

# MODELLING AND SIMULATION OF ORDER SORTATION SYSTEMS

Fahrettin Eldemir<sup>(a)</sup>, Elif Karakaya<sup>(b)</sup>

<sup>(a)</sup>Industrial Engineering Department, Fatih University, Istanbul

<sup>(b)</sup>Industrial Engineering Department, Fatih University, Istanbul

<sup>(a)</sup>[feldemir@fatih.edu.tr](mailto:feldemir@fatih.edu.tr), <sup>(b)</sup>[elif.krky@gmail.com](mailto:elif.krky@gmail.com)

## ABSTRACT

The Order Accumulation and Sortation Systems (OASS) are getting more important as distribution centers try to gain competitive advantages. The parameters that affect the sorting time in OASS are analyzed in this study. The length and speed of conveyors, sending the packages within a wave, the wave size, number of the sorting lanes and the sorting strategy are the main parameter in OASS. The time required to sort mixed items depends on the sortation strategy used as well. Different sorting strategies and different conveyor models are analyzed in this study. Available analytical models assume that all orders are at the same size (quantity). In this study this assumption also is relaxed. Simulation models have been developed to compare different design alternatives and design strategies. For different order combinations and for various design choices, simulation is used to compare sortation strategies. The results have been given in tables that show which strategy should be used under which scenario. AutoMod Software is used as the simulation tool.

Keywords: sortation strategies, inventory management, simulation

## 1. INTRODUCTION

In today's competitive world, it is desirable that a distribution center runs at its optimal settings to gain a competitive advantage. More efficient distribution centers are needed to respond to the increasing competition and to an increased emphasis placed on time-based service. In distribution centers, long list of orders are put together in an intensive way. Each customer order can be full of various items at different quantities. In classical order picking procedure, each order is collected by an assigned picker and the products in this list might being kept at different storage addresses. Therefore, picker may end up traveling to far distances in a warehouse in order to complete the list and searching the items all over the warehouse. This situation often causes unnecessary transportation costs and ineffective worker utilizations. To overcome shortages mentioned above, zone picking method widely used in warehouses. In this picking method, the

items from different orders are arranged over again (batch orders) and the same product types collected by the same workers. With this method, order pickers are assigned to a specific zone. In this way, unproductive travel time will disappear. However, although this situation saves time and speed, the items of accumulated orders completely mixed. Therefore the items collected by different pickers arrive to the packing area at different times. To wait the other items from the same order, the ready packages are accumulated in accumulation zone. There is no doubt that these products (items) have to be sorted according to the product type and quantity before shipment. At this point, sortation systems (these are often automated systems) are used.

The optimal condition for a given system studied would be one in which the rate of sortation (i.e., throughput rate) is maximized, so minimizing the wave sortation time without increasing the capital and operating costs. There is a trade-off between the rate and cost. Using more resources such as labor and machines can increase the rate of sortation; however, the cost of sortation thus increases. This study focuses on maximizing the throughput rate of a given system and assumes that the other variables, such as cost and operating design parameters, are held within satisfactory limits.

There are different sortation strategies available. Fixed Priority Rule, Next Available Rule, and Earliest Completion Rule. In the literature a few analytical models have been developed for these sortation strategies. However the sortation models are limited to the one induction lane and one sortation lane.

## 2. LITERATURE REVIEW

Order Accumulation and Sortation System (OASS) related publications are very few. The first example related sortation strategies comes from (Bozer et al., 1988) developed Fixed Priority Rule (FPR) for lane assignment by simulating different wave of orders. Johnson (1998) developed a dynamic sortation strategy which is called Next Available Rule (NAR) and compared it with "FPR". Eldemir (2006) developed an alternative sortation strategy called Earliest Completion Rule (ECR) by using order statistics.

Closed-loop simple conveyor design researches contain different number of induction lane and number of sortation lanes. Especially the first studies are related one induction and one sortation lane. However later on, because of the variability in products and order sizes, the conveyor designs seen in the literature adapt into many induction and sortation lanes. Following table summarizes the literature on closed-loop conveyor system analysis according to number of its induction and sortation lane.

Table 1: Literature Review about Conveyor Design

| Sortation Literature Summary |                 |                 |           |           |            |
|------------------------------|-----------------|-----------------|-----------|-----------|------------|
| Citation                     | Method          | Problem Setting |           |           |            |
|                              |                 | One Ind.        | Many Ind. | One Sort. | Many Sort. |
| Bozer and Sharp (1985)       | Simulation      | √               |           | √         |            |
| Bozer et al (1988)           | Simulation      | √               |           |           | √          |
| Johnson and Lofgren(1994)    | Simulation      | √               |           |           | √          |
| Johnson (1998)               | Analytical      | √               |           |           | √          |
| Meller (1997)                | Analytical      | √               |           |           | √          |
| Schmidt and Jackman(2000)    | Analytical      | √               |           | √         |            |
| Johnson and Meller(2002)     | Analytical      |                 | √         |           | √          |
| Russell and Meller(2003)     | Descriptive Mdl |                 | √         |           | √          |
| Bozer (2004)                 | Analytical      |                 | √         |           | √          |
| Eldemir (2006)               | Analytical      | √               |           | √         |            |

### 3. SORTATION SYSTEM DESIGN

#### 3.1 One-One Model

In this design model, one induction lane and one sortation lane is available. When the literature is evaluated thoroughly, it is observed that this model is the first applied model to the re-circulating conveyor. For instance, Bozer and Sharp (1985) have carried out this model in order to develop sortation strategies.

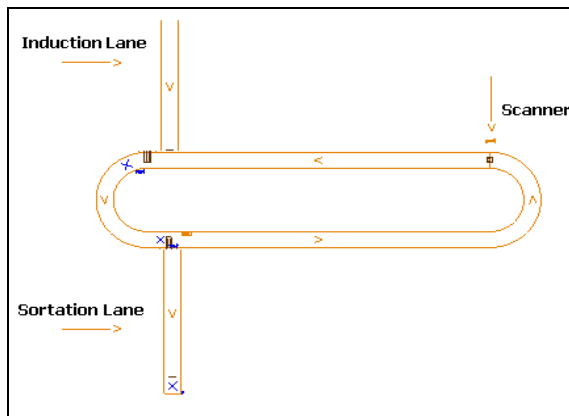


Figure 1: One- One Model Conveyor Design

#### 3.2 One-Many Model

One –Many Model differs from previous model since it has more than one sortation lanes. When it is compared with others, this model is the most applied one. For instance, Johnson and Lofgren (1994), Johnson (1998), Meller (1997) have used this model in their studies.

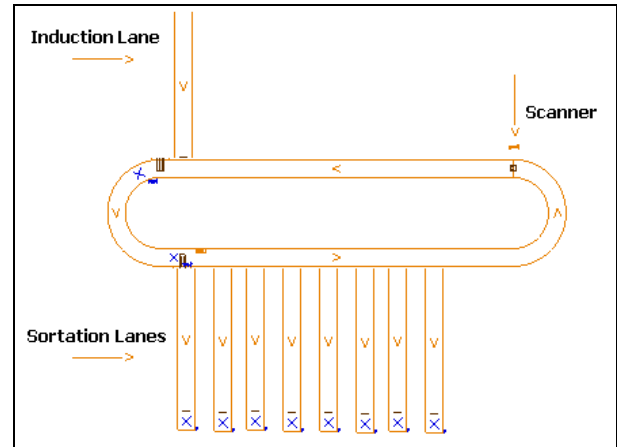


Figure 2: One- Many Model Conveyor Design

### 4. SORTATION STRATEGIES

Sortation strategies can be classified into two families, fixed priority rules (FPR) and dynamic assignment rules. In fixed priority rules, the orders are prioritized before sortation based on a certain rule. Dynamic assignment rules are assignment strategies that consider the item locations on the conveyor. The most common examples of this family are the next available rule (NAR) and the earliest completion rule (ECR). All parameters are determined below:

Table 2 : Notation

|   |  |
|---|--|
| y | Number of items within an order                                  |
| m | Number of orders within a wave                                   |
| l | Length of the closed-loop conveyor                               |
| v | Speed of the conveyors   |
| T | The time for an item to circulate around the main sortation line |
| n | Number of accumulation lanes                                     |
| i | Item index within an order                                       |
| j | Order index within a wave  |
| q | The number of orders sorted thus far                             |

#### 4.1 Fixed Priority Rule (FPR)

Sortation time evaluation by using Fixed Priority Rule is given follows. The number of accumulation lane is accepted as one, and the number of items within the order is assumed to be constant.

Under FPR, The sorting time for all orders within the specific wave will be the summation of all the gaps and spreads as follows:

$$T_{FPR} = \frac{m \cdot T \cdot y}{y + 1} \quad (1)$$

#### 4.2 Next Available Rule (NAR)

In this Next Available Rule, the expected sorting time each order depends on the number of orders which stays behind to be sorted. If it is supposed that the location of the items in the remaining orders are independent and uniformly distributed, and  $q$  the number of orders sorted thus further.

Under NAR, the sorting time for all orders within the specific wave will be as follows:

$$T_{NAR} = T \cdot \sum_{q=0}^{m-1} \left( 1 - \frac{m-q}{y(m-q)+1} \right) \quad (2)$$

#### 4.3 Earliest Completion Rule (ECR)

In dynamic assignment category, another sortation strategy model is Earliest Completion Rule (ECR). When sortation of an order is finished, the next order is determined based on the location of the last items. The order with the last item being closest to the accumulation lane is selected as next order to be sorted. Like NAR, the sortation time will be dependent on the number of orders which are going around on the main sortation lane. Assuming that all items are randomly and uniformly distributed and on the closed-loop conveyor and the item locations are independent of each other, from order statistics.

In Earliest Completion Rule, the total wave sortation time is given:

$$T_{ECR} = \sum_{q=0}^{m-1} \left( \frac{y(m-q)}{T^{y(m-q)}} \cdot \int_{l=0}^T [l^y \cdot (T^y - l^y)^{m-q-1}] dl \right) \quad (3)$$

where  $(l)$  is the location of last item on conveyor with the length of  $(T)$ .

### 5. EXPERIMENTATION

#### 5.1 One-One Model

##### 5.1.1 Analytical Model

To compare ECR, FPR and NAR, an empirical method is used. In developing the analytical models, several assumptions are made to facilitate the analysis. To illustrate the expressions for the three sorting strategies, the time to traverse the re-circulating conveyor is  $T = 100$  seconds and there are  $m = 10$  orders in each wave with  $y = 5$  boxes per order. For analytical model experimentations MAPLE software is used.

Table 3: Sorting Times for Numerical Examples

| Sorting Sequence<br>(Order number) | Order Sorting time (seconds) |        |        |
|------------------------------------|------------------------------|--------|--------|
|                                    | FPR                          | NAR    | ECR    |
| 1                                  | 83,33                        | 80,39  | 57,26  |
| 2                                  | 83,33                        | 80,43  | 58,40  |
| 3                                  | 83,33                        | 80,49  | 59,70  |
| 4                                  | 83,33                        | 80,56  | 61,19  |
| 5                                  | 83,33                        | 80,65  | 62,94  |
| 6                                  | 83,33                        | 80,77  | 65,04  |
| 7                                  | 83,33                        | 80,95  | 67,64  |
| 8                                  | 83,33                        | 81,25  | 71,02  |
| 9                                  | 83,33                        | 81,82  | 75,76  |
| 10                                 | 83,33                        | 83,33  | 83,33  |
| Total                              | 833,30                       | 810,64 | 662,29 |

#### 5.1.2 Simulation Model

In order to compare ECR, FPR and NAR, a simulation method is used as well. Several assumptions are made to facilitate the simulation analysis. To illustrate the expressions for the three sorting strategies, the time to traverse the re-circulating conveyor is  $T = 222$ , 8 seconds and there are  $m = 10$  orders in each wave with  $y = 5$  boxes per order. A hundred repetitions are done for each simulation experiment. Then, the average of these repetitions is taken.

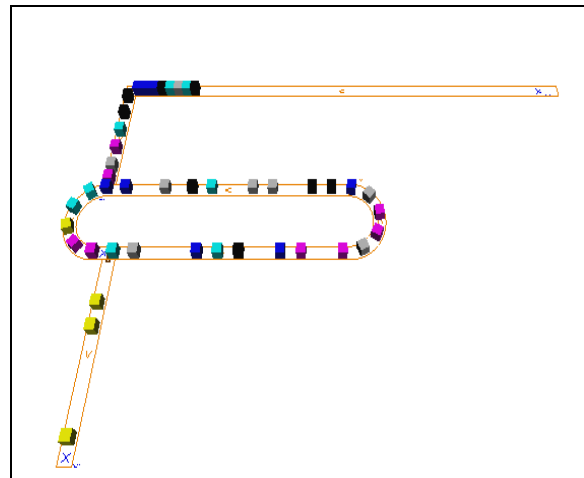


Figure 3: One-One Design Model Simulation Screenshot

For simulation model experimentations, AUTOMOD software is used. Figure 3 is the screenshot of the Automod software for One-One Design Model.

Table 4: Sorting Time Comparison for One-One Model by Using Simulation Model

| Model                | FPR      | NAR      | ECR             |
|----------------------|----------|----------|-----------------|
| <b>One-One Model</b> | 2.248,78 | 2.179,25 | <b>1.841,70</b> |

### 5.1.3 Simulation Model versus Analytic Model

Simulation model and Analytical model outputs, according to different scenarios are illustrated in following Table 5.

Table 5 : Sorting Time Comparison for One-One Model Both Simulation and Analytical Model

| Orders/<br>Wave | Items/<br>Orders | Wave Sorting<br>Time (seconds) |      |     | Wave Sorting<br>Time (seconds) |      |      |
|-----------------|------------------|--------------------------------|------|-----|--------------------------------|------|------|
|                 |                  | <i>Analytical Model</i>        |      |     | <i>Simulation Model</i>        |      |      |
|                 |                  | FPR                            | NAR  | ECR | FPR                            | NAR  | ECR  |
| 24              | 1                | 2676                           | 214  | 214 | 2915                           | 442  | 442  |
| 12              | 2                | 1784                           | 1478 | 992 | 2033                           | 1724 | 1484 |
| 8               | 3                | 1338                           | 1246 | 974 | 1574                           | 1507 | 1268 |
| 6               | 4                | 1070                           | 1033 | 878 | 1298                           | 1279 | 1120 |
| 4               | 6                | 765                            | 754  | 695 | 1003                           | 991  | 920  |
| 3               | 8                | 595                            | 591  | 563 | 823                            | 829  | 792  |
| 2               | 12               | 412                            | 411  | 403 | 640                            | 643  | 634  |
| 1               | 24               | 214                            | 214  | 214 | 442                            | 442  | 442  |

It can be realized above Table 5 that Simulation Model's results are greater than Analytical Model in every case. The reason of this situation is that in simulation model, there are some additional spent times. The following shape points out spending time locations on the simulation system.

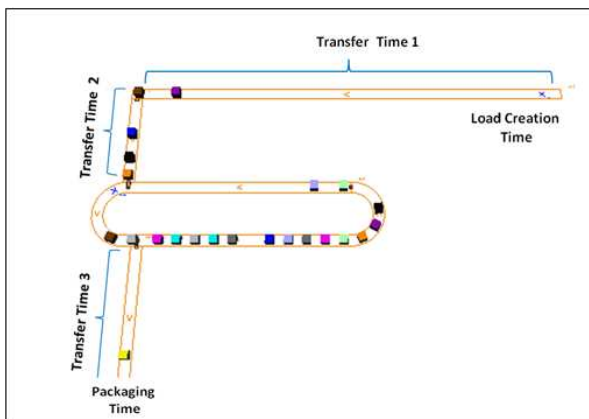


Figure 4: Extra Times Spending for Simulation

In Table 5 there are averages of extra time for each spending point which are shown in preceding shape. Besides, if subtraction is taken from simulation model to analytical model, the average difference is approximately 239 seconds. Also, summation of the extra spending time is 234.86 second. Thus, we can say that these two numbers are too close to each other.

Table 6: Sort of Spending Time for Simulation

| Spending Time      | Duration      |
|--------------------|---------------|
| Transfer Time 1    | 49,5          |
| Transfer Time 2    | 29,76         |
| Transfer Time 3    | 35,2          |
| Load Creation time | 69,7          |
| Packaging Time     | 50,7          |
| <b>Total Time</b>  | <b>234,86</b> |

## 5.2 One-Many Model

### 5.2.1 Simulation Model

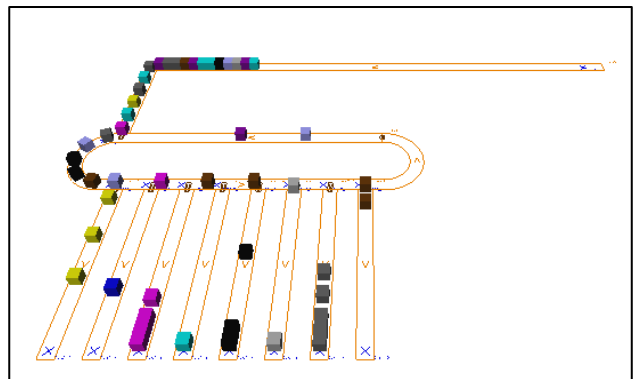


Figure 5: One- Many Design Model Simulation Screenshot

As it can be seen clearly, the best one is ECR model as One-Many Model. Since, the lowest value which emphasizes the average of the total sorting time is for ECR model.

Table 7: Sorting Time Comparison for One-Many Many by Using Simulation Model

| Model                 | FPR    | NAR    | ECR           |
|-----------------------|--------|--------|---------------|
| <b>One-Many Model</b> | 690,61 | 678,72 | <b>651,21</b> |

### 5.3 Random and Equal Number of Items in the Order

Before studies assumed that number of item in an order are same. For Example, In Johnson (1998)'s article an accepted item number is  $y=5$  for any event. In practice, it is known that it cannot be provided for every wave. Item number varies from one order to another order.

Table 8 : Sorting Time Comparison of Sorting Strategies According to Number of y

|                       | Number of y | FPR            | NAR             | ECR             |
|-----------------------|-------------|----------------|-----------------|-----------------|
| <b>One-One Model</b>  | Random      | <b>2142,65</b> | <b>2.116,97</b> | <b>1.818,30</b> |
|                       | Equal       | 2248,78        | 2.179,25        | 1.841,70        |
| <b>One-Many Model</b> | Random      | <b>668,95</b>  | <b>650,04</b>   | <b>636,46</b>   |
|                       | Equal       | 690,61         | 678,72          | 651,21          |

From above shapes, random item size provides more time saving than equal item size in addition to, it does not reflect reality.

### 5.4 Number of Orders versus Number of Items

Different numbers of items and orders combinations are designed in order to comprehend the sortation strategies behavior for various situations. After preparing 8 combinations, for example, 24-1 means that there are 24 different orders within a wave and all orders have only one item, Table 5 represents the strategies' results:

Table 9: Total Sortation Time for Different Sortation Lane in O-OM

| Orders/<br>Wave | Items/<br>Orders | Wave Sorting Time (seconds) |          |          |
|-----------------|------------------|-----------------------------|----------|----------|
|                 |                  | FPR                         | NAR      | ECR      |
| 24              | 1                | 2.915,41                    | 441,8    | 441,8    |
| 12              | 2                | 2.032,74                    | 1.723,56 | 1.484,25 |
| 8               | 3                | 1.574,19                    | 1.506,52 | 1.268,18 |
| 6               | 4                | 1.297,91                    | 1.279,31 | 1.119,80 |
| 4               | 6                | 1.003,05                    | 990,58   | 919,74   |
| 3               | 8                | 822,74                      | 828,92   | 792,45   |
| 2               | 12               | 639,51                      | 642,73   | 634,12   |
| 1               | 24               | 441,8                       | 441,8    | 441,8    |

As can be seen from Figure 5, great savings can be accomplished in total sortation time for every experiment by using ECR

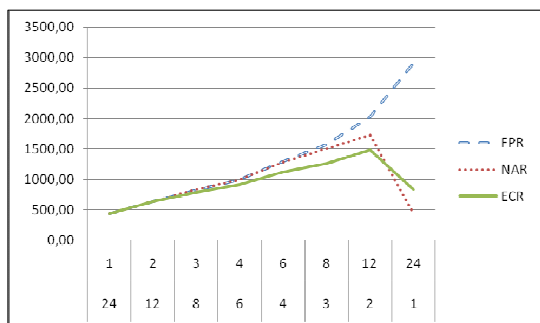


Figure 6: Total Sortation Time for Different Sortation Lane in O-OM

## 6. CONCLUSION

Available sortation strategies are compared and a set of modeling approach in simulation and in analytical is developed for the design and analysis of conveyor sortation system. Consequently, the following contributions are made:

Based on simulation models, FPR, NAR and ECR sortation strategies are compared. Overall outputs are represented as follows in Figure 7.

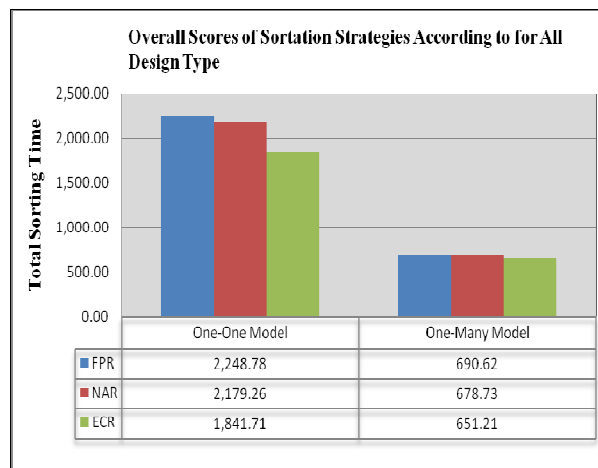


Figure 7: Effect of the Distance between Sortation Lanes

Simulation models are developed for all designs. Therefore, the results of simulation models are compared with analytical models and in this way, the validation of simulation is provided.

Different scenarios are simulated by varying design and operational parameters. For instance, despite of the literature, random item size in an order supports better results. Besides, it is more appropriate for a real case.

## REFERENCES

Bozer, Y. A., and Hsieh, Y. J., 2004, Expected waiting times at loading stations in discrete-space closed-loop conveyors, *European Journal of Operational Research*, vol. 155(2), pp. 516-532

Bozer, Y.A., and Quiroz, M. and Sharp, G.P., 1988, An evaluation of alternate control strategies and design issues for automated order accumulation and sortation system, *Material Flow*, vol. 4, pp. 265-282

Demongodin, I. And Prunet, F., 1993, Simulation modeling for accumulation conveyors in transient behavior, *COMPEURO 93*, Paris, France, pp. 29-37

Dotoli M., Iacobellis, G, Stecco, G., Ukovich W., 2009, Performance Analysis and Management of an Automated Distribution Center” *IEEE*

Gagliardi, J, Angel R, Renauld, J. 2010, A simulation modeling framework of multiple-Aisles Automated Storage and Retrieval System” *CIRRELT -57*

Harit, S., Taylor G. D., 1995, Framework for the Design and Analysis of Large Scale Material Handling Systems” *Winter Simulation Conference*

Jayaraman, A., R. Narayanaswamy, et al. 1997, A sortation system model, *Winter Simulation Conference*

Jing, G. and Kelton, W. D. and Arantes, J.C., 1998, Modeling a controlled conveyor network with merging configuration, *Proceeding of the Winter Simulation Conference*, pp. 1041-1048

Johnson, M. E., 1998, The impact of sorting strategies on automated sortation system performance”, *IIE Transactions*, vol. 30(1), pp. 67-77

- Johnson, M. E. and Russell, M., 2002, Performance analysis of split-case sorting systems, *Manufacturing & Service Operations Management*, vol. 4(4), pp 258-274
- Johnson, M. E. and Lofgren, T., 1994 Model Decomposition speeds distribution center design”, *Interfaces*, vol. 24(5), pp. 95-106
- Kale, N., Zottolo, M., Ülgen, O.M., &Williams, E.J. 2007, Simulation improves end-of line sortation and material handling pickup scheduling at appliance manufacturer. *Proceedings of the 2007 winter simulation conference* pp.1863–1868, Washington, D.C., USA
- Koster, R., Le-Duc., T., Roordbergen, K., J., 2007 Design and control of warehouse order picking: A literature review, *European Journal of Operational Research* 182, 481–501
- Le-Duc, T., and de Koster, R. 2005, Determining Number of Zones in a Pick-and-pack Order picking System, *ERIM Report Series Research in Management*, Rotterdam
- Maxwell, W. and Wilson, R., 1981, Deterministic models of accumulation conveyor dynamics, *International Journal of Production Research*, vol. 19(6), pp. 645-655
- Meller, R. D. 1997, Optimal order-to-lane assignments in an order accumulation/sortation system, *IIE Transactions* 29: 293-301
- Roodbergen, K.J., and Vis, I.F.A., 2009, A survey of literature on automated storage and retrieval system. *European Journal of Operational Research*, Vol.194, pp.343, 362
- Russell, M. L. and Meller, R. D., 2003, Cost and throughput modeling of manual and automated order fulfillment systems, *IIE Transactions*, 35, 589-603
- Schmidt, L. C. and Jackman, J., 2000, Modeling recirculating conveyors with blocking, *European Journal of Operational Research*, vol. 124, pp. 422-436
- Sonderman, D., 1982, An analytic model for recirculating conveyors with stochastic inputs and outputs, *Internal Journal of Production Research*, vol. 20(5), pp.591-605

## AUTHORS BIOGRAPHY

**Fahrettin Eldemir** is an Assistant Professor in the Department of Industrial Engineering at Fatih University, Istanbul, Turkey. He has a Ph.D. in Decision Sciences and Engineering Systems, an M.E. in Operations Research and Statistics B.S in Industrial and Management Engineering from Rensselaer Polytechnic Institute. Before joining to Fatih University, he served as an engineer and a consultant at Omega Advanced Solutions.