, at the same time, we want to reuse it in other Petri nets.

In this case we have a problem, and once hidden net work disappears half the information input or output arcs of the same. In particular, we do not know the source nodes and arcs that leave the target nodes of the arcs that enter the network until no visible again. But if we want to reuse it on other networks, can not wait to make it visible. Should remain hidden, but should be able to connect to other networks.

We will try to solve this problem. This way we can reuse hidden networks like plug-in modules on other networks. However, we will not need the actual implementation of the source or destination nodes of the arcs that leave or enter the network, respectively. The solution is to define a facade or front-end input and output of the network. This front-end will contain the information needed to interact with the network hidden, but hide the specifics of implementation. To define this behavior going from some assumptions.

8.1. Previous definitions

Let $R = \langle P, T, \alpha, \beta\rangle$ be a Petri net and let $P = \{R_1, R_2\}$ be a partition of R.

Definition 8 (Input place). Let p_i a place of R_1. p_i is an input place of R_1 if it is the destination of an arc coming from a R_2 transition, ie,

p_i is an input place of R_1 if $\exists t_j \in R_2 | c_{ij} > 0$

Definition 9 (Input transition). Let t_i a transition of R_1. t_i is an input transition of R_1 if it is the destination of an arc coming from a R_2 place, ie,

t_i is an input place of R_1 if $\exists p_j \in R_2 | c_{ji} < 0$

Definition 10 (Input node). An input node of R_1 is an input place or transition of R_1.

Definition 11 (Output place). Let p_i be a place of R_1. p_i is an output place of R_1 if an arc leaves it towards a transition of R_2, ie,

p_i is an output place of R_1 if $\exists t_j \in R_2 | c_{ij} < 0$

Definition 12 (Output transition). let t_i be a transition of R_1. t_i is an output transition of R_1 if an arc leaves it towards a place of R_2, ie,

t_i is an output place of R_1 if $\exists p_j \in R_2 | c_{ji} > 0$

Definition 13 (Output node). An output node of R_1 is an output place or transition of R_1.

After defining these concepts, we can define the sets thereof.

Notation. We denote the sets of the elements defined above:

- Let IP $(R) \subseteq \overline{P}$ (Input Places) be the set of input places of a subnet.
- Let IT $(R) \subseteq \overline{T}$ (Input Transitions) be the set of input transitions of a subnet.
- Let IN $(R) \subseteq \overline{P} \cup \overline{T}$ (Input Nodes) be the set of input nodes of a subnet.
- Let OP $(R) \subseteq \overline{P}$ (Output Places) the set of output places of a subnet.
- Let OT $(R) \subseteq \overline{T}$ (Output Transitions) be the set of output transitions of a subnet.
- Let ON $(R) \subseteq \overline{P} \cup \overline{\mathrm{T}}$ (Output Nodes) be the set of output nodes of a subnet.

Note. Recall that a node in a Petri net can be both a place and a transition, depending on the context.

Notation. Denote as n_i to a node of a Petri net.

As we have generic definitions, no problem in applying to a network divided into H, V, HN and HT, as the set $\{ H, V \}$ is a partition of R.

8.2. Subnet Front-end

Once all these concepts, we create the front-end input/output of a Petri subnet. A front-end of the Petri net will be a intermediate facade that allows us to physically divide that subnet from the rest of the net. Thus, in order to enter or leave the subnet, you need to make it through this front-end.

Let IA (input arcs) the set of arcs that enter the subnet R_1 and let OA (output arcs) the set of arcs leaving R_1.

Definition 14 (Input gate of a net). Let $a_i \in IA$ an arc of entrance to R_1. We define an input gate to R_1, and denote by ig_i, as a new logical node that is identified with an arc of entrance to the net. For each input arc, defines an input gate, regardless of the origin and destination of the arc. If the source is a transition, we denote igt_i and if a place, igp_i.

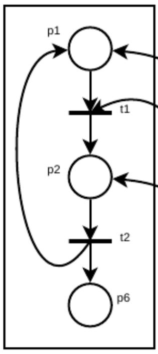
Definition 15 (Output gate of a net). Let a_i *in*OA output arc R_1. We define an output gate of R_1, and denote by og_i, as a new logical node that is identified with an exit arc of the net. For each exit arc is defined an output gate, regardless of the origin and destination of the arc. If the source is a transition, we denote ogt_i and if it is a place, ogp_g.

In this way we can divide the input arcs and output into two parts: a R_1 internal and external to R_1. If we take an arc of entrance ai that has an origin in n_j and destination in n_k, we define an input gate through a point of entry so that the original arc ai is divided into two parts.

- a_{i1} (external to R_1) with origin in n_j and destination in igt_i or igp_i depending on if n_j is a transition or a place.
- a_{i2} (internal to R_1) with destination in igt_i or igp_i depending on if n_j is a transition or a place respectively.

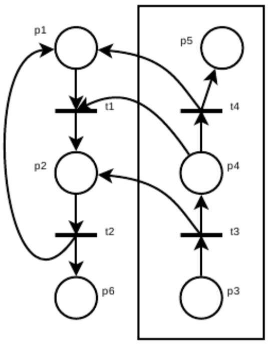
Similarly, if we take a exit arc a_i that has an origin in n_k and destination in n_j, we define an output gate og_i so that the original arc a_i is divided into two parts:

- a_{i1} (internal to R_1) with origin in n_k and destination in igt_i or igp_i depending on if n_j is a transition or a place.



$$C = \begin{array}{c} \\ p_1 \\ p_2 \\ p_6 \\ p_3 \\ p_4 \\ p_5 \end{array} \begin{array}{cc|cc} t_1 & t_2 & t_3 & t_4 \\ -1 & 1 & 0 & 1 \\ 1 & -1 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ \hline 0 & 0 & -1 & 0 \\ -1 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{array}$$

a) Sinkhole subnet

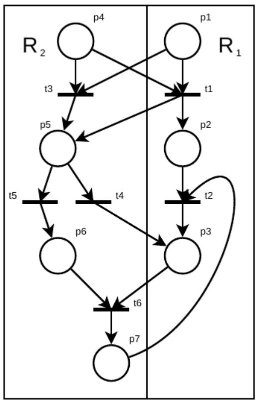


$$C = \begin{array}{c} \\ p_3 \\ p_4 \\ p_5 \\ p_1 \\ p_2 \\ p_6 \end{array} \begin{array}{cc|cc} t_3 & t_4 & t_1 & t_2 \\ -1 & 0 & 0 & 0 \\ 1 & -1 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ \hline 0 & 1 & -1 & 1 \\ 1 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{array}$$

b) Source subnet

Figure 4 – Sinkhole and Source subnets

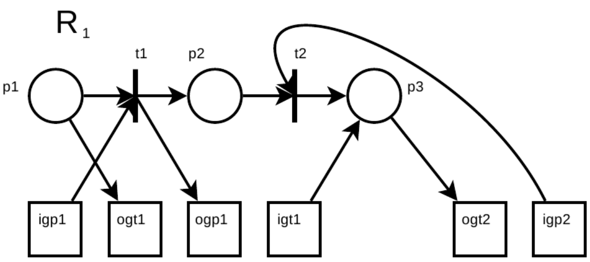
- a_{i1} (internal to R_1) with origin in n_k and destination in *igt*$_i$ or *igp*$_i$ depending on if n_j is a transition or a place.
- a_{i2} (external to R_1)with destination in n_j and origin in *igt*$_i$ or *igp*$_i$ depending on if n_j is a transition or a place respectively.



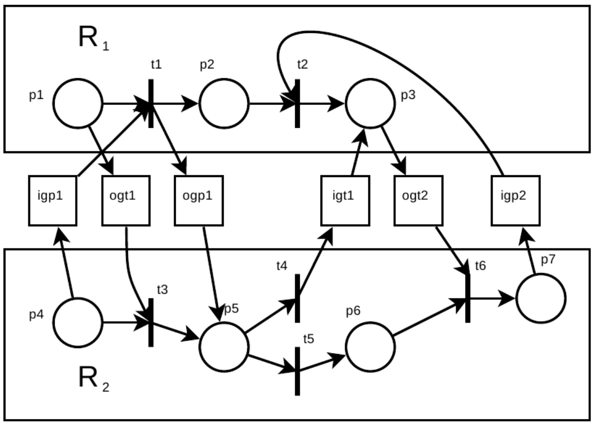
$$C = \begin{array}{c} \\ p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \\ p_6 \\ p_7 \end{array} \begin{array}{cc|cccc} t_1 & t_2 & t_3 & t_4 & t_5 & t_6 \\ -1 & 0 & -1 & 0 & 0 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & -1 \\ \hline -1 & 0 & -1 & 0 & 0 & 0 \\ 1 & 0 & 1 & -1 & -1 & 0 \\ 0 & 0 & 0 & 0 & 1 & -1 \\ 0 & -1 & 0 & 0 & 0 & 1 \end{array}$$

Figure 5 – Subnets with input and output nodes

Example 8. Consider the net in figure 5. In this network we have three arcs entering and leaving three arcs. For each of those emerging define output gates and each coming, we define input gates. The subnet R_1 becomes:



and in the complete net, arcs entering and leaving are divided into two pieces:



Definition 16 (Input Front-end of a net). The input front-end (or input interface) of a subnet R_1 is the set of all input gates of R_1. We denote by *IF* of R_1.

Definition 17 (Output Front-end of a net). The output front-end (or output interface)of a subnet R_1 is the set of all output gates of R_1. We denote by *OF* of R_1.

Definition 18 (Front-end of a net). The front-end (or interface) of a net R_1 is the pair of *IF* and *OF* of R_1. We denote by *F* of R_1.

$$F = \langle IF, OF\rangle$$

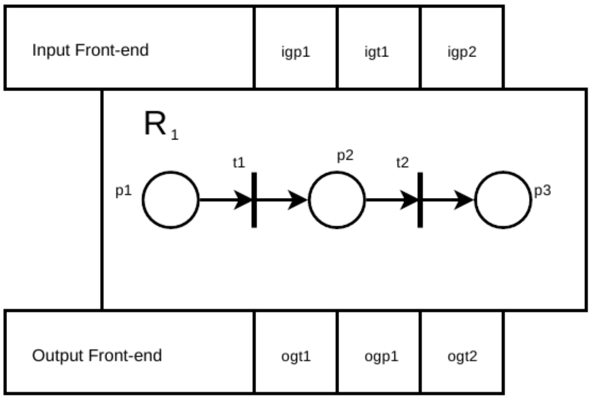


Figure 6 – Front-end of a net

Example 9. Taking the net of the example 8 and applying these new definitions, we would have R_1 net along with its front end as shown in Figure 6.

8.3 Input/output functions

Once all these input and output concepts defined, we will introduce a few key concepts for our purpose.

Let R a Petri net and let $\{R_1, R_2\}$ a partition of R. Let $F = \langle IF, OF\rangle$ the front-end of R_1.

Definition 19 (Petri net Input function). We define the input function f_i of R_1 as:

$$f_i : F \longrightarrow IN$$

such that for each input gate igt_i you mapped one or no input place R_1 and each input gate igp_j you mapped one or no input transition R_1.

$$f_o : ON \longrightarrow F$$

such that each output place R_1 you mapped one or no output gate ogt_i of R_1 and each output transition R_1 you mapped one or no output gate ogp_j to R_1

The input function can be defined for all the input gates and the output function should be surjective because if not, some door would not be connected. Anyway that is not essential. If a front-end door is not connected with any element of your network, simply by solving the final network, the arcs connected to that door disappear. Note also that the input function is not necessarily injective: Multiple input gates can be associated to the same node of R_1.

Example 10. Consider the net R_1 in figure 5 with its front-end in figure 6. The input and output functions are:

• Input function:

F	igp_1	igt_1	igp_2
IN	t_1	p_3	t_2

• Output function:

ON	p_1	t_1	p_3
F	ogt_1	ogp_1	ogt_2

8.4. Attachable net

By joining the subnet R_1 along with its front-end and its input and output functions f_i and f_o we grouped both the internal network with external communication. This way we can " extract" a subnet and "implant" it in another net. You only need this destination network is to communicate with the front-end. So naturally appears the following definition.

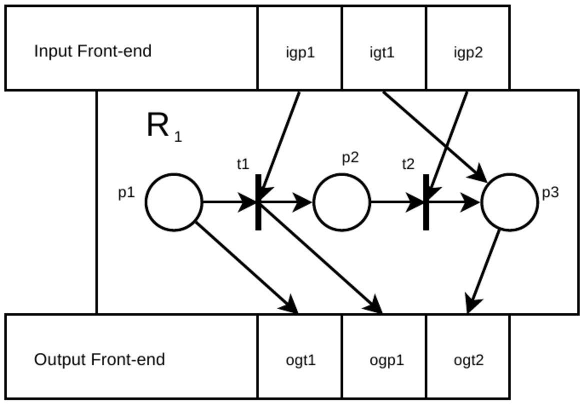
Definition 21 (Attachable Petri net). An [Attachable Petri net is a quadruple $R_a = \langle R, F, f_i, f_o\rangle$

From these definitions, it is clear that you can create attachable subnets taking a subnet of another given and applying the whole process we have defined. But it is also possible to create from scratch, starting from a network, defining a front end for that network and declaring the input and output functions. So you can create Petri nets modules providing functionality and out through a front-end without requiring the actual implementation.

Example 11. The attachable net in figure 5 would be the next:

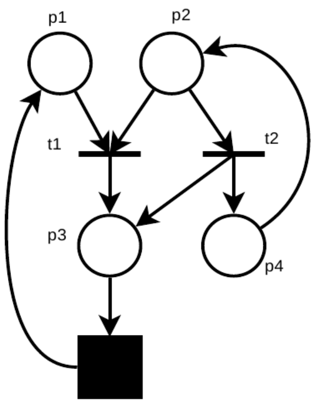
It can be seen as a private black box with visible input and output connectors that are "plugged" to other networks. In a attachable net, the private part would be R, f_i and f_o. The public part of the front-end would be F. All a net need to know is the input/output front-end.

A utility of these nets is that its definition is simple, since only the front-end is needed to define its operation. This makes possible to create nets using



attachable nets in certain areas where they do not know their actual implementation, but its behavior. Additionally, it is possible to use different implementations of "network providers" of the same attachable nets, using at each moment the most appropriate one.

Example 12. Consider now the following Petri net



to which we want to connect an attachable net in the black box. Let's assume we have two equivalent alter natives described in Example 6:

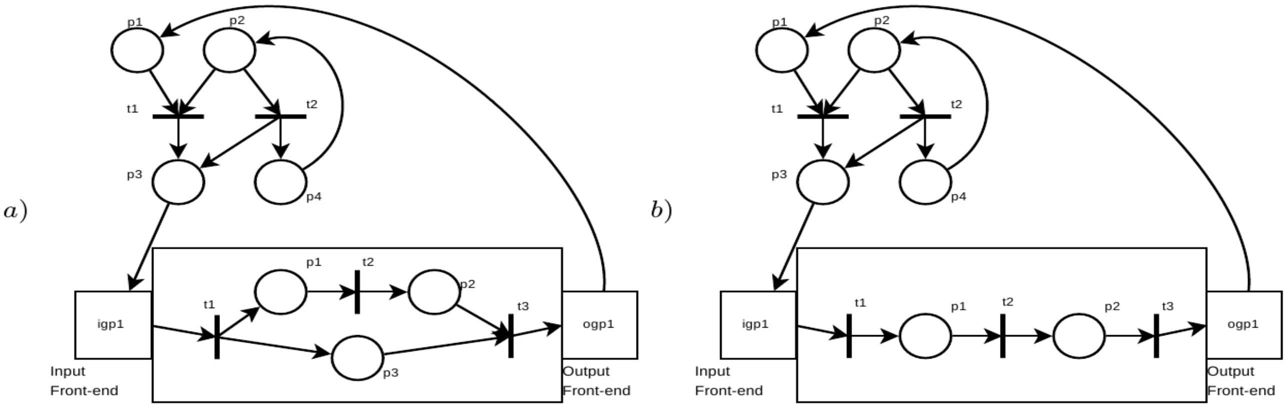
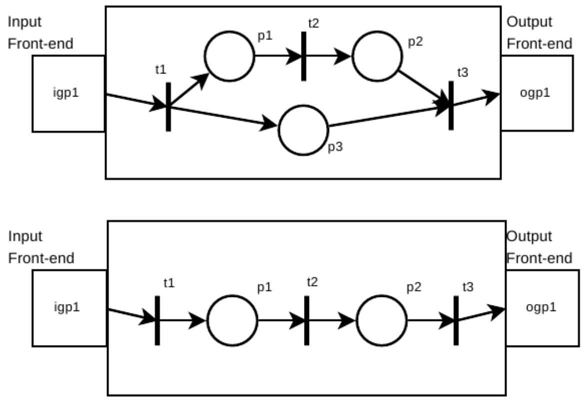


Figure 7 – Two different implementations of attachable nets



We Can "plug" either because their front ends are equivalent and remains in figure 7.

In this case, the behavior of the net will be the same, but does not have to be. That will decide who connects nets. For example, you could create a " silly" net that does nothing at first and replace it later by the real one.

9. CONCLUSIONS

Throughout this paper we have presented Petri nets with definitions and basic properties. From this initial presentation, a series of elements have been building as a basis for further investigation. In particular, a type of subtents has been defined, the subnets classifications have been studied, and the front-ends (interfaces) for those subnets have defined.

From this point a further study of these subnets (their properties, utilities, ...) is possible, and constitutes the line of continuity of this piece of research.

Therefore, the main contribution of this work has been to establish the basis for the methodological study of hiding parts of Petri nets.

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