# MODELING AND SIMULATION OF A ONE-WAREHOUSE, *N*-RETAILER INVENTORY SYSTEM: REASSESSING A NEGATIVE BINOMIAL APPROXIMATION

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## ABSTRACT

Some studies in the multi-echelon inventory systems literature have used a negative binomial distribution to approximate that of a critical random variable arising in the inventory model. Graves (1996) developed a model with fixed replenishment intervals where each site follows a base stock policy. He proposed – in the one-warehouse, *N*-retailer case – a negative binomial distribution to approximate a random variable which he referred to as "uncovered demand". Computational evidence was provided to demonstrate the effectiveness of the approximation. Graves then suggested search procedures for approximately optimal base stock levels at the warehouse and *N* identical retailers under two customer service criteria: (i) probability of no stockout and (ii) fill rate. A separate analytical evaluation of the negative binomial approximation has also been reported elsewhere. In the current study, we apply a modeling and simulation approach to assess whether the approximation-based search procedures, in fact, lead to optimal stock levels.

Keywords: one-warehouse and *N*-retailer inventory system, multi-echelon inventory model, base stock policy, negative binomial approximation, modeling and simulation

## 1. INTRODUCTION

Since the seminal work by Clark and Scarf (1960), many studies on multi-echelon inventory systems have appeared in the literature. These systems involve two or more levels of entities handling or storing inventory of an item or items. Typical entities in a distribution network, for example, would be distribution centers or warehouses, at national, regional, or sub-regional levels, as well as retail sites.

Simplifying assumptions which have been made in some multi-echelon models have allowed for mathematical tractability. However, in order to better capture the complexities of multi-echelon systems that actually exist in practice, those restrictive assumptions have had to be relaxed in favor of more realistic ones (e.g., random demand or stochastic leadtimes). Some of the more complex elements of the resulting models have sometimes required the introduction of approximations to continue to permit analytical investigation.

The negative binomial distribution (NBD), with discrete density function (e.g., Mood, Graybill, and Boes 1974):

$$f(x;r,p)=\binom{r+x-1}{x}p^r(1-p)^x I_{\{0,1,2,\ldots\}}(x), \qquad (1)$$

has parameters $p$ and $r$. The parameter $p$ is a probability of "success" ($0<p\le 1$) and the parameter $r$ (a positive integer) is a target number of successes. A realization $x$ of the random variable $X$ represents a number of failures before the $r^{th}$ success is attained.

An NBD was earlier used by Graves (1985) as an approximation to the distribution of outstanding orders for a repairable item under one-for-one replenishment in a two-echelon system involving $N$ operating sites supported by a repair depot. The distribution of $Q_i(t)$, the outstanding orders at operating site $i$ at time $t$, is observed to be unimodal, with its variance greater than its mean. Graves proposed to approximate the distribution of $Q_i$ by a negative binomial distribution with the same mean and variance as the exact distribution. He reported that the approximation erred in only 0.9% of 1,968 test cases in specifying stockage levels that would minimize inventory holding and shortage costs. Graves reported that, in comparison, the METRIC model developed by Sherbrooke (1968) understated stockage requirements in 11.5% of the cases.

Lee and Moinzadeh (1987) confirmed the effectiveness of the negative binomial approximation under a more general setting: a two-echelon model for a repairable item with batch ordering at the operating sites. The approximation is excellent, with a maximum percentage of cost deviation of 2% for the special case when the batch size is one, which corresponds to the one-for-one replenishment policy assumed by Graves (1985). The performance of the approximation appears to deteriorate for larger batch sizes, but the maximum cost deviations are below 9% up to a batch size of seven.

Graves (1996) reported the negative binomial approximation of a random variable referred to as *uncovered demand* to be “a very accurate approximation.” However, he provided only an illustration of the accuracy of the approximation, while citing that the effectiveness of such an approximation had been shown by Graves (1985) and Lee and Moinzadeh (1987) for the systems they considered.

In all three multi-echelon inventory studies cited above that have proposed a negative binomial approximation, *computational* evidence has been offered in support of the proposed approximation. A first *analytical* evaluation of the negative binomial approximation was reported by Solis, Schmidt, and Conerly (2007), applying to the latest of the three models (Graves, 1996). In the current study, we apply the modeling and simulation approach to evaluate Graves’ NBD approximation. In effect, Modeling and Simulation have proved to be one of the most powerful approaches when dealing with complex stochastic systems (Bruzzone, 2002; Bruzzone, 2004); in particular, the authors have a long experience in using Modeling and Simulation based approaches for inventory management problems (see for instance, De Sensi et al., 2008; Longo and Mirabelli, 2008; Curcio and Longo, 2009).

This paper is organized as follows. In section 2, we present a summary of Graves’ (1996) model. We discuss our modeling and simulation approach, as well as our preliminary simulation results, in section 3. In the final section, we present our conclusion and expected directions for further study.

## 2. GRAVES’ MODEL

In this section, we present a slightly modified version of a summary, as earlier prepared by Solis and Schmidt (2009), of the major assumptions and results in Graves’ (1996) model.

The model involves an arborescent system with $M$ inventory sites each having a single internal supplier, with the exception of site 1, a central warehouse (CW) whose inventory is replenished by an external supplier. Customer demand occurs only at retail sites, at the lowest echelon. All other sites are storage and/or consolidation facilities, called transshipment sites. The unique path linking a retail site to the CW is the *supply chain* for the retail site.

The analysis involves a single item of inventory. The demand at each retail site $j$ is an independent Poisson process with demand rate $\lambda_j$. $D_j(s,t)$ represents the demand over the time interval $(s,t]$ for site $j$. The item under study is included in a multi-item distribution system, with each shipment being a consolidation of orders for various items. Site $j$ places its $m^{th}$ replenishment order on its supplier at preset times $p_j(m)$ with fixed intervals. Fixed positive leadtimes $\tau_j$ are assumed for shipments to site $j$ from its supplier; the $m^{th}$ order is thus received at time $r_j(m) = p_j(m)+\tau_j$. When inventory is in short supply, the supplier will ship less than the quantity ordered and make up for the shortfall on later shipments. Customer demand is fully backordered. The external supplier is fully reliable and fills every order by the CW with a fixed leadtime $\tau_1$. Each site $j$ follows a base stock policy. Initial inventory (at time 0) at site $j$ is the base stock level $B_j$ for site $j$. $T_j(m)$ represents the *coverage* provided by the supplier to site $j$ on its $m^{th}$ order, with $D_j[T_j(m–1),T_j(m)]$ units shipped by the supplier.

Graves’ model assumes *virtual allocation*. Whenever a unit demand occurs at the retail site, each site on the supply chain increases its next order quantity by one. At the same time, each site on the supply chain commits one unit of its inventory, if available, for shipment to the downstream site on the latter’s next order occasion. Virtual allocation, while possible under current information technology, is not the common practice but is assumed for mathematical tractability. It is found by Graves to be near-optimal in many cases.

A random variable requiring attention is $A_j(t)$, which denotes the *available inventory at site j* at time $t$—on-hand inventory not yet committed for shipment to another site. $A_j(t) < 0$ indicates backorders. Graves establishes that, if $r_j(m) \le t < r_j(m+1)$, then

$$A_j(t) = B_j - D_j[T_j(m),t]. \qquad (2)$$

Let site $i$ be the internal supplier to site $j$. If $T_j(m) < p_j(m)$, then $T_j(m)$ equals the time when site $i$ would run out of available inventory to allocate to site $j$. Consider the *relevant* shipment to the supplier $i$ such that, at $p_j(m)$, site $i$ has received its $n^{th}$ shipment but not yet its $(n+1)^{th}$ shipment. Suppose that, based upon its receipt of this $n^{th}$ shipment, site $i$ is able to cover the demand processes of its successor sites up through time $S_i(n)$. We call $S_i(n)$ the *depletion* or *runout time* for this $n^{th}$ shipment to site $i$. It follows that

$$T_j(m) = min\{p_j(m),S_i(n)\}. \qquad (3)$$

Then $S_i(n) - T_i(n)$ is the *buffer time* provided by $B_i$, and

$$S_i(n) - T_i(n) \sim \text{gamma}(\lambda_i,B_i). \qquad (4)$$

Graves then focuses on a two-echelon system consisting of sites 1 (the CW) and $j$ ($N$ retailers). A *single-cycle* ordering policy is in place: each retailer orders a fixed number of times for every order placed by the CW. If $\theta_1$ and $\theta_j$ respectively denote the CW and retail site order cycle lengths, $\theta_1/\theta_j$ is a positive integer. The ordering policy is also *nested*: every time the CW receives a shipment, all retail sites place an order.

Consider an arbitrary ($n^{th}$) CW order cycle. Graves simplifies the analysis by setting time zero equal to $p_1(n)$. Graves draws attention to the last, or $(\theta_1/\theta_j)^{th}$, retail site order within the CW order cycle, placed at time $p_j = \tau_1+\theta_1–\theta_j$ and received at time $p_j+\tau_j$. The resulting available inventory will be used to cover demand until the next order, placed at time $p_j+\theta_j$, arrives at the retail site at time $t_r = \tau_1+\theta_1+\tau_j$. The instant of time $t_r^-$ *just before* this replenishment proves crucial to the analysis. In this case, $r_j(m) \le t_r^- < r_j(m+1)$ holds.

Treating this $(\theta_1/\theta_j)^{th}$ order as the $m^{th}$ order for the retail site $j$ within the $n^{th}$ CW order cycle, the indices $m$ and $n$ are henceforth dropped for notational convenience. Rewriting (3), $T_j = min\{p_j,S_1\}$ is the coverage provided by the $(\theta_1/\theta_j)^{th}$ shipment to retail site $j$. From (4), $S_1 \sim \text{gamma}(\lambda_1,B_1)$.

Graves defines a random variable $D_j[T_j,t]$, which he calls *uncovered demand* (up to some specified point in time $t$): demand at retail site $j$ not covered by the $(\theta_1/\theta_j)^{th}$ shipment from the CW. He derives the following mean and variance:

$$E\{D_j[T_j,t]\} = \lambda_j\,(t - E[T_j]) \qquad (5)$$

$$Var\{D_j[T_j,t]\} = \lambda_j\,(t - E[T_j]) + \lambda_j^2\,Var[T_j]. \qquad (6)$$

He reports computationally finding the NBD having the same first two moments to be a fairly accurate approximation to the distribution of $D_j(T_j,t)$. Graves presents very limited evidence in support of his assertion, however. Based on (5) and (6), the parameters of the NBD approximation are determined as follows:

$$r = (t - E[T_j])^2 / Var[T_j] \qquad (7)$$

$$p = (t - E[T_j]) / \{(t - E[T_j]) + \lambda_j Var[T_j]\}. \qquad (8)$$

Graves proposes a procedure for each of the two most commonly specified service level criteria (Silver, Pyke, and Peterson 1998) that would search for a base stock policy $\langle B_1,B_j\rangle$ that minimizes expected on-hand inventory in the system. The first service criterion is an average *probability of no stockout* $\alpha$. The other service measure is an average fraction of demand to be satisfied from stock on hand, or *fill rate* $\beta$.

**2.1. Probability of No Stockout as Service Criterion**

The probability of the retail site stocking out is greatest for the $(\theta_1/\theta_j)^{th}$ retail order within the CW order cycle. To set base stock levels to achieve a given probability $\alpha$ of the retail site *not* stocking out within the CW order cycle,

$$\Pr\{A_j(t_r^-)\geq 0\} \geq \alpha, \qquad (9)$$

needs to be assured. The distribution of uncovered demand $D_j(T_j,t_r)$, where $t_r = \tau_1+\theta_1+\tau_j$ as discussed above, comes into play and leads to a computational procedure that searches over possible settings of the CW base stock level $B_1$. For each $B_1$, the minimum retail site base stock level $B_j$ that would yield (9) is to be determined. The antecedent $r_j(m) \leq t_r^- < r_j(m+1)$ of (1) being satisfied, it follows that requirement (9) translates into

$$\Pr\{D_j(T_j,t_r)\leq B_j\} \geq \alpha. \qquad (10)$$

We apply the negative binomial approximation $\sum_{x=0}^{B_j}\binom{r+x-1}{x}p^r(1-p)^x$ for the left hand side of (10) – starting with $B_j = 1$, and incrementing $B_j$ by 1 until (10) is first satisfied. The base stock level $B_1$ which yields the lowest average system inventory is selected. (In the case of ties, the smallest value of $B_1$ is preferred, there being no difference assumed between holding costs at the CW and the retail sites.)

Graves provides an approximation to expected system on-hand inventory:

$$\text{Avg. inventory} = B_1 + \sum_1^N B_j - 0.5\lambda_1\theta_1 - \lambda_1\tau_1. \qquad (11)$$

Strictly speaking, (11) should be corrected for counting retail backorders at negative inventory. For reasonable service levels, however, the expected backorder component is very small and is ignored.

**2.2. Fill Rate as Service Criterion**

For "realistic" fill rates (> 0.95), expected backorders over a CW order cycle may be approximated by expected backorders pertaining to the $(\theta_1/\theta_j)^{th}$ retail order, since effectively all backorders occur at this last retail order. $E[\{A_j(t_r)\}^-]$, where $y^- = max\{0,-y\}$, represents expected backorders at time $t_r$ (just before the next order arrives). A computational procedure similar to that in sub-section 2.1 arises. For each $B_1$, a minimum retail site base stock level $B_j$ is sought that would yield

$$E\left[\{A_j(t_r)\}^-\right] = \left\{\sum_{x=0}^{B_j}\left[(B_j - x)\,f(x)\right]\right\} - \{B_j - \lambda_j(t_r - E[T_j]\}$$
$$\leq (1-\beta)\,\lambda_j\theta_1, \qquad (12)$$

where $\lambda_j\theta_1$ represents mean demand at the retail site over the CW order cycle. The base stock level $B_1$ which yields the lowest average system inventory is chosen.

**2.3. Graves' Computational Study**

In his computational study, Graves used test scenarios all based on a single system demand rate $\lambda_1 = 36$. Identical retail sites are assumed, with the number $N$ of retail sites being 2, 3, 6, or 18. Hence, the retail site demand rates $\lambda_j$ are 18, 12, 6, or 2, respectively. The length of the retail site order cycle is fixed at $\theta_j = 1$ time unit. Four different parameter combinations $\langle\theta_1,\tau_1,\tau_j\rangle$ are tested. This resulted in 16 test scenarios, summarized in Table 1.

Table 1: Summary of Graves' Test Scenarios

| Scenario | $\theta_1$ | $\tau_1$ | $\tau_j$ | $N$ | $\lambda_j$ |
|---|---|---|---|---|---|
| 1 | 2 | 1 | 1 | 18 | 2 |
| 2 | | | | 6 | 6 |
| 3 | | | | 3 | 12 |
| 4 | | | | 2 | 18 |
| 5 | 2 | 1 | 5 | 18 | 2 |
| 6 | | | | 6 | 6 |
| 7 | | | | 3 | 12 |
| 8 | | | | 2 | 18 |
| 9 | 5 | 4 | 1 | 18 | 2 |
| 10 | | | | 6 | 6 |
| 11 | | | | 3 | 12 |
| 12 | | | | 2 | 18 |
| 13 | 5 | 4 | 5 | 18 | 2 |
| 14 | | | | 6 | 6 |
| 15 | | | | 3 | 12 |
| 16 | | | | 2 | 18 |

For the probability of no stockout criterion, four levels of α were used: 0.80, 0.90, 0.95, and 0.975. Similarly, four fill rate levels β were tested: 0.95, 0.98, 0.99, and 0.999. Thus, for each service criterion, 64 test cases were considered by Graves.

## 3. MODELING AND SIMULATION

### 3.1. Simulation Models

Solis, Schmidt, and Conerly (2007) reported the very first *analytical* evaluation of the effectiveness of the negative binomial approximation used in Graves' (1996) model. Prior to their analytical evaluation, only *computational* evidence had been offered in all three multi-echelon inventory studies earlier cited (Graves 1985; Lee and Moinzadeh 1987; Graves 1996) in support of the proposed NBD approximation to a crucial random variable.

In the current study, we apply the modeling and simulation (M&S) approach to evaluate the NBD approximation as proposed by Graves (1996). We create simulation models using the AnyLogic platform for the sixteen scenarios as summarized in Table 1. For each simulation model, we test the "optimal" base stock policy $<B_1,B_j>$ as determined by Graves' search procedure (based upon the NBD approximation of uncovered demand). Each simulation run is over 100 CW order cycles – i.e., 200 retail site order cycles for scenarios 1-8 or 500 retail site order cycles for scenarios 9-16. Our simulation experiments involve 100 replications each; hence, 20,000 or 50,000 retail site order cycles for scenarios 1-8 or 9-16, respectively. Depending upon whether the simulated service level is below or above the target service level, we increase or decrease $B_1$ or $B_j$ one unit at a time until the simulated α or β is at or just over the target level. The "optimal" base stock policy from Graves' search procedure is then compared against the optimal policy obtained using M&S. To attain comparability of simulated service levels under different pairs of $B_1$ and $B_j$ values being assessed, we apply fixed random number seeds in generating the Poisson demand streams at the retail sites.

### 3.2. Simulation Results for the Probability of No Stockout Criterion

In Table 2, we compare Graves' "optimal" base stock policies $<B_1,B_j>$ under the probability of no stockout criterion for scenarios 4, 8, 12, and 16 (where $N$ = 2 retail sites) against the optimal policies using the M&S approach. In our simulation experiments, we find that Graves' "optimal" policies meet the target (minimum) α in only seven of the 16 test cases, and are thus equal to the M&S optimal policies. For the remaining nine test cases, the simulated α from Graves' optimal policy is below the target α. However, the echelon base stocks arising from the M&S optimal policy is not more than two units over that resulting from Graves' "optimal" policy.

Table 2: Comparison of Simulation Results under the Probability of No Stockout Criterion when $N$ = 2

| | "Optimal" Policy Using Graves' Search Procedure | | | Simulated | Optimal Policy Using Simulation | | | Simulated |
|---|---|---|---|---|---|---|---|---|
| Scenario | $B_1$ | $B_j$ | Ech Base St | α | $B_1$ | $B_j$ | Ech Base St | α |
| **α = 0.80** | | | | | | | | |
| 4 | 44 | 56 | 84 | 0.7996 | 45 | 56 | 85 | 0.8164 |
| 8 | 7 | 150 | 235 | 0.8045 | 7 | 150 | 235 | 0.8045 |
| 12 | 245 | 66 | 143 | 0.7959 | 246 | 66 | 144 | 0.8085 |
| 16 | 248 | 139 | 292 | 0.7991 | 249 | 139 | 293 | 0.8092 |
| **α = 0.90** | | | | | | | | |
| 4 | 53 | 55 | 91 | 0.9000 | 53 | 55 | 91 | 0.9000 |
| 8 | 37 | 140 | 245 | 0.9019 | 37 | 140 | 245 | 0.9019 |
| 12 | 272 | 57 | 152 | 0.8942 | 273 | 57 | 153 | 0.9015 |
| 16 | 268 | 135 | 304 | 0.8962 | 269 | 135 | 305 | 0.9017 |
| **α = 0.95** | | | | | | | | |
| 4 | 55 | 57 | 97 | 0.9520 | 55 | 57 | 97 | 0.9520 |
| 8 | 61 | 132 | 253 | 0.9499 | 62 | 132 | 254 | 0.9540 |
| 12 | 283 | 55 | 159 | 0.9411 | 283 | 56 | 161 | 0.9524 |
| 16 | 272 | 138 | 314 | 0.9496 | 273 | 138 | 315 | 0.9529 |
| **α = 0.975** | | | | | | | | |
| 4 | 60 | 57 | 102 | 0.9762 | 60 | 57 | 102 | 0.9762 |
| 8 | 53 | 140 | 261 | 0.9759 | 53 | 140 | 261 | 0.9759 |
| 12 | 287 | 56 | 165 | 0.9681 | 290 | 56 | 168 | 0.9757 |
| 16 | 271 | 143 | 323 | 0.9755 | 271 | 143 | 323 | 0.9755 |

For scenario 3 (with $N$ = 3 retail sites), Graves' "optimal" policies meet the target α in all four test cases. However, we have found that Graves' optimal policies meet the target α in only about one-fourth of all the 64 test cases. Moreover, we have observed that the deviations between simulated and target service levels tend to become larger as $N$ increases – and are thus largest for scenarios 1, 5, 9, and 13 (with $N$ = 18 retail sites) than in corresponding scenarios with fewer retail sites. These deviations also tend to be larger with a longer CW order cycle (i.e., when $\theta_1 = 5$, as compared to $\theta_1 = 2$). Furthermore, these deviations become more pronounced with lower target α levels, particularly when α = 0.90 and 0.80.

### 3.3. Simulation Results for the Fill Rate Criterion

We show in Table 3 the comparisons between Graves' "optimal" and the M&S optimal base stock policies under the fill rate criterion for scenarios 4, 8, 12, and 16 (with $N = 2$ retail sites). In our simulation experiments, the policies determined using Graves' search procedure meet the target β in only five of the 16 test cases. The simulated β from Graves' "optimal" policy is below the target β in the remaining 11 test cases. In one of these test cases, the echelon base stocks arising from the M&S optimal policy is four units more than that from Graves' "optimal" policy, but not more than two units in the remaining cases.

None of Graves' "optimal" policies meets the target β in any of the four test cases under scenario 3 (where $N = 3$). The echelon base stocks for the M&S optimal policies in these four test cases, however, are only either one or two units more than for Graves' "optimal" policies.

We have found that Graves' optimal policies meet the target β in only five of the 64 test cases. As in the probability of no stockout service criterion, we have observed that the deviations between simulated and target β levels tend to become larger as $N$ increases – and are thus largest for scenarios 1, 5, 9, and 13 (where $N = 18$) relative to corresponding scenarios with smaller $N$. These deviations also tend to be larger with a longer CW order cycle. Moreover, these deviations are less pronounced with higher target β levels, and are more pronounced as target β levels become lower.

Table 3: Comparison of Simulation Results under the Fill Rate Criterion when $N = 2$

**β = 0.95**

| | "Optimal" Policy Using Graves' Search Procedure | | | Simulated | Optimal Policy Using Simulation | | | Simulated |
|---|---|---|---|---|---|---|---|---|
| Scenario | $B_1$ | $B_j$ | Ech Base St | β | $B_1$ | $B_j$ | Ech Base St | β |
| 4 | 56 | 47 | 150 | 0.9477 | 57 | 47 | 151 | 0.9523 |
| 8 | 53 | 125 | 303 | 0.9460 | 52 | 126 | 304 | 0.9504 |
| 12 | 259 | 50 | 359 | 0.9486 | 260 | 50 | 360 | 0.9513 |
| 16 | 207 | 151 | 509 | 0.9506 | 207 | 151 | 509 | 0.9506 |

**β = 0.98**

| | "Optimal" Policy Using Graves' Search Procedure | | | Simulated | Optimal Policy Using Simulation | | | Simulated |
|---|---|---|---|---|---|---|---|---|
| Scenario | $B_1$ | $B_j$ | Ech Base St | β | $B_1$ | $B_j$ | Ech Base St | β |
| 4 | 55 | 52 | 159 | 0.9804 | 55 | 52 | 159 | 0.9804 |
| 8 | 51 | 132 | 315 | 0.9798 | 52 | 132 | 316 | 0.9813 |
| 12 | 267 | 53 | 373 | 0.9795 | 268 | 53 | 374 | 0.9810 |
| 16 | 259 | 133 | 525 | 0.9791 | 260 | 133 | 526 | 0.9803 |

**β = 0.99**

| | "Optimal" Policy Using Graves' Search Procedure | | | Simulated | Optimal Policy Using Simulation | | | Simulated |
|---|---|---|---|---|---|---|---|---|
| Scenario | $B_1$ | $B_j$ | Ech Base St | β | $B_1$ | $B_j$ | Ech Base St | β |
| 4 | 64 | 50 | 164 | 0.9892 | 65 | 50 | 165 | 0.9906 |
| 8 | 47 | 138 | 323 | 0.9904 | 47 | 138 | 323 | 0.9904 |
| 12 | 275 | 53 | 381 | 0.9891 | 276 | 53 | 382 | 0.9900 |
| 16 | 269 | 133 | 535 | 0.9888 | 269 | 134 | 537 | 0.9905 |

**β = 0.999**

| | "Optimal" Policy Using Graves' Search Procedure | | | Simulated | Optimal Policy Using Simulation | | | Simulated |
|---|---|---|---|---|---|---|---|---|
| Scenario | $B_1$ | $B_j$ | Ech Base St | β | $B_1$ | $B_j$ | Ech Base St | β |
| 4 | 65 | 57 | 179 | 0.9991 | 65 | 57 | 179 | 0.9991 |
| 8 | 58 | 143 | 344 | 0.9991 | 58 | 143 | 344 | 0.9991 |
| 12 | 288 | 56 | 400 | 0.9984 | 288 | 58 | 404 | 0.9990 |
| 16 | 279 | 141 | 561 | 0.9988 | 280 | 141 | 562 | 0.9990 |

## 4. CONCLUSION AND FURTHER WORK

Departing from the traditional computational and analytic approaches to looking into the effectiveness of distributions used to approximate exact distributions of random variables arising in multi-echelon inventory models, we have applied the M&S approach in the evaluation of a negative binomial approximation as proposed by Graves (1996) in a one-warehouse, N-retailer inventory system.

Computational evidence offered by Graves (1996) suggests the NBD approximation to be fairly accurate, and an analytical investigation (Solis, Schmidt, and Conerly 2007) has suggested why the approximation is effective in certain instances.

In our simulation studies to date, we have found Graves' search procedures, based on his NBD approximation, to be less effective when the number of retail sites is larger, the CW order cycle is longer, or when the target service level is lower. At the time of the conference, we will provide a more thorough report of our findings.

Solis and Schmidt (2007, 2009) have introduced stochastic leadtimes $\tau_j$ between the CW and the retail sites and investigated how optimal base stock policies differ between deterministic and stochastic leadtime cases when the CW does not or does carry stock. In the former situation where the CW does not carry stock (as in a distribution center with cross-docking), an analytical investigation was reported (Solis and Schmidt 2007). In the latter situation where the CW actually carries stock, with the model becoming mathematically intractable, Solis and Schmidt (2009) applied an M&S-based heuristic taking off from Graves' search procedures. We will soon apply the M&S approach to the stochastic leadtime $\tau_j$ case, whether the CW carries stock or not.

Further, we will also use the M&S approach to investigate the implications of stochastic leadtimes $\tau_1$ between the external supplier and the CW. In practice, $\tau_1$ is probably more prone to randomness than the internal leadtimes between the CW and retail sites, over which retail firms would expectedly be able to exercise greater control. In investigating stochastic leadtimes $\tau_1$, we would be interested in finding out whether the model is more sensitive to randomness in $\tau_1$ or to randomness in $\tau_j$.

## REFERENCES


Bruzzone A.G., (2002). Introduction to the Special Issue on Simulation in Supply Chain Management. Simulation, vol. 78, p. 284-286, ISSN: 0037-5497.

Bruzzone A.G. (2004). Preface to modeling and simulation methodologies for logistics and manufacturing optimization . Simulation, vol. 80 , p. 119-120 , ISSN: 0037-5497

Clark, A.J. and Scarf H., 1960. Optimal policies for a multi-echelon inventory problem. *Management Science*, 6 (4), 475-490.

Curcio D., Longo F., (2009). Inventory and Internal Logistics Management as Critical Factors Affecting the Supply Chain Performances. International Journal of Simulation & Process Modelling, vol. 5(4), p. 278-288, ISSN: 1740-2123

De Sensi G, Longo F, Mirabelli G (2008). Inventory policies analysis under demand patterns and lead times constraints in a real supply chain. International Journal of Production Research, vol. 46, p. 6997-7016, ISSN: 0020-7543, doi: 10.1080/00207540701528776

Graves, S.C., 1985. A multi-echelon inventory model for a repairable item with one-for-one replenishment, *Management Science*, 31 (10), 1247-1256.

Graves, S.C., 1996. A multiechelon inventory model with fixed replenishment intervals. *Management Science*, 42 (1), 1-18.

Lee, H.L. and Moinzadeh, K., 1987. Two-parameter approximations for multi-echelon repairable inventory models with batch ordering policy. *IIE Transactions*, 19, 140-149.

Longo F., Mirabelli G., (2008). An advanced supply chain management tool based on modeling and simulation. Computers & Industrial Engineering, vol. 54, p. 570-588, ISSN: 0360-8352, doi: 10.1016/j.cie.2007.09.008.

Mood, A.M., Graybill, F.A., and Boes, D.C., 1974. *Introduction to the Theory of Statistics*, 3rd ed. New York, USA: Mc-Graw-Hill.

Sherbrooke, C.C., 1968. METRIC: A multi-echelon technique for recoverable item control. *Operations Research*, 16 (1), 122-141.

Silver, E.A., Pyke, D.F., and Peterson, R., 1998. *Inventory Management and Production Planning and Scheduling*. New York, USA: John Wiley & Sons.

Solis, A.O. and Schmidt, C.P., 2007. Stochastic leadtimes in a one-warehouse, N-retailer inventory system with the warehouse not carrying stock. *European Journal of Operational Research*, 181 (2), 1004-1013.

Solis, A.O. and Schmidt, C.P., 2009. Stochastic leadtimes in a one-warehouse, N-retailer inventory system with the warehouse actually carrying stock. *International Journal of Simulation and Process Modelling*, 5 (4), 337-347.

Solis, A.O., Schmidt, C.P., and Conerly, M.D., 2007. On the effectiveness of the negative binomial approximation in a multi-echelon inventory model: a mathematical analysis. *Proceedings of the International Conference of Numerical Analysis and Applied Mathematics*, American Institute of Physics Conference Proceedings 936, pp. 531-534, September 16-20, Corfu (Greece).

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