PETRI NET TRANSFORMATION FOR DECISION MAKING: COMPOUND PETRI NETS TO ALTERNATIVES AGGREGATION PETRI NETS.

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ABSTRACT
The design and operation of discrete event systems (DES), including their control, require taking decisions in order to guarantee an expected behavior. Usually, this behavior can be characterized by means of performance measurements. To take decisions may require choosing the best solution that optimizes a cost function and complies with certain restrictions, i.e. solving an optimization problem. In the field of DES modeled by Petri nets, it is a classical problem to optimize the initial marking and the sequence of priority assignment to the firing of transitions involved in conflicts (Piera and Music 2011). This problem may be solved by means of simulation and by optimization based on simulation (Piera *et al.*, 2004). The second approach can use a heuristic search to find the best configurations to solve. On the other hand, in the first approach is a human operator who should make this choice and can skip the best solutions to be tested. In the cases, less studied in the literature, of requiring an optimization of the structure of the Petri net, the classical approach is similar to the simulation in the previous problem: several feasible structures are chosen and they are simulated or optimized. If the human operator does not choose the best solutions, the result of the decision taking may be poor. A field of research that has taken the interest of the authors consists of applying a heuristic search to find the best structure for a Petri net (Latorre *et al.* 2009). This kind of optimization problem requires an adequate formalism as the compound Petri nets or the alternatives aggregation Petri nets to perform an efficient solving process (Latorre *et al.* 2011). The transformation from the first formalism to the second one is presented in this paper and illustrated with an example. Its utility arises when it is required to compare the performance of both formalisms for a particular case or when it is easier to model a DES as compound Petri net but the optimization process is based on the second formalism.

Keywords: decision support system, compound Petri net, alternatives aggregation Petri net, simulation, optimization.

1. INTRODUCTION

Decision making on discrete event systems is a common and difficult task associated to the design and operation of discrete event systems. The complexity in the behaviour of most discrete event systems associated to technological solutions requires the development and use of decision support systems (Jiménez *et al.*, 2005). Even with the help of computers, the analysis of the behaviour of DES under any possible scenario is usually intractable. (Piera *et al.*, 2004). For this reason diverse techniques have been created and applied from state space reduction to the use of metaheuristics to perform efficient searches in the state space. One interesting methodology broadly analysed and with important results that improve the verification, validation and performance measurement of the DES is the net transformation (Berthelot, 1987), (Silva, 1993).

This paper is focussed on the net transformation applied to two formalisms that allow representing an undefined Petri net. The undefined Petri nets constitute an abstraction of the model of an undefined discrete event system with alternative structural configurations among which one should be chosen for the definition of the DES. The two formalisms that will be considered are the compound Petri nets and the alternatives aggregation Petri nets (Latorre *et al.* 2009). Both formalisms may arise from a natural approach to model a DES with alternative structural configurations: the alternative Petri nets. When there are structural similarities between the alternative structures it is likely that the representation of the undefined

Petri net in the form of a compound Petri net or an alternatives aggregation Petri net is more compact than considering the equivalent set of alternative Petri nets, that is to say different models for every structure. A more compact model usually implies that the exploration of the space state of the model of the DES is more efficient than the search performed by means of the set of alternative Petri nets (Latorre *et al.*, 2010b).

On the other hand, the research is active in the field of determining the conditions where the compound Petri net is more efficient than an equivalent alternatives aggregation Petri net and vice versa. For this reason, the comparison between the two formalisms may lead to transformations among them. Furthermore, the compound Petri nets may serve of “formalism bridge” or “origin” towards the process of obtaining an alternatives aggregation Petri net or a disjunctive coloured Petri net. This last formalism can be obtained almost immediately from an alternatives aggregation Petri net and is very useful because of the software developed for the CPN that can be applied for the validation, verification or performance optimization of this formalism.

2. DEFINITIONS

The compound Petri nets can be defined in the following way:

Definition 1. Compound Petri net.
A compound Petri net is a 7-tuple $R^c = \langle P, T, F, \mathrm{w}, \mathbf{m_0}, S_\alpha, S_{val\alpha} \rangle$, where

i) S_α is the set of undefined parameters of R^c.
ii) $S_{str\alpha} \neq \emptyset$ is the set of undefined structural parameters of R^c, such that $S_{str\alpha} \subseteq S_\alpha$. Notice that S_α is the set of undefined parameters of R^c.
iii) $S_{val\alpha}$ is the feasible combination of values for the undefined parameters .

□

A compound Petri net can be considered as a parametric Petri net with undefined structural parameters.
The structural parameters refer to the elements of the incidence matrix of a Petri net. If a Petri net has undefined structural parameters it has a structure with certain freedom degrees that should be specified by a decision from the set of feasible combinations of values for them. In summary, a the undefined structural parameters are present in models that correspond with DES with undefined structure, in process of being designed, modified or controlled.
On the other hand, an alternatives aggregation Petri net may be defined as indicated below:

Definition 2. Alternatives aggregation Petri net system.
An alternatives aggregation Petri net system, $\boldsymbol{R^A}$, is defined as the 8-tuple:

$R^A = \langle P, T, \mathrm{pre}, \mathrm{post}, \mathbf{m}_0, S_\alpha, S_{val\alpha}, S_A, \mathrm{f_A} \rangle$

Where,

- P is the set of places.
- T is the set of transitions.
- pre is the pre-incidence matrix, also called input incidence matrix.
- post is the post-incidence matrix, also called output incidence matrix.
- $\mathbf{m}_0$ is the initial marking that represents the initial vector of state and is usually a function of the choice variables.
- S_α is a set of undefined parameters.
- $S_{val\alpha}$ is the set of feasible combination of values for the undefined parameters in S_α.
- S_A is a set of choice variables such that $S_A \neq \emptyset$ and $|S_A| = n$.
- $\mathrm{f_A}$: $T \rightarrow \mathrm{f}(a_1, \ldots, a_n)$ assigns a function of the choice variables to each transition t such that type[$\mathrm{f_A}(t)$] = Boolean.

□

Where a set of choice variables is given by:

Let $c_{str} \in C_{str} = \{1, 2, \ldots, m_{strq}\} \subseteq \mathrm{N}^*$.

A set of choice variables can be defined as $S_A = \{a_1, a_2, \ldots, a_{mstrq} \mid \exists!\ a_i{=}1,\ i \in C_{str} \wedge a_j{=}0\ \forall\, j{\neq}i, j \in C_{str}\}$

Furthermore, the dynamic behaviour of an alternatives aggregation Petri net is given by an enabling rule that differs slightly from most of the formalisms based on Petri nets. The firing rule is the one of a generalized Petri net.

Definition 3. Enabled transition.
Given an alternatives aggregation Petri net R^A with an associated set of choice variables $S_A = \{ a_1, a_2, \ldots, a_n \}$, let us consider the following decision:

$$a_i = 1 \Rightarrow a_i = 0\ \forall\, j \in \mathrm{N}^* \text{ such that } 1 \leq j \leq n \wedge j \neq i$$

A transition $t_j \in T$ in an alternatives aggregation Petri net is said to be enabled if

$$m_i \geq \mathrm{pre}(p_i, t_j)\ \forall\, p_i \in {}^{o}t_j \wedge \mathrm{f_A}(t_j) = 1$$

□

3. TRANSFORMATION ALGORITHM

The transformation from a compound Petri net to an alternatives aggregation Petri net can be performed by the following algorithm:

A compound Petri net is a very compact representation of a discrete event system with an undefined structure. Nevertheless, it contains some undefined structural parameters that require to complement the model with a set of feasible combinations of values for the undefined structural parameters. On the other hand, the alternatives aggregation Petri net, might be more compact for certain systems and this fact may lead to more efficient optimization algorithms. The main reason is that the alternatives aggregation Petri net can profit from similarities in the subnets of the different structures that can be chosen for the original DES and on the other hand it can be constructed in a way that it lacks completely of undefined structural parameters. This last property implies that the model does not require an additional set of feasible combinations for the undefined structural parameters.

For these reasons it is going to be presented an algorithm to perform the transformation from a compound Petri net into an alternatives aggregation Petri net, where both models are equivalent or, what is the same, their graphs of reachable markings are isomorphous.

Let us consider a compound Petri net R^c .

Step 1.
Define
$\{ S_{valstr\alpha}(R_1), S_{valstr\alpha}(R_2), \ldots , S_{valstr\alpha}(R_{n_r}) \}$,
a partition of $S_{valstr\alpha}(R^c)$.
Define a set of choice variables from R^c and $\{ S_{valstr\alpha}(R_1), S_{valstr\alpha}(R_2), \ldots , S_{valstr\alpha}(R_{n_r}) \}$ in the way $S_A = \{a_1, a_2, \ldots, a_{n_r} \mid \exists!\ a_i=1,\ 1 \le i \le n_r,\ a_j=0\ \forall\ j\neq i \}$, where (by definition) $\mathrm{card}(\{S_{valstr\alpha}(R_1), S_{valstr\alpha}(R_2), \ldots, S_{valstr\alpha}(R_{n_r})\}) = \mathrm{card}(S_A) = n_r$

Step 2.
Create a bijection between the elements of the partition $\{ S_{valstr\alpha}(R_1), S_{valstr\alpha}(R_2), \ldots , S_{valstr\alpha}(R_{n_r}) \}$ and the elements of S_A (choice variables).

Step 3.
Replicate every transition t_i into a set $\{t_i^1, t_i^2, \ldots, t_i^{n_r}\}$, where $n_r=\mathrm{card}(S_A)$ and

i) $\mathrm{pre}(p_j, t_i^q)$, $\mathrm{post}(t_i^q, p_k) \in S_{valstr\gamma}(R_q)$, where $S_{valstr\alpha}(R_q) \cup S_{valstr\beta}(R_q) = S_{valstr\gamma}(R_q)$ and the couple of sets $S_{valstr\alpha}(R_q)$ and $S_{valstr\beta}(R_q)$ stand for the set of feasible combination of values for the undefined structural parameters of R^c and for the defined ones respectively.

ii) The choice variable a_q is associated to the transition t_i^q as a boolean condition to allow its enabling.

Step 4.
Transform the resulting AAPN from **step 3** into an equivalent PN by removing the non-connected places and transitions.

Step 5.
Apply a reduction rule to quasi-identical transitions associated to different choice variables, obtaining an equivalent AAPN with a reduced set of transitions.

Step 6.
Apply simplification rules to remove the unnecessary choice variables and functions of choice variables to the transitions of the AAPN. As a result a simpler AAPN than the original basic AAPN is expected.

□

4. APPLICATION EXAMPLE OF THE TRANSFORMATION ALGORITHM

In order to illustrate the application of this algorithm, the following examples can be considered.

4.1. Example 1.

In the **figure 1** it can be seen a compound Petri net in both representations, a graphical one and another matrix-based one based on the incidence matrix.

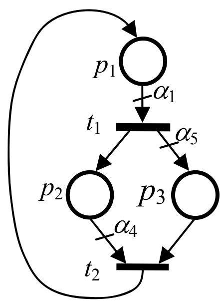

Fig. 1. Compound PN.

The set of structural parameters of the compound Petri net R^c is:

$S_{str\gamma}(R^c) = \{ \alpha_1 , \beta_2 , \beta_3 , \alpha_4 , \alpha_5 , \beta_6 \} =$
$S_{str\alpha}(R^c) \cup S_{str\beta}(R^c)$, where
$S_{str\alpha}(R^c) = \{ \alpha_1 , \alpha_4 , \alpha_5 \}$ and $S_{str\beta}(R^c) =$
$\{ \beta_2 , \beta_3 , \beta_6 \}$

On the other hand, the set of feasible values for the structural parameters of R^c can be written as follows:

$S_{valstr\beta}(R^c) = \{ (1,1,1) \}$
$S_{valstr\alpha}(R^c) = \{ (1,0,1), (0,1,0), (2,0,1), (0,1,2) \}$

$$\mathbf{W}(R^c) = \begin{array}{cc} \begin{array}{cc} t_1 & t_2 \end{array} & \\ \begin{pmatrix} -\alpha_1 & \beta_2 \\ \beta_3 & -\alpha_4 \\ \alpha_5 & \beta_6 \end{pmatrix} & \begin{array}{c} p_1 \\ p_2 \\ p_3 \end{array} \end{array}$$

$$= \begin{array}{cc} \begin{array}{cc} t_1 & t_2 \end{array} & \\ \begin{pmatrix} -\alpha_1 & 1 \\ 1 & -\alpha_4 \\ \alpha_5 & 1 \end{pmatrix} & \begin{array}{c} p_1 \\ p_2 \\ p_3 \end{array} \end{array}$$

Fig. 2. Matrix-based representation of the compound Petri net.

Finally, it is possible to determine the set of feasible values for every undefined structural parameter of R^c.

$S_{val\alpha_1}(R^c) = \{ 0,1,2 \}$
$S_{val\alpha_4}(R^c) = \{ 0,1 \}$
$S_{val\alpha_5}(R^c) = \{ 0,1,2 \}$

The first partition of $S_{valstr\alpha}(R^c)$ has order two and can be represented as follows:

$\prod_1(S_{valstr\alpha}(R^c)) = \{ S_{1valstr\alpha}(R^c) , S_{2valstr\alpha}(R^c) \}$
$S_{valstr\alpha}(R^c) = S_{1valstr\alpha}(R^c) \cup S_{2valstr\alpha}(R^c)$
$S_{valstr\alpha}(R^c) = \{ (1,0,1), (0,1,0), (2,0,1), (0,1,2) \}$
$S_{1valstr\alpha}(R^c) = \{ (1,0,1), (0,1,0) \}$
$S_{2valstr\alpha}(R^c) = \{ (2,0,1), (0,1,2) \}$

In order to know the number of undefined structural parameters associated to every subset of the partition, it is necessary to analyse every parameter of $S_{valstr\alpha}(R^c)$.

On one hand, the first subset of the partition will be considered:

$S_{1val\alpha1}(R^c) = \{ 0,1 \}$
$S_{1val\alpha4}(R^c) = \{ 0,1 \}$
$S_{1val\alpha5}(R^c) = \{ 0,1 \}$

As a consequence there will be three undefined structural parameters associated to this subset of the partition, since any of them can take values from a set with more than one element:

$S_{1str\alpha}(R^c) = \{ \alpha_1^1 , \alpha_4^1 , \alpha_5^1 \}$

On the other hand, the second subset of the partition will lead to:

$S_{2val\alpha1}(R^c) = \{ 0,2 \}$
$S_{2val\alpha4}(R^c) = \{ 0,1 \}$
$S_{2val\alpha5}(R^c) = \{ 1,2 \}$

In this case there will be another new three undefined structural parameters associated to this subset of the partition, since any of them can take values from a set with more than one element:

$S_{2str\alpha}(R^c) = \{ \alpha_1^2 , \alpha_4^2 , \alpha_5^2 \}$

As a result, it is possible to see how this partition of $S_{valstr\alpha}(R^c)$, $\prod_1(S_{valstr\alpha}(R^c))$, from a compound Petri net with three undefined structural parameters leads to a representation with six undefined structural parameters. This representation can be a set of compound alternative PN or an aggregations alternative Petri net. The AAPN that results from the replication of the transitions of the compound PN R^c according to this partition is represented in **figure 3**.

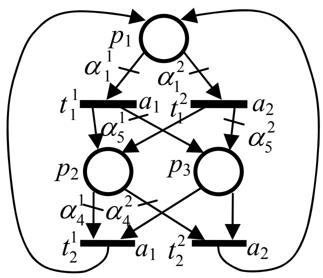

Fig. 3. Graphical representation of an AAPN obtained from a first partition of $S_{valstr\alpha}(R^c)$.

$$\mathbf{W}(R^A) = \begin{array}{cc} \begin{array}{cccc} t_1^1 & t_1^2 & t_2^1 & t_2^2 \end{array} & \\ \begin{pmatrix} -\alpha_1^1 & -\alpha_1^2 & 1 & 1 \\ 1 & 1 & -\alpha_4^1 & -\alpha_4^2 \\ \alpha_5^1 & \alpha_5^2 & 1 & 1 \end{pmatrix} & \begin{array}{c} p_1 \\ p_2 \\ p_3 \end{array} \\ \begin{array}{cccc} a_1 & a_2 & a_1 & a_2 \end{array} & \end{array}$$

Fig. 4. Matrix-based representation of an AAPN obtained from a first partition of $S_{valstr\alpha}(R^c)$.

As a conclusion, it is possible to state that despite the fact that the original compound Petri net R^c has only three undefined structural parameters $S_{str\alpha}(R^c) = \{ \alpha_1 , \alpha_4 , \alpha_5 \}$, the resulting AAPN obtained by a replication of the transitions based on this first partition of

$S_{valstr\alpha}(R^c)$ has six undefined structural parameters:

$S_{str\alpha}(R^A) = S_{1str\alpha}(R^c) \cup S_{2str\alpha}(R^c) =$
$\{ \alpha_1^1 , \alpha_1^2 , \alpha_4^1 , \alpha_4^2 , \alpha_5^1 , \alpha_5^2 \}$

4.2. Example 2.
The second partition of $S_{valstr\alpha}(R^c)$ has order two and can be represented as follows:

$\prod_2(S_{valstr\alpha}(R^c)) = \{ S_{1valstr\alpha}(R^c) , S_{2valstr\alpha}(R^c) \}$
$S_{valstr\alpha}(R^c) = S_{1valstr\alpha}(R^c) \cup S_{2valstr\alpha}(R^c)$
$S_{valstr\alpha}(R^c) = \{ (1,0,1), (0,1,0), (2,0,1), (0,1,2) \}$
$S_{1valstr\alpha}(R^c) = \{ (1,0,1), (2,0,1) \}$
$S_{2valstr\alpha}(R^c) = \{ (0,1,0), (0,1,2) \}$

In order to know the number of undefined structural parameters associated to every subset of the partition, it is necessary to analyse every parameter of $S_{valstr\alpha}(R^c)$.

On one hand, the first subset of the partition will be considered:

$S_{1val\alpha 1}(R^c) = \{ 1,2 \}$
$S_{1val\alpha 4}(R^c) = \{ 0 \} \Rightarrow$ In this subset of the second partition α_4 is no longer an undefined structural parameter but a defined one: β_4^1.
$S_{1val\alpha 5}(R^c) = \{ 1 \} \Rightarrow$ In this subset of the second partition α_5 is no longer an undefined structural parameter but a defined one: β_5^1.

As a consequence there will be only one undefined structural parameters associated to this subset of the partition, since only α_1^1 can take values from a set with more than one element:

$S_{1str\alpha}(R^c) = \{ \alpha_1^1 \}$

On the other hand, the second subset of the partition will lead to:

$S_{2val\alpha 1}(R^c) = \{ 0 \} \Rightarrow$ In this subset of the second partition α_1 is no longer an undefined structural parameter but a defined one: β_1^1.
$S_{2val\alpha 4}(R^c) = \{ 1 \} \Rightarrow$ In this subset of the second partition α_4 is no longer an undefined structural parameter but a defined one: β_4^1.
$S_{2val\alpha 5}(R^c) = \{ 0,2 \}$

This case will provide with another single undefined structural parameters associated to the corresponding subset of this second partition, α_5^2, since it is the only one that can take values from a set with more than one element:

$S_{2str\alpha}(R^c) = \{ \alpha_5^2 \}$

As a result, it is possible to see how this second partition of $S_{valstr\alpha}(R^c)$, $\prod_2(S_{valstr\alpha}(R^c))$, from a compound Petri net with three undefined structural parameters leads to a representation with only two undefined structural parameters. This representation can be a set of compound alternative PN or an aggregations alternative Petri net. The AAPN that results from the replication of the transitions of the compound PN R^c according to this partition is represented in **figure 5**.

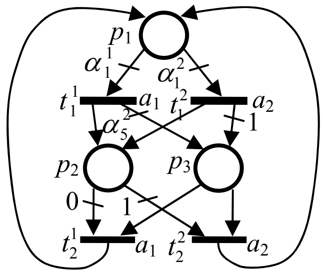

Fig. 5. Graphical representation of an AAPN obtained from a 2nd partition of $S_{valstr\alpha}(R^c)$.

$$\begin{array}{cc}
 & \begin{array}{cccc} t_1^1 & t_1^2 & t_2^1 & t_2^2 \end{array} \\
\mathbf{W}(R^A) = & \begin{pmatrix} -\alpha_1^1 & -\alpha_1^2 & 1 & 1 \\ 1 & 1 & 0 & -1 \\ 1 & \alpha_5^2 & 1 & 1 \end{pmatrix} \begin{array}{c} p_1 \\ p_2 \\ p_3 \end{array} \\
 & \begin{array}{cccc} a_1 & a_2 & a_1 & a_2 \end{array}
\end{array}$$

Fig. 6. Matrix-based representation of an AAPN obtained from a second partition of $S_{valstr\alpha}(R^c)$.

As a conclusion, it is possible to state that despite the fact that the original compound Petri net R^c has only three undefined structural parameters $S_{str\alpha}(R^c) = \{ \alpha_1 , \alpha_4 , \alpha_5 \}$, the resulting AAPN obtained by a replication of the transitions based on this second partition of $S_{valstr\alpha}(R^c)$ has only two undefined structural parameters:

$S_{str\alpha}(R^A) = S_{1str\alpha}(R^c) \cup S_{2str\alpha}(R^c) = \{ \alpha_1^1 , \alpha_5^2 \}$

It is interesting to notice that it depends on the parameters of the transformation algorithm (in this case the chosen partition), that the size of the resulting model is more or less compact.

5. CONCLUSIONS AND FURTHER RESEARCH

In this paper it has been described a transformation algorithm between two formalisms that represent an undefined Petri net. This algorithm develops a link that allow to obtain an alternatives aggregation Petri net and the subsequent disjunctive coloured Petri net from a compound Petri net or even from another formalism that had been previously transformed in the former. This transformation constitutes an important step in the research of the conditions where one of the two involved formalisms is more efficient in the application of a decision making algorithm related to a discrete event system. The transformation allows as well transforming in a certain case a less promising formalism to a more promising one.

The future research leads to the characterization of the decision problems to forecast the performance of the different formalisms in the exploration of the state space that requires the decision making based on discrete event systems.

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