# OBJECT ORIENTED MODELING AND SIMULATION OF BATCH SUGAR CENTRIFUGES 

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#### Abstract

Sugar crystallizers deliver a slurry where the sugar grains are suspended in the so called mother liquor. The centrifuges are the units in charge of mechanically separating the former form the latter guaranteeing the quality of the final product and the efficiency of the process. In the typical Sugar House there are batch and continuous types of centrifuges. Batch centrifuges are deployed in the stage directly concerned with the production of the commercial product. In what follows the hybrid character models of batch sugar centrifuges are described as part of a reusable Sugar House library specially designed for the training of control room personnel.


Keywords: process modeling, simulators, simulation
languages, process equipment

## 1. INTRODUCTION

The conduction of sugar factories has experimented over the years a significant improvement by putting in place automatic control procedures wherever possible. But the role of control room operators remains decisive due mainly to the lack of measurements regarding the main quality related variables but also to the intrinsic complexity of the process.

In this context, the correct training of technical personnel is critical, and even more so due to the seasonal character of the industry which imply long periods of inactivity along the year.

An object oriented hybrid library specifically dedicated to the purpose of building simulators for operators training which features the main units to be found in the last department of sugar factories, the Sugar House, has been developed (Mazaeda, 2010) as part of larger effort to model the whole factory (Merino, Mazaeda, Alves, Acebes, Prada, 2006).

The present paper will describe the model of batch sugar centrifuges and will present some simulation results.

## 2. THE SUGAR HOUSE

The sugar fabrication process involves the extraction of the sucrose from the beets by diffusion, the ulterior conditioning, in a purification stage, of the obtained
juice in order to remove the maximum possible amount of the impurities that where inevitably pulled out along with the sucrose and also the elimination of water in a cascade of industrial evaporators. The resulting concentrated syrup is fed to the last department of the factory, the Sugar House, where the crystallization of the solute is carried out to deliver the sugar crystals with the average size, uniformity, and purity which are commercially appreciated.

Crystallization is conducted in batteries of semibatch operated vacuum pans (Georgieva, Meireles, and Feyo de Azevedo 2003, Mazaeda y Prada 2007). In each one of the crystallizers, the migration of the sucrose molecules in the solution to the faces of the population of crystals which are suspended in the contained magma or slurry is made to happen by keeping the concentration of the dissolved substance at the right value, carefully controlling the rate of water evaporation and the input of syrup to the equipment. The supersaturation of sucrose, that is the ratio of its actual concentration to the one defining the solubility at the current temperature and purity, should be higher than unity for the crystallization to be possible but it must be prevented to exceed the so called metastable limit if the uncontrolled apparition of new crystals with its negative impact on the uniformity of the grains, is to be avoided.

The resulting magma should be processed by centrifuges where the mother liquor (the remains of the original impure solution containing the non crystallized sucrose) gets expelled out of a rotating basket through conveniently sized holes while the sugar crystals are retained.

Sugar Houses are typically organized in three stages of similar structure (fig.1), with the first one dedicated to the production of the commercial white sugar and the remaining two, to the exhaustion of the syrups delivered by the previous stages in the flowsheet. A scheme of this type allows a technologically convenient separation of concerns: the first " A " stage is devoted to guaranteeing the quality of the marketable crystals, while the " B " and " C " exhausting stages should recover as much sucrose as possible by recurring to the same mechanism of crystallization. The sugar obtained in the " B " and "C" stages gets finally recycled
to conform the fed liquor of the "A" stage while the unrecoverable syrup separated at the end of the "C" stage, the so called molasses, is a by-product of the factory, still useful for alcohol production (Azucarera Ebro. S.A 2002-2008, Van der Poel, Schiweck and Schwartz 1998).


Figure 1: Sugar House three stages scheme.
The fact that the sucrose solution to be processed is contaminated with an assortment of other inorganic and organic substances which are termed collectively as impurities is the single most important quality and technological hurdle that should be sorted out. The purity of the syrups, for example, steadily decreases from the " $A$ " to the " $C$ " stage and this is the intended effect achieved as the dissolved sucrose is progressively crystallized. But the solubility of sucrose in water increases with the presence of impurities and this fact implies the need for a greater concentration of the syrups to achieve a similar degree of supersaturation and consequently the presence of, harder to handle, more viscous flows. The sugar production logic then favors the use of batch type of units in the "A" stage: the batch crystallizers for delivering an slurry containing a sugar crystals population with the right average size and spread and of batch centrifuges for performing a sharp separation of the grains and the syrup so as to offer a high purity commercial product. In the exhausting stages, however, the use of simpler, more robust continuous processing units is stipulated, since their smooth operation around a slowly varying reference, is more appropriate for the handling of viscous streams. The quality requirements of continuous equipment is relaxed in the " B " and " C " stages even
though they shouldn't be neglected completely: a bad functioning of the continuous centrifuges, for instance, could easily compromise the quality of first product by inducing the recycling of a too impure recovered sugar.

## 3. SUGAR CENTRIFUGES

In what follows, batch sugar centrifuges will be described along with their mathematical models.

The developed model needs to be in tune with the declared purpose of the Sugar House Library of being instrumental in the creation of operators training simulators. The mathematical and logical description attempted should be complete in the sense that all the elements over which the trainee has access in the real equipment and the main phenomena taking place there, are to be represented. The generic model should capture the correct behavior when deployed in different conditions of the simulated factory and so the right relations between the properties of the manipulated masses and the performing abilities of the equipment should be adequately accounted for.

The intention is for the components of the library to be put to use as part of higher hierarchy larger models representing the flowsheet of a complete beet sugar factory, because only in this broader context the operators under training can get the full implication of the different operation policies. So, the need to keep the complexity of the individual unit models under check as a means of controlling the numerical integration effort for the final aggregated simulation job, has been an important conscious aim in the design of the library.

### 3.1. Batch sugar centrifuge description.

The main constructive elements of a regular sugar batch centrifuge are depicted in figure 2.a. There is a electric motor driven rotating basket with perforated walls whose angular velocity profile can be specified by programmable speed controller. The drum is housed in an exterior case intended for receiving the filtrate expelled from the former. The basket bottom is provided with gates which should be opened at the end of the cycle to discharge the wet mass of sugar crystals on the appropriate conveyor to be transported to the factory sugar dryer.

In batch centrifuges, the basket is charged, at the beginning of every cycle, with the designated amount of magma. From the very beginning, the mother liquor starts to be expelled out of the drum through the holes in the basket's wall, while the crystals, trapped inside the case, rapidly build a solid porous sugar cake. In a first moment, when there is still mass on top of the sugar bed, in the so called filtering mode, the outgoing syrup must overcome the resistance represented by the cake. The existence of the two separated phases, the sugar cake and the supernatant slurry, which are ideally here considered as perfect concentric cylindrical rings of radii $\boldsymbol{r}_{c}$ and $\boldsymbol{r}_{l}$ respectively, is represented in figure 2.b.


Figure 2. Batch centrifuge.
Eventually, since there is a finite amount of magma to be processed at each cycle, the supernatant slurry disappears, and a so called draining mode is entered. Centrifuging is, nevertheless, prolonged with the purpose of eliminating the maximum possible amount of the syrup which remains trapped in the space represented by the pores of the bed. Not all the liquid can be drained off the cake by centrifugal force alone, so it is important, at a certain moment of the cycle, to introduce water to reduce the amount of impurities of the final product below the maximum established by the quality requirements for the first product. But the injection of water provokes the dissolution of part of the already crystallized sugar and so, the increasing of the purity of the syrup which is pushed out after the introduction of water. So the need to meet the demanded purity of sugar implies that the efficiency of the equipment is compromised. As a result, the batch centrifuges delivers not only the mass of sugar grains but two additional streams of syrups differing in purity: the so called poor syrup, of relatively low concentration of sucrose and then the rich syrup, of a higher purity, since it contains the contribution of the dissolved crystals. Poor syrup is processed in the following stages but the rich syrup of higher purity gets recycled to the same "A" stage.

Critical to the performance of the centrifuge is the resistance offered by the sugar bed which depends on the characteristics of the grain grown in the crystallizers (Bruhns, 2004). A uniform population of crystals with the right average size implies a more permeable bed and consequently the possibility of obtaining the desired level of purity with less water and thus a much better conservation of crystals. There would be also a less amount of rich syrup and, therefore, less recycling with a greater efficiency in the use of the installed base.

## 4. BATCH SUGAR CENTRIFUGE MODEL.

The model to be developed for the batch centrifuge not only need to state the dynamical mathematical equations describing the physicochemical phenomena taking place in the unit. It is also important to the describe the sequencing program that drives the stage by stage evolution for each cycle.

### 4.1.1. Modeling the batch program.

Each working cycle of the batch centrifuge consists of a number of stages including the loading of the drum with the nominal amount of magma, the administration of two water washing phases, the discharge of the sugar obtained and finally the cleaning of the basket.


Figure 3: The batch centrifuge program.
The transition from one stage to the next under the control of the centrifuge program is decided mostly based on the time elapsed since the beginning of the phase as shown in the state machine of fig. 3a. There are two concurrent extra threads of execution of the control program: one is in charge of establishing the adequate angular velocity profile for the basket (fig. 3.b) and the other decides the position of the syrup output gate used to redirect the expelled filtrate from the circuit of poor syrup at the beginning of the cycle to the pipes transporting rich syrup at some instant after the application of the first washing (fig. 3.c). The exact switching time is defined by a parameter of the program, $\boldsymbol{t}_{s w}$, which decides the relative purities of poor and rich syrups.


Figure 4. A batch centrifuge cycle.
There are a few points of synchronization between the three state machines, for example: the discharge state is not entered in fig. 3.a until the basket speed doesn't reach a predefined low angular velocity in fig.3.b. Whereas the accelerating ramp from a low angular speed, adequate to avoid the abrasion of sugar crystals while the basket is being filled with slurry, to the high centrifuging velocity appropriate for getting rid of as much syrup as possible, isn't entered till the moment the loading stage is finished.

Figure 4 gives, at a glimpse, the main activities which are carried out in a cycle of the centrifuge. It shows the rotating drum angular speed profile in fig. 4.a, the material flows into and out of the unit at different points in the evolution of the program in fig 4.b and the evolution of the radii of the bed $\left(\boldsymbol{r}_{\boldsymbol{c}}\right)$ and of the slurry on top $\left(\boldsymbol{r}_{l}\right)$ of the first in fig. 4.c.

### 4.1.2. Mass balances

In the filtering mode, the dynamic mass balances to each of the species concerned must be carried out for the supernatant space and for the cake. But when the state event triggered by the disappearance of the slurry on top of the bed marks the beginning of the draining mode ( $\boldsymbol{r}_{\boldsymbol{l}}=\boldsymbol{r}_{\boldsymbol{c}}$ ), the balances which describes the evolution of each of the species in the bed are the only ones needed. In the model here described, both mentioned volumes are considered as perfectly mixed. This last requirement, in the case of particle systems, include not only the assumption of homogeneous composition and temperature in the continuous, liquid phase but the more stringent MSMPR (Mixed-Solids Mixed-Products Removal Reactor) condition stipulating that the properties of the discrete, crystal phase, that is, its size distribution, is the same everywhere in the considered volume and that the output flow contains a representative sample of grains. The real situation is, of course, more complex (Barr, White 2006). There is a continuous slip of the particles with respect to the solution, describable on time and on the radial dimension and whose degree depends on their density difference. So that the boundary between what is considered slurry and cake are not well defined and it is more like a convenient convention. The MSMPR supposition is specially severe in the description of the cake. In this space there isn't even a suspension but a network of sugar grains in physical contact with the solution flowing through the tortuous conductions formed by the empty spaces between the crystals. In any case, the assumption here adopted is compatible with the numerical simplicity requirement previously stated.


Figure 5: The batch centrifuge.

In (1) $M_{s}(\mathrm{~kg})$ stands for the total mass of slurry in top of the cake, while $\boldsymbol{M}_{s_{-}}$suc, $\boldsymbol{M}_{s_{-} i m p}$ and $\boldsymbol{M}_{\boldsymbol{s}_{-} \text {cris }}$ are the masses of sucrose, impurities and crystals. The mass flow rates prefixed with in correspond to the input of slurry to the centrifuges, $\boldsymbol{W}_{\text {water }}(\mathrm{kg} / \mathrm{s})$ is the flow rate of water, while $\boldsymbol{W}_{\boldsymbol{s}_{-} \text {out }}$ (and the similar terms corresponding
to each of the mentioned species) are the flows than abandons the supernatant space to be incorporated to the bed.

Similar mass balances are carried out in the cake space (2), where now $\boldsymbol{W}_{s_{-} \text {out }}$ terms are input flow rates to the volume considered, $\boldsymbol{W}_{s_{-} \text {water }}$ the flow rate of water that enters the bed. The terms prefixed with un are the flow rates leaving the basket with its final discharge, while the ones with the filt prefix are the mother liquor components that are filtered out. The term $\boldsymbol{W}_{\text {dis }}(\mathrm{kg} / \mathrm{s})$ is the mass flow rate of sucrose representing the partial dissolution of the sugar crystals which is mostly important during both water washing stages.

$$
\left[\begin{array}{c}
M_{c}^{\prime} \\
M_{c_{-}^{\prime} \text { suc }}^{\prime} \\
M_{c_{-} i m p}^{\prime} \\
M_{c_{-}^{\prime} \text { cris }}^{\prime}
\end{array}\right]=\left[\begin{array}{c}
W_{s_{-} \text {out }}+W_{s_{-} \text {water }}-W_{\text {filt }}-W_{u n^{\prime}} \\
W_{s_{-} \text {out_suc }}+W_{\text {dis }}-W_{\text {filt_suc }}-W_{u n_{-} \text {suc }} \\
W_{s_{-} \text {out_imp }}-W_{\text {filt_imp }}+W_{\text {un_i }} \\
W_{s_{-} \text {out_cris }}-W_{\text {dis }}-W_{u n_{-} \text {cris }}
\end{array}\right]
$$

In the filtering mode (Wakeman and Tarleton 1999), the mass flow of syrup out of the unit can be described by the Darcy law according to (3.a), where $\boldsymbol{\eta}_{\boldsymbol{m} l}(\mathrm{~kg} / \mathrm{ms})$ and $\boldsymbol{\rho}_{\boldsymbol{m} \boldsymbol{l}}\left(\mathrm{kg} / \mathrm{m}^{3}\right)$ are the dynamic viscosity and the density of the syrup, $\boldsymbol{H}(\mathrm{m})$ is the height of the drum, $\boldsymbol{R}\left(\mathrm{m}^{-1}\right)$ the resistance of the basket perforated wall. The resistance to the flow of the latter is represented by the inverse of the permeability $\boldsymbol{k}\left(\mathrm{m}^{2}\right)$.

$$
W_{\text {filt }}=\left\{\begin{array}{l}
\frac{\rho_{m l}^{2} \omega^{2}\left(r_{0}^{2}-r_{l}^{2}\right)}{2 \cdot\left(\frac{\eta_{m l}}{2 \pi H} \frac{1}{k} \ln \left(\frac{r_{0}}{r_{c}}\right)+\frac{\eta_{m l} R}{2 \pi r_{0} H}\right)}  \tag{a}\\
\frac{\rho_{m l}^{2} \omega^{2}\left(r_{0}^{2}-r_{c}^{2}\right)}{2 \cdot\left(\frac{\eta_{m l}}{2 \pi H} \frac{1}{k_{r l} k} \ln \left(\frac{r_{0}}{r_{c}}\right)+\frac{\eta_{m l} R}{2 \pi r_{0} H}\right)}
\end{array}\right.
$$

In the draining mode, the situation changes, however. Now the movement of the syrup out of the bed is governed by the relation between the hydrostatic pressure of the liquid and the surface tension in the capillaries formed by the pores. The description of this process could be quite involved, but here the model suggested in Ambler, (1988) is adopted. The flow of each phase, liquid and air, is thought to individually follow Darcy law (3.b). The applicable permeability for the draining of syrup will now be affected by a factor $\boldsymbol{k r l}$ which will decrease as the relative volume $\left(\boldsymbol{S}_{\boldsymbol{R}}\right)$ occupied by the syrup in the pores diminishes (4).
$k_{r l}=S_{R} \frac{(2+3 \lambda)}{\lambda}$
In (4) $\lambda$ is a so called pores distribution index to be adjusted and $\boldsymbol{S}_{R}$ is defined as in (5). The pores saturation term $\boldsymbol{S}_{\text {sat }}$ gives how much of their volume ( $\boldsymbol{V}_{\text {pores }}$ ) is occupied by the liquid. The volume taken by
the latter $\left(\boldsymbol{V}_{\boldsymbol{m}}\right)$ can be easily determined form the mass balance (2).

$$
\begin{align*}
& S_{R}=\frac{S_{\text {sat }}-S_{\infty}}{1-S_{\infty}}  \tag{5}\\
& S_{\text {sat }}=\frac{V_{m l}}{V_{\text {pores }}}  \tag{6}\\
& S_{\infty}=k_{\infty} L_{m m}^{-0.5}\left(\frac{\omega^{2} r_{0}}{g}\right)^{-0.5} \rho_{m l}^{-0.25} \tag{7}
\end{align*}
$$

The draining will never be complete, since there always remains a residual saturation $\boldsymbol{S}_{\infty}$ which had been here characterized by equation (7) as function of the crystal average size $\boldsymbol{L}_{\boldsymbol{m} \boldsymbol{m}}$, the angular speed and the density of the liquor.

The characterization of the permeability as a function of the characteristics of the grains in the suspension can be attempted with the following expression due Carman-Kozeny:

$$
\begin{equation*}
k=\frac{\varepsilon^{3}}{5 S_{0}^{2}(1-\varepsilon)^{2}} \tag{8}
\end{equation*}
$$

Where $\boldsymbol{S}_{0}\left(\mathrm{~m}^{-1}\right)$ is the specific surface of the bed, which is defined as the relation of the area to the volume of the particles, and $\varepsilon\left(\mathrm{m}^{3} / \mathrm{m}^{3}\right)$ its voidage: the relation between the volume of the pores to the bulk volume of the cake.

### 4.2. Dissolution of sugar crystals

Dissolution will mainly occur when the application of water makes that the concentration of sucrose in the solution ( $\boldsymbol{C}_{\text {suc }}$ in $\mathrm{kg} / \mathrm{m}^{3}$ ) that goes through the pores of the bed fall below the solubility $\left(\boldsymbol{C}_{\text {suc_sat }}\right)$ of that substance in water calculated at the existing temperature (Bubnik, Kadlec, Urban and Bruhns, 1995). For simplicity, the dissolution is not considered in the supernatant space and this can be justified because in the normal functioning of the centrifuge the two washing stages are performed when the mentioned phase has already disappeared. So the mass rate of dissolving sucrose ( $\boldsymbol{W}_{\text {dis }}$ ) in the bed is to be described as:
$W_{\text {dis }}=J_{\text {cris }} A_{\text {cris }}=\beta\left(C_{\text {suc_sat }}-C_{\text {suc }}\right) A_{\text {cris }}$
In (9) $\boldsymbol{A}_{\text {cris }}$ is the aggregate area of sugar crystals and $\boldsymbol{\beta}$ the mass transfer coefficient ( $\mathrm{m} / \mathrm{s}$ ) that could be estimated by the following Frössling correlation that relates the Sherwood ( $\boldsymbol{S h}$ ), Schmidt ( $\boldsymbol{S c}$ ) and particle referred Reynolds ( $\boldsymbol{R}_{e p}$ ) non dimensional numbers, enhanced by factor $\boldsymbol{f}_{\boldsymbol{\varepsilon}}$ which depends on the voidage $\boldsymbol{\varepsilon}$ :

$$
\begin{align*}
& S h=f_{\varepsilon}\left(2+0.664 R_{e p}^{1 / 2} S c^{1 / 3}\right) \Rightarrow \\
& \beta=\frac{D_{m l}}{L_{e}} f_{\varepsilon}\left[2+0.664\left(\frac{u L_{e} \rho_{m l}}{\eta_{m l}}\right)^{\frac{1}{2}}\left(\frac{\eta_{m l}}{D_{m l} \rho_{m l}}\right)^{\frac{1}{3}}\right]  \tag{10}\\
& f_{\varepsilon}=[1+1.5(1-\varepsilon)] \tag{11}
\end{align*}
$$

In the above expressions $\boldsymbol{L}_{e}$ is the crystals average size equivalent to the sphere of same volume, $\boldsymbol{D}_{\boldsymbol{m}}$ the coefficient of diffusion ( $\mathrm{m}^{2} / \mathrm{s}$ ) and $\boldsymbol{u}$ is slip velocity between the grains and the liquid. An estimation of the diffusion coefficient for sucrose solution have been compiled in Bubnik, Kadlec, Urban and Bruhns (1995). The slip velocity $\boldsymbol{u}$ applicable can be obtained readily from superficial filtrate velocity $\boldsymbol{v}$ and the voidage of bed by using (12).
$u=\frac{v}{\varepsilon}=W_{\text {filt }} / \varepsilon \cdot 2 \pi \cdot r_{0} \cdot H \cdot \rho_{m l}$

### 4.3. Energy balances.

Energy balances are likewise performed for each of the considered volumes. In the case of the slurry on top of the cake, the rate of change of the specific enthalpy, $\boldsymbol{h}_{s}(\mathrm{~J} / \mathrm{kg})$, can be determined form:
$\frac{d h_{s}}{d t}=\frac{W_{\text {in }} \cdot h_{\text {in }}+W_{\text {water }} \cdot h_{\text {water }}-W_{s_{-}} \text {out } \cdot h_{s}-h_{s} \cdot \frac{d M_{s}}{d t}}{M_{s}}$
The value of temperature can be recovered from the specific enthalpy applying known relations depending of massecuite composition (Bubnik, Kadlec, Urban and Bruhns, 1995).

For the cake, the energy balances are stated independently for the crystal (14) and mother liquor (15) phases.

$$
\begin{align*}
& \frac{h_{\text {cris }}}{d t}=\frac{\left[\begin{array}{l}
W_{s_{-} \text {out_cris }} h_{s_{-} \text {out_cris }}-W_{\text {dis }} h_{m m_{-} c}+Q \\
-W_{\text {un_cris }} h_{\text {cris }}-h_{\text {cris }} \frac{d M_{c_{-} c r i s}}{d t}
\end{array}\right]}{M_{c_{-} c r i s}}  \tag{14}\\
& \frac{h_{m l}}{d t}=\frac{\left[\begin{array}{l}
W_{s_{-} m m} h_{s_{-} m l}+W_{\text {dis }} h_{m m_{-} c}-W_{\text {un_ml }} h_{m l} \\
-Q-W_{\text {filt_ml }} h_{m m}-h_{m l} \frac{d M_{m m}}{d t}
\end{array} M_{m l}\right.}{} \tag{15}
\end{align*}
$$

In the preceding expressions, $\boldsymbol{h}_{\boldsymbol{m} m_{-} c}$ accounts for the specific enthalpy of the dissolved sucrose, while $\boldsymbol{Q}$ (W) stands for the heat energy exchange between the crystals forming the bed and the filtrate which could be expressed according to (16).

$$
\begin{equation*}
Q=\alpha_{T} \cdot A_{\text {cris }} \cdot\left(T_{m m}-T_{\text {cris }}\right) \tag{16}
\end{equation*}
$$

Where $\boldsymbol{T}_{\boldsymbol{m} \boldsymbol{m}}$ and $\boldsymbol{T}_{\text {cris }}$ are the temperatures of the filtrate and of the crystals in the bed, obtained from the respective specific enthalpy values given by (15) and (14) through known relations. The term $\boldsymbol{\alpha}_{\boldsymbol{T}}\left(\mathrm{W} / \mathrm{m}^{20} \mathrm{C}\right)$ represents the heat transfer coefficient, a parameter that could be estimated by the non dimensional expression (17) relating the Nusselt, Prandtl and Reynolds particle numbers and which is the exact analogous to the one used in the mass transfer coefficient determination for dissolution (Baehr and Stephan, 2006).
$N u=f_{\varepsilon}\left(2+0.664 R_{e p}^{1 / 2} P_{r}^{1 / 3}\right)$

### 4.4. Population balance

Terms related to the crystal size distribution (CSD) are described appealing to a population balance equation. The interest here is on following the evolution of the first moments of the number density function describing the characteristic size of particles which can be described by the following set of ODEs (Ramkrishna, 2000).

$$
\begin{align*}
\frac{d}{d t} \mu_{k}= & \frac{Q_{s_{-} m l} \mu_{k}^{i n}-Q_{u n_{-} m l} \mu_{k}-\frac{d V_{m l}}{d t} \mu_{k}}{V_{m l}}  \tag{18}\\
& +k G \mu_{k-1}
\end{align*}
$$

Where $\boldsymbol{\mu}_{\boldsymbol{k}}$ and $\boldsymbol{\mu}_{\boldsymbol{k}}^{\text {in }}$ are the moments of order $\boldsymbol{k}$ in the bed and of the incoming mass, the latter suspended in the syrup flow represented by $\boldsymbol{Q}_{s_{-} m l}$. The term $\boldsymbol{Q}_{u n_{\_} m l}$ stands for the discharge volumetric flow rate while $\overline{\boldsymbol{V}}_{\boldsymbol{m} \boldsymbol{l}}$ is the volume occupied by the syrup in the pores. The linear growing velocity of crystals, $\boldsymbol{G}(\mathrm{m} / \mathrm{s})$, which can be obtained as function of the already known mass transfer flux $\boldsymbol{J}_{\text {cris }}$ (9) using (19), will be negative, representing dissolution. The terms $\boldsymbol{f}_{a}$ and $\boldsymbol{f}_{\boldsymbol{v}}$ are form factors expressing, respectively, the relation between the square and the cube of the crystal characteristic dimension and its surface and volume.

$$
\begin{equation*}
G=\frac{f_{a} \cdot J_{c r i s}}{3 \cdot f_{v} \cdot \rho_{\text {cris }}} \tag{19}
\end{equation*}
$$

From the moments of the CSD it is possible to obtain the total crystal surface of bed (20), its specific surface (21), the mass average size (22) which corresponds to the magnitude which is usually measured in the sugar industry by performing sieve analysis, the sphere equivalent average size (23) needed in the Frössling correlation (10) and a quantity that evaluates the spread of the population sizes, the coefficient of variation ( $\boldsymbol{C V}$ ), defined as the ratio between the standard deviation and the average size, which is determined as in (24) .
$A_{\text {cris }}=f_{a} \mu_{2} V_{m l}$
$S_{0}=\frac{A_{\text {cris }}}{V_{\text {cris }}}=\frac{f_{a} \mu_{2}}{f_{v} \mu_{3}}$
$L_{m m}=\frac{\sum L_{i} \text { mass }_{i}}{\sum \text { mass }_{i}}=\frac{\mu_{4}}{\mu_{3}}$
$L_{e}=\sqrt[3]{\frac{6}{\pi}} f_{v} \frac{\mu_{3}}{\mu_{0}}$
$C V=\frac{\sigma}{L_{m m}}=\sqrt{\frac{\mu_{5} \mu_{3}}{\left(\mu_{2}\right)^{2}}-1}$

A key element of the model is the determination of the voidage $\varepsilon$ and the description of its dependence on the crystal size distribution on the incoming suspension. It is known that grains with a smaller average size $\left(\boldsymbol{L}_{\boldsymbol{m} \boldsymbol{m}}\right)$ or with a too great dispersion $(\boldsymbol{C V})$, would create a more compact bed with a smaller voidage and, consequently, with a reduced permeability. The impact on the increase of the degree of bed packaging due to the presence of a substantial mass of too small crystals, the so called false grain or fines $(\boldsymbol{F} \boldsymbol{G})$, is also very important. The threshold for classifying crystals as fines is arbitrary. A reasonable value for the present case, where the nominal average size of crystals is about $550 \mu \mathrm{~m}$, could be established around $135 \mu \mathrm{~m}$.

So, in accordance with the previous discussion, the effect of CSD characteristic on voidage will be modeled with a the power law of the kind shown in (25). The parameters $\boldsymbol{c}, \boldsymbol{e M a}, \boldsymbol{e} \boldsymbol{C V}, \boldsymbol{e} \boldsymbol{G F}$ should be adjusted with experimental data.
$\varepsilon=f\left(c L_{m m}{ }^{e M A} C V^{e C V} F G^{e F G}\right)$

### 4.5. Calibration and Validation

Calibration and validation of the batch centrifuge model has had to be performed appealing to historical data and in a context of lack of enough measurements.

The batch centrifuge used as reference has been a FIVES-CAIL's COMPACT, C41 unit.

Calibration has been conducted fixing the known physical dimensions, drum speed profile and typical stage timing.


Figure 6. Data to determine voidage of bed.

The important functional dependence between the voidage of the bed and the characteristics of the grain has been fixed at the values shown in table 1, that were obtained by adjusting the free parameters in expression (25) to reproduce the data shown in figure 6. The data in the latter graphic has been extracted from the experimental results obtained by Bruhns (2004) which establishes the relation between voidage, average size and the presence of false grain. Information regarding the possible influence of the coefficient of variation has been assumed, reflecting the known tendency (Heffels 1986).

Table 1. Assumed parameters.

| $\boldsymbol{c}$ | $\boldsymbol{e} \boldsymbol{L}$ | $\boldsymbol{e} \boldsymbol{C} \boldsymbol{V}$ | $\boldsymbol{e F G}$ |
| :---: | :---: | :---: | :---: |
| 0.7114 | 0.1037 | -0.0283 | -0.1244 |

Other model parameters ( $\boldsymbol{R}, \boldsymbol{k}_{\infty}$ ) of difficult identifiability due to scarcity of measurements have been fixed to reasonable values extracted from the literature. The final average measured humidity of $0.805 \%$, corresponding to the discharged sugar has been reproduced by adjusting the pores distribution index parameter, $\lambda$, to a value of 15 .

Table 2. A massecuite characteristics.

| $\boldsymbol{B}_{\boldsymbol{m} \boldsymbol{m}}$ <br> $(\boldsymbol{\%})$ | $\boldsymbol{P}_{\boldsymbol{m} \boldsymbol{m}}$ <br> $(\boldsymbol{\%})$ | $\boldsymbol{c} \boldsymbol{c}$ <br> $\mathbf{( \% )}$ | $\boldsymbol{F G}$ <br> $(\boldsymbol{\%})$ | $\boldsymbol{L}_{\boldsymbol{m} \boldsymbol{m}}$ <br> $(\boldsymbol{\mu m})$ | $\boldsymbol{C v}$ <br> $(\boldsymbol{\%})$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 79 | 84.6 | 55 | 10 | 55 | 35 |

Table 3. "A" poor syrup characteristics.

| "A" poor syrup |  |  |  |
| :---: | :---: | :---: | :---: |
| $\boldsymbol{B}_{\boldsymbol{m} \boldsymbol{m}}(\%)$ |  | $\boldsymbol{P}_{\boldsymbol{m} \boldsymbol{m}}(\%)$ |  |
| Measured | Simulated | Measured | Simulated |
| 77.1 | 77.91 | 85.0 | 85.3 |
| 78.0 |  | 86.8 |  |

Table 4. "A" rich syrup characteristics.

| "A" rich syrup |  |  |  |
| :---: | :---: | :---: | :---: |
| $\boldsymbol{B}_{\boldsymbol{m} \boldsymbol{m}}(\%)$ |  | $\boldsymbol{P}_{\boldsymbol{m} \boldsymbol{m}}(\%)$ |  |
| Measured | Simulated | Measured | Simulated |
| 67.9 | 71.6 | 94.6 | 94.8 |
| 71.8 |  | 95.3 |  |

Table 2 offers average data describing the characteristics of the " $A$ " stage massecuite which is fed to the centrifuges and that have been used in the calibration-validation exercise. The variables recorded are the mass fraction (or Brix) of dissolved substances in the mother liquor $\left(\boldsymbol{B}_{\boldsymbol{m} \boldsymbol{m}}\right)$ and its purity $\left(\boldsymbol{P}_{\boldsymbol{m} \boldsymbol{m}}\right)$, the mass fraction of crystals referred to the whole suspension (cc), the amount of fines which has been assumed and the average size $\left(\boldsymbol{L}_{\boldsymbol{m} \boldsymbol{m}}\right)$ and $\boldsymbol{C V}$ of CSD.

On the other hand, tables 3 and 4 presents measured and simulated data regarding the poor and rich syrups, respectively. The measured data
corresponds to various laboratory analysis performed during the period here considered. Rather than stating a single value, the interval where the true average value for each variable would reside with a $95 \%$ confidence, is recorded. Static simulated final values for the Brix and purity of both types of syrups, and which corresponds to variables not specifically adjusted during calibration, are well within the correct range, so contributing to increase the trust on the model.

It had been not viable to conduct a conventional dynamic validation, but the fact that the model is able to reproduce the workings of the represented unit in its evolution through strictly timed stages adds to credit of the former. For example, it is known that the supernatant slurry had disappeared by the time the first washing arrives, since otherwise it would be pointless. It is also common knowledge in industrial sugar practice, that the rate of crystal dissolution is swift enough so as to guarantee the maximum possible degree of dissolution according to the amount of water introduced meaning that at each wash stage, the mother liquor is able to return to the equilibrium state defined by the solubility. The above type of behaviors, with a marked a dynamic character, and whose gross violation would have cast severe doubts on the validity of the model, have been dutifully verified.

## 5. SOME SIMULATION RESULTS

A wise conduction of the batch centrifuges is key in guaranteeing the quality and economical objectives of the factory. The operator must pay attention to the measured variables and to the reports periodically arriving from the factory laboratory to make the required adjustments. A prejudicial decrease in the purity of white sugar is probably an indication that the amount of water introduced in the unit is not enough given the characteristics of the processed grain and so should be increased.


Figure 7. Three simulation experiments illustrating effects of false grain presence.

In fig. 7, the evolution of the radii of the supernatant slurry $\left(\boldsymbol{r}_{l}\right)$ and of the bed $\left(\boldsymbol{r}_{c}\right)$ are shown for three experiments. In the cycle labeled as (I), the grain delivered by the first stage crystallizers is good, in particular the presence of false grain is a moderate $10 \%$.

The following cycle (II), the same variable has risen to a high value of $30 \%$ and this fact, that can be indirectly appreciated as a more closely packed bed (label 3) implies a worse filtering capability. This effect is evidenced in the higher peak reached by the mass of slurry (1) and, more importantly, by the increased presence of mother liquor which remains saturating the pores of the bed at the end of the draining phase (fig. 8). The extra amount of residual traces of mother liquor, if the purity of the massecuite and the amount of water applied are kept constants, implies a lower total purity $\left(\boldsymbol{P}_{\boldsymbol{m} c}\right)$ of the commercial product as reflected in table 5.

The preferred solution to the above situation implies the improvement of the uniformity of the crystals delivered by the crystallizers. An immediate remedy, however, would be the increase of the amount of water introduced in the unit. In the processing of the batch (III) (fig. 7), the water to mass ratio has been increased from the original $3.5 \%$ to a value of $5.6 \%$. A greater relative presence of water implies a correspondingly higher mass of dissolved sugar crystals as can be observed in the reduction of the radii of the bed which is obtained after washing (label 4 if fig. 7). Thus, the purity of the residual syrup, and consequently of the total humid sugar mass, is higher, an effect not only due to the above mentioned greater dissolution but to the fact that its composition is more favorable: the amount of syrup finally remaining is relatively insensitive to the increase on water but the presence of impurities is lower since it is more diluted.

Table 5. Simulated experiments.

|  | W/M <br> $\mathbf{( \% )}$ | FG <br> $\mathbf{( \% )}$ | $\mathbf{P}_{\mathbf{m c}}$ <br> $\mathbf{( \% )}$ | $\mathbf{C}_{\text {con }}$ <br> $\mathbf{( \% )}$ | $\boldsymbol{\varepsilon}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| (I) | 3.7 | 10 | 99.94 | 86.2 | 0.45 |
| (II) | 3.7 | 10 | 99.93 | 86.2 | 0.45 |
| (III) | 5.6 | 30 | 99.97 | 77.0 | 0.39 |

Table 6. Simulated experiment syrup characteristics.

|  | Poor syrup |  |  |  | Rich syrup |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathrm{P}_{\mathrm{mm}}$ <br> $(\%)$ | $\mathrm{B}_{\mathrm{mm}}$ <br> $(\%)$ | M <br> $(\mathrm{kg})$ | $\mathrm{P}_{\mathrm{mm}}$ <br> $(\%)$ | $\mathrm{B}_{\mathrm{mm}}$ <br> $(\%)$ | M <br> $(\mathrm{kg})$ |  |
| (I) | 86.4 | 81.5 | 373 | 92.7 | 77.0 | 184 |  |
| (II) | 86.4 | 81.5 | 368 | 92.5 | 77.0 | 188 |  |
| (III) | 86.4 | 81.4 | 371 | 94.5 | 76.0 | 261 |  |



Figure 8. Effect of false grain on bed saturation.

So the increase of the water to mass ratio implies the reduction of the efficiency associated to a lesser conservation of crystals which is also evidenced as an increase of the purity of the expelled syrups (table 6). The extra mass of water also ends up increasing the amount of rich syrup (M) which needs to be recycled, meaning a consequent reduction of the capacity of the installed base to process the incoming fresh concentrated juice. The operator can counterbalance this latter effect by acting on the parameter which determines the poor to rich syrup switching time $\left(\boldsymbol{t}_{\boldsymbol{s w}}\right)$. An increase of $\boldsymbol{t}_{s w}$ from the nominal situation labeled in fig. 9 as (i) to the value represented in (ii) implies the simultaneous increase of the purities of both syrups. The extra time allotted to the syrup classified as poor means a corresponding reduction of the amount of rich juice to be recycled as evidenced in the results shown in table 7. The operator should weigh the relative importance of the benefit so obtained, with the risk of finally losing too much sucrose in the molasses due to the increase in purity of the poor syrup which gets sent to the "B" and "C" exhausting stages (see also table 7).


Figure 9. Implication of poor to rich syrup switch policy.

Table 7. Simulated experiment modifying syrups switching time.

|  | Poor syrup |  |  | Rich syrup |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | $\mathrm{P}_{\mathrm{mm}}$ <br> $(\%)$ | $\mathrm{B}_{\mathrm{mm}}$ <br> $(\%)$ | M <br> $(\mathrm{kg})$ | $\mathrm{P}_{\mathrm{mm}}$ <br> $(\%)$ | $\mathrm{B}_{\mathrm{mm}}$ <br> $(\%)$ | M <br> $(\mathrm{kg})$ |
| (i) | 86.4 | 81.4 | 371 | 94.5 | 76.0 | 261 |
| (ii) | 87.5 | 80.5 | 477 | 97.5 | 75.5 | 154 |

## 6. THE OPERATORS TRAINING LIBRARY AND TRAINING SIMULATION TOOL

The centrifuges model, along with those corresponding to the other main units to be found in a typical Sugar House, are offered as part of an object oriented library. In the current state of the art of the modeling and simulation discipline for dynamical continuous or hybrid systems, the OO paradigm (Cellier, and Kofman 2006) is the one that offers the best adapted tools for dealing with the complexity posed by the need of representing large plants. Furthermore, in the case here addressed, the very nature of a project aiming at creating a general library of reusable components,
practically claims for the use of the OO paradigm with its provision for allowing the hierarchical creation of models and its ability for closely mimicking the plant to be represented by connecting the individual components reflecting the real life processing units. In figure 10, this kind of physical modeling is used to represent a battery of three batch centrifuges which has been carried out by deploying a matching number of instances of the corresponding model class and by connecting them, trough compatibles ports, to objects modeling other typical process industry equipment such as tanks and conveyors.

The modeling and simulation software chosen for implementing the Sugar House Library project has been EcosimPro (EA International 1999). This state of the art tool features EL, its own full-fledged OO language, able to deal with dynamical continuous processes but that also has the ability of managing hybrid type of systems.

This last capability is badly needed for implementing batch kind of models, like the one here described. The existence, for example, of a controlling program that defines a recipe whose stages are mostly specified in terms of the their duration, implies the need for the simulation to respond to time defined discrete events.

Moreover, the fact that a finite amount of material is to be process each cycle, means that the model must represent widely different situations, probably requiring a discrete type of logic for switching between different sub-models. Occurs, for instance, that the need for the equations which describes mass balances in the supernatant space, eventually disappears. The discrete event which requires attention in this latter case, is different than the first, and more difficult to handle, since it is triggered, not by time, but by the state of the simulation.

EcosimPro (EL) offers the language constructs that can adequately deal with both type of events, and this is useful for alerting the numerical integration machinery of their occurrence since otherwise the simulation could be severely slowed down or even disrupted.

Another advantage of the simulation concerns more specifically the particular project here reported EcosimPro is an open tool in the sense it offers the final model, ready to be numerically simulated, as $\mathrm{C}++$ class that could be driven from third party systems. This characteristic is relevant for the purpose of creating stand-alone simulation application software. A deed which is imperative if the purpose is that of providing an effective training experience by reproducing an interface similar to the one to be found in the control room of the factory.

The simulation code delivered by EcosimPro is wrapped (Alves, Normey-Rico, Merino, Acebes and Prada 2005) as an OPC (Ole for Process Control) server, so that the values of simulated variables can be transparently accessed from any client implementing the
mentioned standard communication protocol, the one currently preferred in process industries.

But the job of training demands the use of specific features such as the need of controlling the pace of the numerical integration for allowing real time or accelerated performance, or the provision for the role of the instructor, an actor which is authorized to change model parameters or introduce a simulated failure mode, to challenge the trainee and so conduct her learning process, among others. This special tasks have been specifically addressed in the development of the HMI/SCADA tool, EDUSCA, (Alves, Normey-Rico, Merino, Acebes y Prada 2006) used in the present project.


Figure 10. EcosimPro connection diagram for "A" stage centrifuges battery.

Figure 11, shows, for example, the interacting panel for dealing with one of the batch centrifuges of the simulated factory.


Figure 11. Training interface for one batch centrifuge.

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