

IMPROVEMENT OF ADVANCED MATHEMATICAL SKILLS AND ABILITIES USING THE COMPUTER ALGEBRA BASED E-LEARNING SYSTEM MAPLE T.A.

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ABSTRACT

This paper is about an e-learning environment based on Maple, called Maple T.A. Maple T.A. for stands for Testing and Assessment with Maple. It is used at the Institute for Analysis and Scientific Computing at the Vienna University of Technology. Over the last four years some improvements have been achieved and the number of courses using this tool increased. Also the way of using the e-learning system has changed. At the beginning it was a support for students practicing their mathematical skills. Now it is also a system for testing and assessing. Depending on the course the students have to pass one to three tests each semester to pass the course.

On the other hand Maple T.A. supports the lecturer and organizer as well. All the users and their test results are stored in this system. Therefore using this environment was enhanced through three different courses.

Keywords: Maple, e-learning, mathematics, simulation, testing, assessment

1. INTRODUCTION

At the Institute for Analysis and Scientific Computing the idea of using Maple T.A. was established due to the refresher course. In 2008 the vice-rector of the Vienna University of Technology requested a refresher course supported by an e-learning system. This course should represent the bridging of the gap of knowledge in basic mathematics which comes up between school and university. Most of the male students have to attend military or civilian service. Also some of the students-to-be take time for travelling or au pair. Additionally, pupils from different types of schools graduate with different mathematical levels. For example pupils of tourism schools and pupils of a technical school differ in their mathematical knowledge because of the different focus of their schools. This refresher course should equalize the basic mathematical knowledge and ease the access to university.

To fulfil the requirements the institute was looking for adequate software. One of the advantages of e-learning systems is the availability at any time. On the one hand this tool should help students to coordinate their learning and to give them a feedback about their skills so the students would be able to control their learning progress and success. On the other hand a benefit should be a simplified administration and organization of students and their grading. Therefore Maple T.A. was established.

2. THE DEVELOPMENT (EXPANSION)

Initially the refresher course was only offered in the beginning of September for students of electrical engineering because the Institute for Analysis and Scientific Computing is also involved in all the other mathematical courses for this field of study. Since then, every year another field of study was added to participate of the refresher course. Since two years Maple T.A. is also used for the regular mathematical courses. In 2011 the refresher course was offered additionally in the last week of September for the first time. That year the students of the following fields of studies attended the refresher course.

- Electrical Engineering
- Technical Physics
- Surveying and Geoinformation
- Mechanical Engineering
- Mechanical Engineering - Economics
- Civil Engineering
- Urban and regional planning
- Some students from other fields of study

Because of the simplification due to Maple T.A. and the positive feedback from the students the Institute for Analysis and Scientific Computing started using the system also in the basic mathematical courses. For the first time students had the opportunity to use the

examples on Maple T.A. to understand the subject and to improve their skills.

Up to now Maple T.A. is used in three different ways, the refresher course, two basic mathematical courses and one advanced mathematical course.

3. ABOUT MAPLE T.A.

3.1. In General

Maple T.A. stands for Maple Testing and Assessment. The system is based on Maple, a computer algebra system. On the one hand Maple supports analytic and algebraic calculations. On the other hand it is a very common programming language at the Vienna University of Technology. Currently Maple T.A. version 6.0 is used.

Maple T.A. is designed to simplify the creation of examples and the arrangement of tests. For the examples there are many different possible question types like multiple choice, fill in the blanks, numeric, multipart or true/false questions. The choice of the proper question type depends on the topic of the question and on the aspired goal of the tutor. There are examples which target the logical thinking of the students and some which enhance their numeracy.

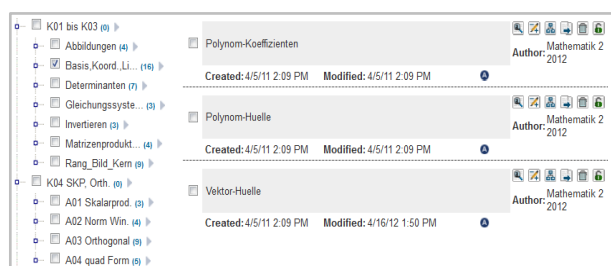


Figure 1: On The Left Side The Question Three Is Shown And On The Right Side The Examples Of The Chosen Subchapter Are Listed.

In general the Maple T.A. system is an interface which contains a Maple kernel. The surface offers many different functions. At first one creates a certain class which is dedicated to the course. This class contains a question repository where all the coded examples are stored and sorted, shown in figure 1. With these questions assignments are created.

Assignment Name	Points	Type
3 UE-Test	16.0	Proctored
2. UE-Test	16.0	Proctored
1 UE-Test	16.0	Proctored
K10 Lokale und globale Verhalten von Funktionen	5.0	Practice
K13 Potenzreihen	5.0	Practice
K12 Integralrechnung	31.0	Practice
K01 Vektorräume	15.0	Practice
K02 Matrizen und lineare Gleichungssysteme	22.0	Practice
K03 Determinanten	7.0	Practice
K04 Skalarprodukt und Orthogonalität	21.0	Practice
K05 Das Eigenwertproblem	11.0	Practice
K06 Differentialrechnung im Mehrdimensionalen Teil 1	12.0	Practice
K06-2 Differentialrechnung im Mehrdimensionalen Teil 2	15.0	Practice

Figure 2: An Overview Of All The Assignments Of Mathematics 2. The First Three Lines Are The Tests.

In figure 2 the assignments, which can help organising the different topics of the lecture, are shown. The students have free access to these assignments if the tutors unlock them. There are different types of assignments. The different topics are anonymous exercises. If one wants to create a test a proctored assignment is necessary. That means that a proctor, in most cases the tutor, has to unlock and close the test again. So the students are not able to make the test at home.

3.2. Maple T.A. Coding

In the Maple T.A. system there are some special commands similar to the Maple ones. But it is also possible and sometimes necessary to use the common and original Maple commands depending on the complexity of the example.

For example an if-command in Maple T.A. looks like the first line of Table 1. The if-query embedded in Maple T.A. including an initialization, which is necessary with an \$-sign, stands below. It is obvious that it is very easy to use the regular Maple commands. The differences are the particular initialization and the active call of Maple. In contrast to the Maple code the same demand within the MTA functions is quite short, as can be seen in the last row of Table 1.

Table 1: The Different Ways For Programming The Same Function In Maple T.A.

Commands	
Maple code	if a<b then a else b end if
Embedded in MTA	\$c = maple("if a<b then a else b end if")
MTA code	\$c:=if(gt(a,b), b, a)

It is also possible to embed a procedure, which means more than one command one after another, separated with a colon, see for example equation 1.

$$\begin{aligned} & \$v1 = \text{maple}("a:=\text{Random}[\text{VecInts}](2,2) : \\ & a(1):=1 : a(2):=0 : \$v1v2.a") \end{aligned} \quad (1)$$

One important advantage of Maple T.A. is that through the creation of one exercise a small pool of different examples results. The principle of the example is every time the same but the numbers or functions are randomized so they change at every request. So the tutors only have to create a few examples to offer a great questionnaire. Additionally the students are not able to cheat at the test that easy because the students sitting next to each other need different results for the same example. The copy of the calculation itself would not help so the student has to think about it although he/she can look at the sheet of paper from the neighbour. So Maple T.A. causes a win-win situation for students, lecturers and tutors.

3.3. Creation Routine

As already mentioned, it is possible to create randomized numbers within Maple T.A. To offer a big variety of examples it is also profitable to create random functions and text as well. Therefore the question designers have developed a new library including some different commands for the creation of examples. There are commands which support the creation of randomized numbers and functions.

For example the command *Random[FromSet]* randomly picks one of the following expressions which are listed in the set like in the following code-line (2).

```
Random[FromSet]({cos(x)^(a), sin(x)^(a)}) (2)
```

For the course Mathematics 2 it was also important to create vectors and matrices in an easy way. In addition to the randomization of the entries of vectors and matrices it is possible to choose the character of the entries as well, for example the amount of zeros or the eigenvalues.

```
Random[MatInts](5, rows = 3, rank = 3) (3)
```

The command above (3) creates a matrix dimensioned 3x3 with integer entries between -5 and 5 except 0. Additionally the matrix will be regular because of the specification *rank=3*, which means the rank of the matrix is the dimension.

This improved library shortens the preparation of examples enormously.

3.4. Grading Routine

Another improvement is the new grading routine. The creating routine supports the designers and tutors more but the development of the grading benefits the students mainly.

This routine enables fair grading for every student. If the question requires a vector with three entries as input one would get no points if only one entry is wrong. The improved grading routine controls every entry so it is possible to get parts of the points. The according command is *Grade[Vec]*. In the same way *Grade[Mat]* works and grades matrices.

The questions concerning the topic of partial fraction expansion causes some complications because they are more difficult to grade. The students enter a long term consisting of many different fractions. The calculation of the factors of these fractions is not so difficult but offers many possibilities for miscalculation. Therefore the improved grading routine checks every single fraction-factor. So the calculation of the students is honoured. This feature makes the e-learning system Maple T.A. more comfortable for the students and so the feedback improves.

4. THE COURSES

As already mentioned, Maple T.A. is used in three different areas. First of all the e-learning system is used in the refresher course in which the mathematical level

of pupils is improved. Secondly there are two basic mathematical courses for the students of Electrical Engineering where this MTA system is used for one year all together. And at last the advanced mathematical course is supported by this system as well. This course is also obligatory for the students of Electrical Engineering. It is obvious, that these students benefit from the system most because they get to know the system in the refresher course. Therefore the usage of MTA in the following lectures does not causes problems or questions anymore.

4.1. Refresher Course

The refresher course was initialized as a “bridging the gap”- course for students of the first semester. The course consists of 8 different topics. They are held in different lectures to offer the students the possibility to visit only the topics, they cannot remember so well. This year there will be ten topics to cover the mathematical requirements of all fields of study.

The course is separated in 2 parts, the lecture and the exercise. The whole course lasts one to two weeks. There are two different timeslots. On the one hand there is the course in the last week of September. An advantage of the September course is that the ordinary courses of the study have not started then. The students can focus on the improvement of their mathematical skills. On the other hand there is one course in the first and second weeks of October. This course is separated in the different fields of study to customize the timetable.

For every module there is a lecture first which takes more than one hour. Afterwards the students have 15 minutes to divide into smaller groups and change the location. There, in the seminar rooms, a tutor is waiting to continue with the exercise part of the module. The practicing takes one and half an hour. The tutor presents many different examples fitting to the lecture. The students also have the possibility to make their first experience at the blackboard in front of the tutor.

Maple T.A. is used in more than one way. On the one hand MTA offers the students the possibility to practice the examples of the different topics whenever they want to. It is also possible to have a look at the examples during the whole semester because the account stays activated. On the other hand there are three voluntary tests. First there is a knowledge test. This test is anonymous so the students have the chance to figure out if they need the refresher course. In the first exercise the students have to make a test to show them and the tutors their current level and knowledge. The last test is in the end of the course. If the students want to receive a mark for that course they have to pass the final exam. Additionally these tests show how much the students profit of this offer.

To get an idea of the level of the Maple T.A. examples the following figure shows a typical example of the refresher course.

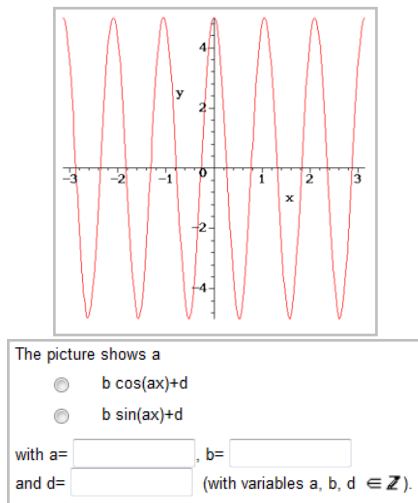


Figure 3: A Typical Example Of The Refresher Course: The Task Of This Question Is The Classification Of The Curve.

In Figure 3 an example is shown which is mostly used in all of the three tests. The identification of mathematical graphs is very important for students of technical fields of studies. There are several variables to figure out. The students have to decide whether the curve is a cosine or a sine. The variables a , b and d stand for the frequency, amplitude and the shift of the sine or cosine. Most of the examples of the refresher course have the same degree of difficulty.

4.2. Basic Mathematical Courses

Another course, where Maple T.A. is used, is obligatory in the first two semesters of Electrical Engineering. Around 300 students attend these two courses, Mathematics 1 and Mathematics 2. Mathematics 1 focuses on Analysis and Mathematics 2 on Linear Algebra. Because of the amount of students the usage of an e-learning system suggests itself, especially an assessment system. It eases the coordination and administration of the students and the verification of their learning progress and skills.

The course itself consists of a lecture and an exercise course. The lecture is held several times a week but is voluntary. The exercise course has more parts. On the one hand they have to prepare eight examples every week. The students should be able to present them at the blackboard and answer questions of the tutor to show their comprehension. If they cannot answer the question they have to correct their achievement with another, hopefully better presentation. On the other hand there are three tests each semester realized in Maple T.A. which are spread over the semester. The students only have to pass two of them. They also get another change with an additional test to achieve two positive results. The points of the test are not important. The minimum to pass the course are fifty percent of the points of each test. For the lecture itself the students have to pass an extra exam. Last year this exam was also supported by the MTA system. In that case there was a written test realized with the Maple T.A. system. about the e-learning

test was basically to check the calculation skills of the students. At the oral exam the students had to prove their theoretical knowledge. Normally the lecture exam consists only of a written exam only containing theoretical question as well as ordinary calculation examples.

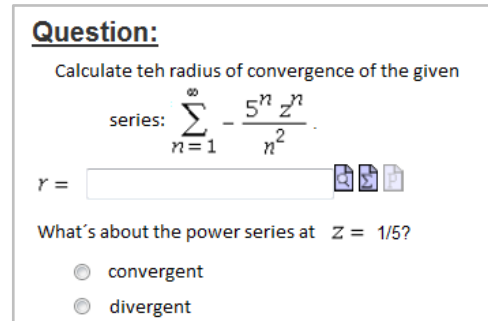


Figure 4: An Example Of The Course Mathematics 1 Which Deals With The Behaviour Of Power Series.

To support the exercises well, the questionnaire in Maple T.A. is enlarged with new examples according to the current topic in the lecture every two weeks. So the students have the facility to practice the fitting examples every time they like to. Additionally the questions on Maple T.A. can help understanding the actual subject. In addition to the questionnaire on Maple T.A. the learning matter for one test usually includes the examples of the last four exercises. Every test consists of four examples.

Figure 4 shows a question of the mathematical course of the first semester. In the first task they have to determine the radius of the convergence of a given power series. In the second part they have to analyse the behaviour of the series at the edge of the convergence area.

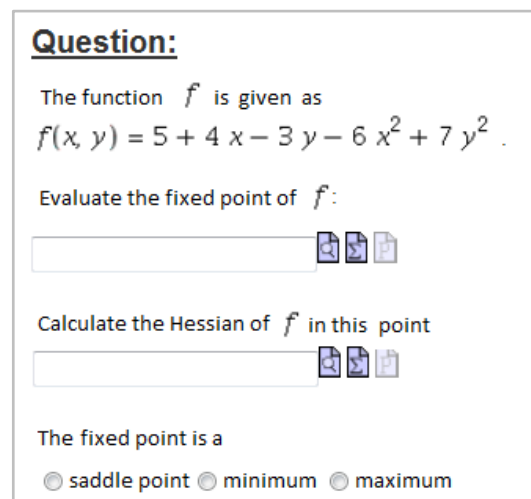


Figure 5: Mathematics 2 Focus On Linear Algebra And Its Usage. The Student Has To Determine The Fixed Point.

In figure 5 an example about the Linear Algebra is given. Given is a function in two variable. The students

have to calculate the fixed point at first. They have to fill in the point as a vector. The second question asks for the Hessian in this certain point. The students should insert a matrix like this, $[a_{11}, a_{12}; a_{21}, a_{22}]$. The last task is a multiple choice question in which the student has to decide if the fixed point is a saddle point, a minimum or a maximum. The vector and the matrix are graded in the way described in chapter three. Most of the examples in the two basic mathematical courses have more than one task so the possibility to earn points without calculating all the right solutions increases.

4.3. Advanced Mathematical Courses

In the last subsection the advanced mathematical course is described. Last year it was the first time that this course was supported with the Maple T.A. system. Last year the examples focused on theoretical knowledge. This year the focus should also be the calculation skills of the students. The questionnaire will be enlarged to offer a greater variety of examples to help the students to cope with the challenges given in the lecture.

This course is a combination of a lecture and an exercise. Most of the time the course is an ordinary lecture and once a week the lecturer calculates some examples in front of the students and explains them. The support of the Maple T.A. is basically voluntary practice at home. The examples which are presented in the lecture can be reconstructed via the questions of Maple T.A. Additionally there are two tests during the course, which are important for the final grade. Additionally there is an oral exam in the end of the course. The final mark consists of the tests and the oral exam.

Question:
Suppose we have a sample of size 12 :
-19, -10, 9, 14, -13, 8, -11, 2, -1, 1, 18, 20.

(1) Calculate:

(a) the sample mean:

(b) the sample variance:

(c) the 0,25-quantile:

(d) the mode:

the 0,75-quantile:

The values must be entered exactly.

Figure 6: An Example For The Advanced Mathematical Course Out Of The Field Of Statistic.

The question in figure 6 shows a task of the statistical part of the lecture. The students receive a

random sample and then have to calculate the mean, the variance and three different quantiles of this sample. Additionally the students have to choose one of the box plots. The right box plot is the one which reflects the random sample.

A random variable Y is given by the transformation of the random variable X :
 $Y = -3X - 6$.

The probability density function of the continuous random variable X is given by

$$f(x) = \begin{cases} \frac{17}{200}, & 1 \leq x < 5, \\ \frac{17}{100}, & 5 \leq x < 7, \\ \frac{8}{175}, & 7 \leq x < 14, \\ 0, & \text{else.} \end{cases}$$

Find the expected value of Y and round your answer to two decimal places.

$E(Y) =$

Figure 7: This Is Another Example Of The Advanced Mathematical Course, One Of The Probability Theory.

Figure 7 presents an example out of the probability theory. The task is the calculation of the expected value of a continuous random variable. This is a very important and common value of the probability theory.

The last example of the advanced mathematical course is one of the chapter vector analysis, shown in figure 8. The students receive a certain function and have to determine the value of the path integral.

Question:

The function f is given as
 $f(x, y) = 5 + 4x - 3y - 6x^2 + 7y^2$.

Evaluate the fixed point of f :

Calculate the Hessian of f in this point

The fixed point is a

saddle point minimum maximum

Figure 8: This Is Another Example For The Advanced Mathematical Course, One Out Of Probability Theory.

5. SUMMARY AND OUTLOOK

On the left side of the figure 9 the results of the test at the beginning of the refresher course are shown. As already mentioned the students take the test in the first exercise of the refresher course in September before the regular semester starts. The results of the first test of the course Mathematics 1 for electrical engineers are found on the right side. It is remarkable that all the marks enhanced enormously. The refresher course recreates the mathematical basics required for further

mathematical lectures and, as one can see in figure 1, really increases their mathematical skills. Additionally most of the students of electrical engineering attend the refresher course, so they are used to the Maple T.A. system at the beginning of the mathematical course.

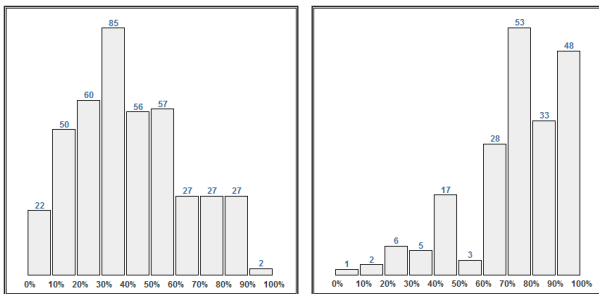


Figure 9: Compares The First Test At The Beginning Of The Refresher Course And The First Test Of Mathematics 1.

The refresher course helps the students to get back their lost mathematical knowledge. In every single technical study mathematic is the basic for all further lectures. Therefore it is very important that the students are aware of all the calculation skills without thinking a lot about it so they can focus on the real challenges of the given problems. With the help of Maple T.A. the students can control their increased skills and can practise also after the end of the courses with immediate feedback from Maple T.A. about the correctness of their calculations.

Figure 10 confirms the gain of mathematical knowledge and shows that the students don't have any big problems handling the e-learning system.

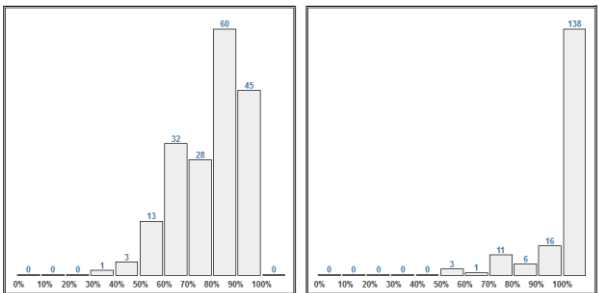


Figure 10: The Two Tests Of The Advanced Mathematical Courses, Left Side – 1st Test And Right Side – 2nd Test.

To put it all in a nutshell the Institute for Analysis and Scientific Computing initiated Maple T.A. to comfort administrators, tutors and students as well. On the one hand it is a time-saving invention for the tutors and lecturers. It eases the creation of the questionnaire which helps students to follow the course by exercising with the examples. The administration and organisation of tests as well as the lecture exam is easier with the support of this e-learning system.

On the other hand the students have the possibility of practicing with immediate control at any time. The obligatory tests should help the students to improve

their numeracy. In the exercise course the students can ask the tutors about incomprehensibilities of the lecture, the examples of Maple T.A. and the examples of the exercises as well. Therefore every student gets any help possible if he uses it. After some primary troubles with the grading of the Maple T.A. system the most important achievement of the improvement concerning the grading routine is a great acceptance by the students.

In the next semester the goal will be the invention of a course for modelling and simulation. Also enlarging and improving the questionnaire of Maple T.A. are important tasks for the next semester.

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