A STOCHASTIC APPROACH TO RISK MODELING FOR SOLVENCY II

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ABSTRACT
Solvency II establishes EU-wide capital requirements and risk management standards for (re)insurers. The capital requirements are defined by the Solvency Capital Requirement (SCR), which should deliver a level of capital that enables the (re)insurer to absorb significant unforeseen losses over a specified time horizon. It should cover insurance, market, credit and operational risks, corresponding to the Value-at-Risk (VAR) subject to a confidence level of 99.95% over one year. Standard models are deterministic, scenario-based or covariance-based, i.e. non-stochastic. They don't optimise the investment portfolios. These are two major deficiencies. A stochastic approach is proposed, which combines Monte Carlo Simulation and Optimisation. This method determines minimal variance portfolios and calculates VAR/SCR using the optimal portfolios' simulation distributions, which ultimately eliminates the standard models' deficiencies. It offers (re)insures internal model options, which can help them to reduce their VAR/SCR providing higher underwriting capabilities and increasing their competitive position, which is their ultimate objective.

Keywords: Solvency II stochastic model, VAR/SCR reduction, portfolio optimisation – minimal variance, Monte Carlo simulation

1. INTRODUCTION

The Solvency II regulations are fundamentally redesigning the capital adequacy regime for European (re)insurers and will be effective from 1st January 2013.

Solvency II establishes two levels of capital requirements: i) Minimal Capital Requirement (MCR), i.e. the threshold below which the authorization of the (re)insurer shall be withdrawn; and ii) Solvency Capital Requirement (SCR), i.e. the threshold below which the (re)insurer will be subject to a much higher supervision. The SCR should deliver a level of capital that enables the (re)insurer to absorb significant unforeseen losses over a specified time horizon. It should cover, at a minimum, insurance, market, credit and operational risks, corresponding to the VAR of the (re)insurer's own basic funds, subject to a confidence level of 99.95% over a one-year period.

Solvency II offers two options for calculating VAR/SCR, i.e. by applying either: i) a standard model, which will be provided by the regulator; or ii) an internal model developed by the (re)insurer's risk department.

The standard models are non-stochastic risk models. They are rather deterministic, scenario-based or covariance models. They are also conservative by nature and generic across the EU so they cannot consider the company's specific factors. Moreover, they do not use optimisation to determine the minimal variance investment portfolios in order to minimise the financial risk for the (re)insurers. Thus the calculated VAR/SCR will be higher. These are apparent most important limitations of the standard models.

For example, the deterministic model applies analytically calculated or estimated input parameters to calculate the results. However, the likelihood of the outcome is not considered at all. Also, the scenario-based models consider the worst, most likely and best case scenarios. However, they fail to answer the two basic questions: i) how likely are the worst and best case scenarios? And more importantly, ii) how likely is the most likely case?

Solvency II offers capital-reduction incentives to insurers that invest in developing advanced internal models, which apply a stochastic approach for risk management and control. Thus, insurers will benefit from using internal models. A very good explanation of developing the Enterprise Risk Management (ERM) frameworks in (re)insurance companies for Solvency II is presented in a handbook edited by Cruz (2009).

There are a number of published examples of recommended internal model, which could be used for Solvency II (Cruz 2009). These suggested internal model examples don't consider optimisation to determine the minimal variance investment portfolios in order to minimise the financial risk, which is a significant deficiency.

The stochastic models usually apply the Monte Carlo Simulation method, which assigns distributions of random variables to the input parameters and the calculated results are presented in the form of a histogram. This allows statistical and probabilistic tools to be used to analyse the results. A comprehensive elaboration of Monte Carlo Simulation in Finance is given by Glasserman (2004).

An investment portfolio is defined by the fraction of the capital put in each investment. The problem of determining the minimum variance portfolio that yields a desired expected return was solved by Markowitz in the 1950's. He received the 1991 Nobel Prize for his work in Economics (Markowitz 1987). Mostly, the Optimisation methodology is used to find the minimum variance portfolio in order to minimise the financial risk.

VAR is a widely used financial risk measure. The approach to calculate VAR is well summarised by Jorion (2011). This approach includes the VAR Parametric method and VAR Monte Carlo Simulation method.

This paper proposes a stochastic approach to risk modelling for Solvency II. This method applies combined Monte Carlo Simulation and Optimisation methodologies. The method uses Optimisation to calculate the minimal variance portfolios that yield desired expected returns to determine the Efficient Frontier of optimal portfolios. Monte Carlo Simulation is used to calculate VAR/SCR for every portfolio on the Efficient Frontier by using the respective portfolios' simulation distributions. Therefore, by using the synergy of Monte Carlo Simulation and Optimisation, the method eliminates the deficiencies and limitations, which are identified above.

This approach can help (re)insures to develop and improve their internal risk models in order to reduce their VAR/SCR. Consequently, this will provide insurers with higher underwriting capabilities and increase their competitive position, which is their ultimate objective.

According to research by Mercer Oliver Wyman, the impact of the four quantifiable risks on the economic capital of insurance companies is: i) 64% Investment Asset Liability Management (ALM) Risk, i.e. Market Risk; ii) 27% Operational Risk; iii) 5% Credit Risk; and iv) 4% Insurance Risk. Considering that the Market (ALM) Risk is the top contributing risk factor, the method is demonstrated by using an example of Market (ALM) Risk Management. Also, in order to facilitate the presentation, a simple Market (ALM) Risk model is demonstrated.

Only the practical aspects of the Market (ALM) Risk modelling are discussed. Microsoft™ Excel® and Palisade™ @RISK® and RISKOptimizer® were used in these experiments.

1.1. Related Work

The following is a summary of some published works related to Market (ALM) Risk modelling for Solvency II.

1.1.1. Market Risk in the GDV Model

The GDV (Gesamtverband der Deutschen Versicherungswirtschaft) Model is the standard model of the German Insurance Association for Solvency II (GDV 2005). This model is to some extent a Static Factor deterministic model, where the risk capital calculation is based on linear combinations of static risk factors. Actually, the model is mostly a Covariance (or VAR) Model, which is a very simplified version of Stochastic Risk Models.

1.1.2. Market Risk in the Swiss Solvency Test (SST) Model

This is the standard model of Swiss Federal Office of Private Insurance. The Market Risk in the SST model is handled by the ALM model. The SST ALM model is a Risk Factor Covariance model complemented with Scenario-Based models (SST 2004).

1.1.3. Bourdeau's Example of Internal Market Risk Model

Michele Bourdeau published an example of an internal model for Market Risk. This model calculates VAR using Monte Carlo Simulation. This is an example of a true Stochastic Risk Model (Bourdeau 2009).

2. ALM RISK MODELLING PROCEDURE

The following sections demonstrate the ALM Risk modelling procedure for Solvency II step-by-step. Actual financial market data are used in the presentation.

2.1. Problem Statement

The following is a simplified problem statement for the demonstrated investment ALM risk model under Solvency II.

Determine the minimum variance investment portfolio that yields a desired expected annual return to cover the liabilities of the insurance company. Calculate the VAR considering all the company's specific factors including their risk appetite. The model should allow the insurer to reduce their VAR (SCR) providing for higher underwriting capabilities and increasing their competitive position. The model should help the company to achieve their ultimate objective.

2.2. Calculating Compounded Monthly Return

The monthly returns of four investment funds are available for a period of seven years, i.e. 1990-1996 (Table 1). Note that the data for the period July/1990-June/1996 are not shown.

Table 1: Monthly Return

Month	Fund 1	Fund 2	Fund 3	Fund 4
Jan/1990	0.048	-0.01	-0.06	-0.01
Feb/1990	0.066	0.096	0.037	0.038
Mar/1990	0.022	0.022	0.12	0.015
Apr/1990	0.027	-0.04	-0.02	-0.04
May/1990	0.112	0.116	0.123	0.075
Jun/1990	-0.02	-0.02	-0.04	-0.01
Jul/1996	0.086	-0.07	-0.12	-0.02
Aug/1996	0.067	0.026	0.146	0.018
Sep/1996	0.089	-0.03	-0.04	0.092
Oct/1996	0.036	0.117	0.049	0.039

The Compounded Monthly Return (*CMR*) is calculated for each month and each investment fund from the given Monthly Return (*MR*) fund using the following formula (Table 2):

$$CMR = ln\ (1 + MR)$$

Table 2: Compounded Monthly Return (CMR)

Month	CMR1	CMR2	CMR3	CMR4
Jan/1990	0.047	-0.01	-0.06	-0.01
Feb/1990	0.063	0.092	0.036	0.038
Mar/1990	0.021	0.022	0.113	0.015
Apr/1990	0.027	-0.04	-0.02	-0.04
May/1990	0.106	0.11	0.116	0.073
Jun/1990	-0.02	-0.02	-0.04	-0.01
Jul/1996	0.082	-0.07	-0.13	-0.02
Aug/1996	0.065	0.026	0.136	0.018
Sep/1996	0.085	-0.03	-0.04	0.088
Oct/1996	0.036	0.111	0.048	0.038

2.3. Fitting Distributions to Compounded Monthly Return

For the Monte Carlo method, we need the distribution of the compounded monthly return for each investment fund. Thus, for each investment fund, we determine the best fit distribution based on the Chi-Square measure. For example, the best fit distribution for the compounded monthly return of Fund 4 (i.e. CMR4) is the normal distribution presented in Figure 1.

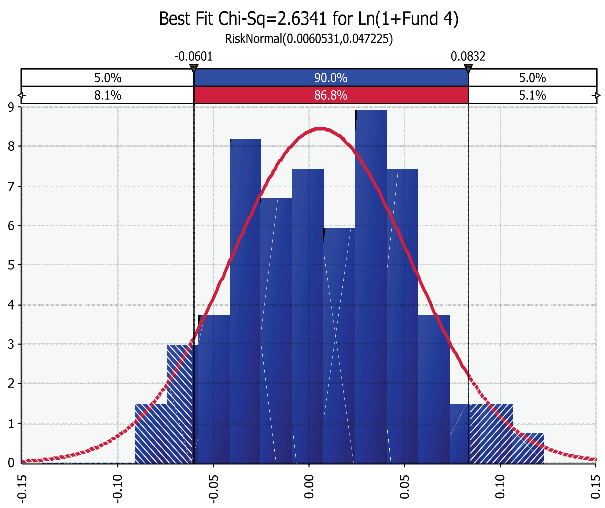

Figure 1: Fund 4 Best Fit Distribution

2.4. Finding Compounded Monthly Return Correlations

The compounded monthly returns of the investment funds are correlated. We need to find the correlation to allow the Monte Carlo method to generate correlated random values for the compounded monthly returns. The correlation matrix is presented in Table 3.

Table 3: Correlation Matrix

	CMR1	CMR2	CMR3	CMR4
CRM 1	1	0.263	0.038	0.0868
CRM2	0.263	1	0.244	0.0895
CRM3	0.038	0.244	1	0.095
CRM4	0.087	0.089	0.095	1

2.5. Generating Compounded Monthly Return

The Compounded Monthly Return (*CAR*) is randomly generated for each investment fund from the best fit distribution considering the correlations. The following distribution functions of the Palisade™ @RISK® are used:

CMR1=RiskLogistic(0.0091429,0.044596))

CMR2=RiskLognorm(1.1261,0.077433,Shift(-1.1203))

CMR3= RiskWeibull(6.9531,0.46395, Shift(-0.42581))

CMR4= RiskNormal(0.0060531,0.047225)

The correlation is applied by using the "*RiskCorrmat*" function of the Palisade™ @RISK®.

2.6. Calculating Compounded Annual Return by Fund

The Compounded Annual Return (*CAR*) is calculated for each investment fund from the respective Compounded Monthly Return (*CMR*), using the following formula:

$$CAR = 12*CMR$$

2.7. Calculating Expected Annual Mean Return on the Portfolio

The expected annual mean return on the portfolio (*EAPR-Mean*) is calculated from the asset allocation weights vector (*Weights-V*) and the vector of compounded annual returns of funds (*CAR-V*) by using the following Excel® formula:

EAR-Mean = SumProduct(Weights-V, CAR-V)

2.8. Calculating Variance, Standard Deviation and VAR of the Portfolio

The variance, standard deviation and VAR of the portfolio are calculated from the distribution of the expected annual mean return on the portfolio (*EAR-Mean*) by using the following Palisade™ @RISK® functions:

Variance = RiskVariance(EAR-Mean)

Standard-Deviation = RiskStdDev(EAR-Mean)

$$VAR = RiskPercentile(EAR\text{-}Mean, 0.005)$$

2.8.1. Portfolio Simulation and Optimisation #1

Palisade™ RISKOptimizer® is used to solve the portfolio simulation and optimisation problem. That is to find the minimal variance portfolio of investments, which yields sufficient return to cover the liabilities. Thus, the aim of the simulation and optimisation model is to minimise the variance of the portfolio subject to the following specific constraints:

- The expected portfolio return is at least 8.2%, which is sufficient to cover the liabilities;
- All the money is invested, i.e. 100% of the available funds is invested; and
- No short selling is allowed so all the fractions of the capital placed in each investment fund should be non-negative.

The model should also calculate the Standard Deviation and VAR of the portfolio.

2.8.2. Finding the Efficient Frontier of Portfolios

Palisade™ RISKOptimizer® is used repetitively to solve the portfolio simulation and optimisation problem in order to find the Efficient Frontier of investment portfolios. That is to find the minimal variance portfolios of investments, which yield expected portfolio returns of at least 8.4%, 8.6%, …, 10% and 10.2%. Thus, the aim of the simulation and optimisation models is to find in ten iterations, the ten minimal variance portfolios subject to the following specific constrains:

- The expected portfolio return is at least 8.4%, 8.6%, …, 10% and 10.2% respectively;
- All the money is invested, i.e. 100% of the available funds is invested; and
- No short selling is allowed so all the fractions of the capital placed in each investment fund should be non-negative.

The model should also calculate the Standard Deviation and VAR of these ten portfolios.

3. RESULTS AND DISCUSSION

3.1. Simulation and Optimisation #1

The optimal portfolio found by this model has the following investment fractions: 14.6% in Fund 1; 11.6% in Fund 2; 18.6% in Fund 3; and 55.2% in Fund 4. The Portfolio Return is 8.2% with Variance of 19.9%, Standard Deviation of 44.6% and VAR of -7%.

The probability distribution of this optimal portfolio is given in Figure 2. From the graph, we can read the confidence levels as follows.

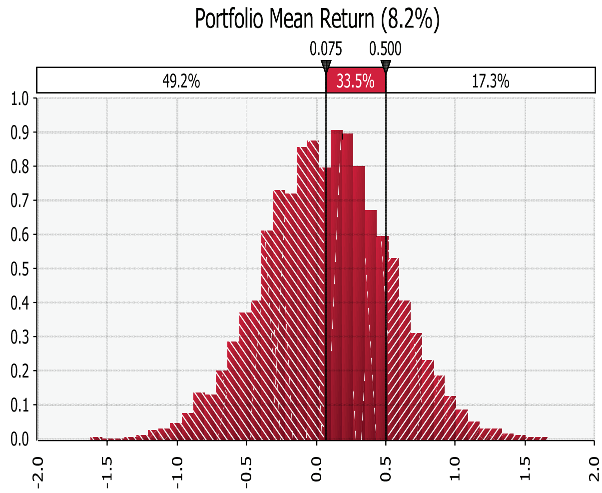

Figure 2: Probability Distribution #1

The probability that the portfolio return is below 7.5% is 49.2%. There is a 33.5% probability that the return is in the range of 7.5%-50%. From the simulation statistics we also find that there is a 43.2% probability that the portfolio return is negative, and 51.4% probability that the return is below 10%.

From the correlation graph (Figure 3), we can conclude that the portfolio return is most dependent on the return of Fund 4 with a correlation coefficient of 77%. The other three funds, i.e. Fund 3, Fund 2 and Fund 1, are less influential with correlation coefficients of 48%, 46% and 44% respectively.

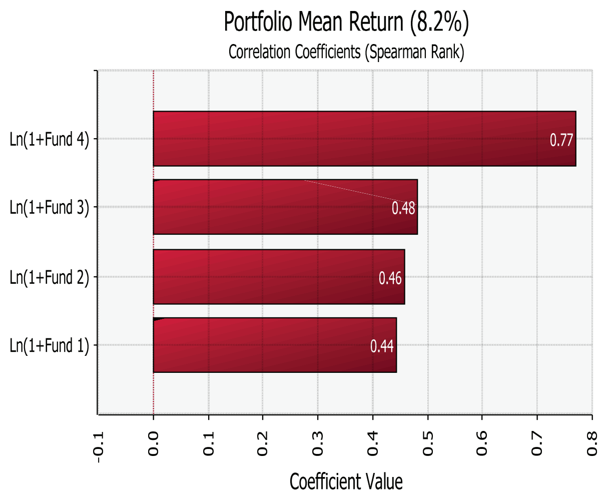

Figure 3: Correlation Sensitivity

The regression sensitivity graph is given in Figure 5. This graph shows how the portfolio mean return is changed in terms of Standard Deviation, if the return of a particular fund is changed by one Standard Deviation.

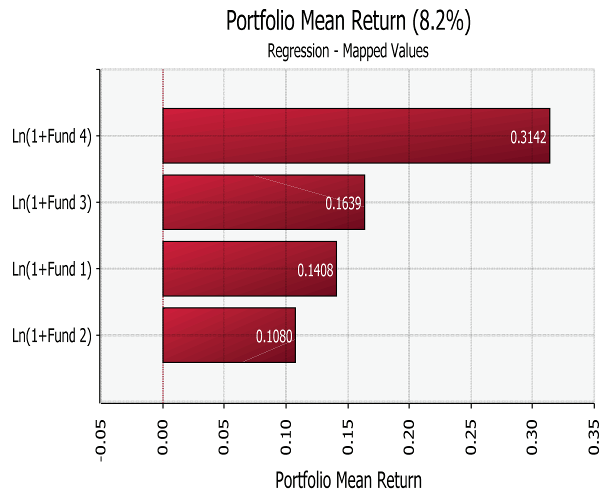

Figure 4: Regression Mapped Values Sensitivity

Therefore, we can read from the graph for example, that if Fund 4 return is changed by one Standard Deviation, the portfolio return will be changed by 0.3142 Standard Deviations (as shown by the regression coefficient of 0.3142). Again, the other three funds, i.e. Fund 3, Fund 2 and Fund 1, are less influential as their regression coefficients are 0.1639, 0.1408 and 0.1080 respectively.

3.2. Overall Simulation & Optimisation Results

The overall results of all the eleven simulation and optimisations are presented in Table 4 showing the Mean Return, Variance, Standard Deviation and VAR of the optimal portfolios.

Table 4: The overall results

Portfolio No.	Mean Return	Variance	Standard Deviation	VAR
1	0.082	0.199	0.446	-0.067
2	0.084	0.202	0.45	-0.088
3	0.086	0.204	0.452	-0.11
4	0.088	0.215	0.464	-0.147
5	0.09	0.222	0.471	-0.223
6	0.092	0.246	0.496	-0.257
7	0.094	0.266	0.516	-0.308
8	0.096	0.295	0.543	-0.403
9	0.098	0.33	0.574	-0.498
10	0.1	0.37	0.608	-0.608
11	0.102	0.42	0.648	-0.753

3.3. Efficient Frontier of Portfolios

Efficient Frontier of the optimal portfolios is presented in Figure 5.

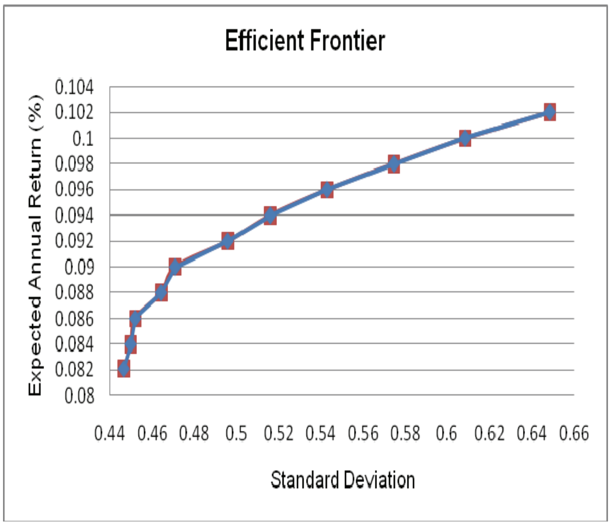

Figure 5: Efficient Frontier of Optimal Portfolios

The Efficient Frontier shows that an increase in expected return of the portfolio causes an increase in portfolio Standard Deviation. Also, the Efficient Frontier gets flatter as expected. This tells us that each additional unit of Standard Deviation allowed, increases the portfolio mean return by less and less.

3.4. Portfolio Expected Mean Return versus VAR

Figure 6 shows the dependency of the expected portfolio returns against VAR. From the graph we can see that an increase in expected return of the portfolio causes an increase in portfolio VAR in terms of money. (It should be noted that mathematically, VAR is a negative number, which actually decreases when the return increases.) Also, the curve on the graph gets flatter, again as expected. This tells us that each additional unit of VAR allowed, increases the portfolio mean return by less and less.

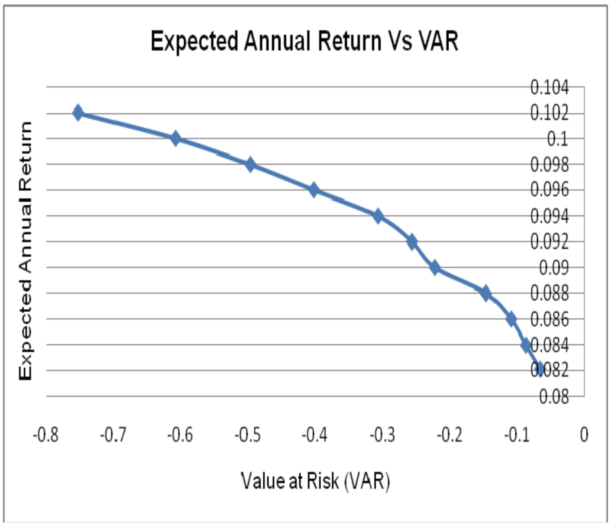

Figure 6: Portfolios Return versus VAR

3.5. Portfolio Standard Deviations versus VAR

Figure 7 shows the dependency of the portfolio Standard Deviation against VAR.

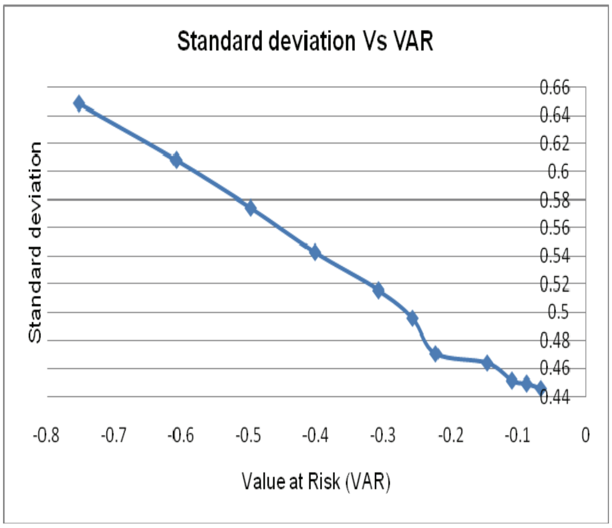

Figure 7: Standard Deviation versus VAR

From the graph we can see that the portfolio VAR is almost linearly proportional to the Standard Deviation. This is also as expected because a higher Standard Deviation translates to a higher risk, thus the VAR also increases in money terms (decreases mathematically).

3.6. Decision Support

The results presented above provide for comprehensive and reliable decision support for the decision makers, i.e. the financial risk executives of the insurance company. In particular, considering the Efficient Frontier of portfolios and the dependencies between portfolio expected return, Standard Deviation and VAR (shown in Figure 5, Figure 6 and Figure 7), the decision maker can decide in which assets to invest according to the desired expected return, risk appetite (i.e. standard deviation) and VAR. These results can help to reduce the SCR as required.

3.7. The Simulation & Optimisation Approach Comparison with the Related Work Examples

A comparison of the Simulation and Optimisation method proposed in this paper with the related work examples summarized in Sec. 1.1 is given below.

3.7.1. The Simulation & Optimisation Method versus the GDV Model

The GDV Model inherits its limitations from the Static Factor deterministic model. Also, this model is a very simplified Stochastic Model, which is an additional limitation. Moreover, the model doesn't use Optimisation to minimise the variance of the investment portfolios of the insurer, which is another major limitation.

In contrast, the proposed method does not have these two major limitations because they are resolved by using the Monte Carlo Simulation and Optimisation methodologies. Thus, the proposed approach is superior to the GDV Model.

3.7.2. The Simulation & Optimisation Method versus the Swiss Solvency Test (SST) Model

The SST ALM model is a Risk Factor Covariance model complemented with Scenario-Based models. Therefore, The SST ALM Model has the same deficiencies as the Scenario-Based models and the Covariance Models, which are not true Stochastic Models. In addition, the SST ALM Model does not apply optimisation to minimise the variance of the investment portfolios, which is another major deficiency.

The proposed method has eliminated these deficiencies by using the synergy of the Mote Carlo Simulation and Optimisation methodologies. Therefore, the proposed approach is also superior to the SST ALM Model.

3.7.3. The Simulation & Optimisation Model versus Bourdeau's Internal Market Risk Model Example

The Market Risk internal model proposed by Michele Bourdeau is a true Stochastic Risk Model. However, it does not use optimisation to minimise the variance of the investment portfolio, which is a main limitation. In this sense, the proposed method has a significant advantage versus this example because it minimises the variance of the investment portfolio, which ultimately minimises the risk and VAR.

4. CONCLUSION

This paper proposed a stochastic method for risk modelling under Solvency II. The method combines Monte Carlo Simulation and Optimisation methodologies in order to manage financial risk. The Optimisation methodology is used to calculate the minimal variance portfolios that yield desired expected returns in order to determine the Efficient Frontier of portfolios. The Monte Carlo methodology is used in order to calculate VAR/SCR for every portfolio on the Efficient Frontier by using the respective portfolios' simulation distributions. Consequently, the synergy of the Monte Carlo Simulation and Optimisation methodologies, which are used by the method, eliminates the identified significant limitations of the standard models. Also, the method has a significant advantage against the internal models, which do not use simulation and optimisation methodologies.

This stochastic approach can help the insurance and reinsurance companies to develop or improve their Solvency II internal risk models in order to reduce their VAR/SCR. Reducing the VAR and SCR will ultimately provide the insurance and reinsurance firms with higher underwriting capabilities, which will increase their competitive position on the market. Moreover, the proposed method can significantly assist the insurance and reinsurance companies to achieve their business objectives.

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Vojo has a very strong background in Mathematics, Operations Research, Modeling and Simulation, Risk & Decision Analysis, Six Sigma and Software Engineering, and a proven track record of delivered solutions applying these methodologies in practice. He is also a specialist in Business Systems Analysis & Design (Banking & Insurance) and has delivered major business solutions across several organizations. He has received several formal awards and published a number of written works, including a couple of textbooks. Vojo has also been featured as a guest speaker at several prominent conferences internationally.

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