SUPPLY CHAIN MANAGEMENT THROUGH P AND PI CONTROLLERS

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ABSTRACT

An important phenomenon in supply chain management, know as the bullwhip effect, suggests that demand variability increases as one moves in the supply chain. In this paper, a discrete time model for the beer game is derived by using the well-known z-transform. The system can be viewed as linear discrete MIMO system with lead times and integrators. We analyze the impact of P and PI controllers as replenishment policies. We also analyze the stability of the system using the characteristic equation and Jury criterion. Finally, a comparison between our proposed control-based strategy and other existing replenishment policies is performed.

Keywords: supply chain, beer game, z-transform bullwhip effect, PI control.

1. INTRODUCTION

Supply chain management has attracted much attention among process system engineering researchers recently. A supply chain includes all the participants and processes involved in the satisfaction of customer demand: transportation, storages, wholesales, distributors and factories Dejonckheere, Disney, Lambrecht and Towill (2002). A large number of participants, a variety of relations and processes, dynamics and the randomness in material and information flow prove that supply chains are complex systems in which coordination is one of the key elements of management. In this paper the focus is on the analysis and control of the material balance and information flow among of the system.

An important phenomenon in supply chain management, known as the bullwhip effect, suggests that demand variability increases as one goes up in the supply chain Hoberg, James, Bradley, Ulrich and Thonemann. (2007). The causes of the bullwhip effect can be due to the forecasting demand, the lead times, order batching, supply shortages and price fluctuations. We will mainly address the non-zero lead times and particularly the forecasting demand.

In order to demonstrate the existence of this effect the beer game was created at the beginning of the sixties School of Management, Massachusetts Institute of Technology (MIT) Dragana, Panić and Vujošević (2007). The game simulates a multi-echelon serial supply chain consisting of a Retailer (R), a Wholesaler (W), a Distributor (D) and a Factory/Manufacturer (M).

We model the basic protocol of the "beer distribution game". The mathematical model used is the transfer function which represents the relation between the input and output of a linear time invariant system (LTI). For a continuous time domain models it is customary to conduct a theoretical analysis using the Laplace transform to convert ordinary differential equations into s-domain transfer functions. In this case we are dealing with discrete signals and systems, therefore our theoretical analysis is achieved with the z-transform.

After the system is modelled one obtains a transfer function of one chain echelon introducing an ordering strategy based on controller design principles. In this work P and PI control structures are proposed and tuned accordingly, to eliminate the bullwhip effect.

We analyse the P and PI controllers parameter range for which the system is stable based on the characteristic equation and the Jury criterion. We also examine the effect of demand forecasting and lead time in such a system using the z-transform technique.

Finally in section 4 we observe the mathematical structure of replenishment strategies approached by other authors Marko and Rusjan (2008), in order to classify them according to the laws of implicit control and the type of feedback that is employed.

2. BEER GAME MODEL

Lets us consider a basic beer supply chain as shown in figure 1. There are four logistic echelons: Retailer (R), wholesales (W), distributor (D), and Factory (F).

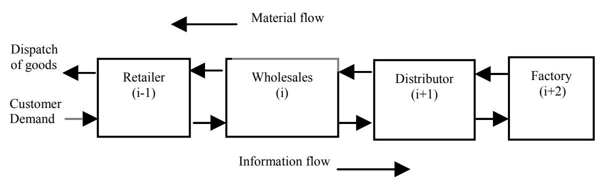

Figure 1. The Block Diagram Of Supply Chain.

Let $I_i(k)$ denote the inventory level of a echelon of chain *I(R,W,D,F)* at any discrete time instant k. We also let $Y_{i,i-1}(k)$ indicate the amount of goods to be delivered to node i-1 by the upstream node i at the instant k, $O_{i-1,i}$ indicates the demand received by node i from downstream i-1. A time delay of L due to transport is assumed for all delivery of goods so that goods dispatched for an upstream $(i+1)$ at time k will arrive at time k+l at node i. However, due to the need for examination and administrative processing, this new delivery is only available to the node i at k+L+T_0. The orders placed in the upstream for the i node is denoted for $O_{i,i+1}(k)$. We assume that the upstream $i+1$ supplier has sufficient inventory so that the orders of node i are always satisfied so that the amount of goods delivered by upstream $i+1$ in instant k $Y_{i+1,i}(k)$ is equal to the order made for the downstream i in a previous time $O_{i,i+1}(k-L-T_O)$. It's also assumed the amount of goods delivered by upstream i in k instant $Y_{i,i-1}(k)$ is equal to the order made for the downstream i-1 in a previous time $O_{i-1,i}(k-L-T_O)$.

The result of the integration of the difference between amount goods that entry from upstream node $i+1$ and amount goods that dispatched for downstream node i-1 is known as inventory balance, this has a role as a buffer to absorb the demand variability. In other words, the inventories should have stabilizing effect in material flow patterns. The equation for inventory balance at node i is given by:

$$I_i(k) = I(k-1) + O_{i,i+1}(k-L-T_0) - O_{i-1,i}(k) \tag{1}$$

A signal control denoted by $U_{i,i+1}$ is the result of a control strategy for compute the orders to upstream $O_{i,i+1}$. For example, a simple P-control can be used hence $U_{i,i+1} = K_p\left(I_0(z) - I_i(z)\right)$. We assume that ordering information is communicated after a time delay T_o. Hence, the order placed by the node i at the upstream $i+1$ is given by:

$$O_{i,i+1}(k) = U_i(k - T_o) \tag{2}$$

The z-transform is a powerful operational method when one works with discrete control systems because the differential equation is converted to an algebraic problem. Using the Time Shifting z property $Z(x(k-n)) = z^{-n} X(z)$ Ogata (1996), on equations (1) and (2), we obtained the z-transform of the above discrete time model, this is given by the equations (3) and (4).

$$I_i(z) - I_i(z)z^{-1} = U_{i,i+1}(z)z^{-(T_o+L)} - O_{i-1,i}(z) \tag{3}$$

$$O_{i,i+1}(z) = U_{i,i+1}(z)z^{-T_o} \tag{4}$$

It is considerer as a MIMO system and its matrix transfer function shown in equation 6.

$$\begin{vmatrix} O_{i,i+1}(z) \\ I_i(z) \end{vmatrix} \quad \begin{vmatrix} 0 & z^{-To} \\ -\dfrac{z}{z-1} & \dfrac{z}{z-1}\left(z^{-To}z^{-L_i}\right) \end{vmatrix} \quad \begin{vmatrix} O_{i-1,i}(z) \\ U_{i,i+1}(z) \end{vmatrix} \tag{5}$$

The corresponding simplified block diagram is given in figure 2.

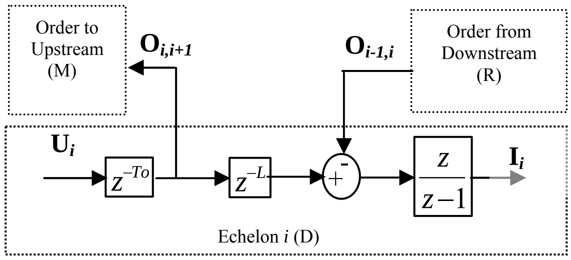

Figure 2. The Block Diagram Of Node *I* Of A Supply Chain.

3. ANALYSIS OF THE EFFECT OF P AND PI CONTROLLERS AS REPLENISHMENT INVENTORY

A control system is a combination of elements (components of the system) which enable us to control the dynamics of the selected process in a certain way. The PID (proportional, integral and derivative) is the controller most commonly used in control engineering because of its flexibility and simplicity. Therefore there will by introductory analysis of this controller as inventory replenishment policy. In this section we analyze the effect of each action (proportional, integral) over the stability in close loop in one level of the supply chain. We perform simulations with different values of parameters (K_p, K_i) and finally we compare the behaviour of the inventory and orders signals using P or PI actions.

3.1 Proportional Control

In this section we obtain the transfer function of the system in close loop with the proportional action, we approach a proportional controller and we analyse the stability of the system and the behaviour of the orders $O_{i,i+1}$ an the inventory level I_i. Our interest is management the of inventory level therefore we uses a feedback-level inventory and a proportional controller. The corresponding simplified block diagram is show below.

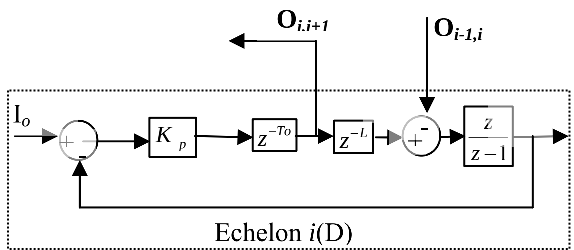

Figure 3. The Block Diagram Of Node *I* Within The Supply Chain With P-Controller.

The orders to upstream $O_{i,i+1}$ is given by the following expression.

$$O_{i,i+1}(z) = K_p z^{-To}\left(I_0(z) - I_i(z)\right) \tag{6}$$

Where K_p the proportional parameter that multiplicity the difference between the desired inventory $I_0(z)$ and physical inventory $I_i(z)$.

The physical inventory is given by:

$$I_i(z) = O_{i,i+1}(z) z^{-L}\left(\frac{z}{z-1}\right) \tag{7}$$

Replacing $I_i(z)$ in (7) obtains the closed loop transfer function or the rate ordering can be derived as the following equation.

$$G_o(z) = \frac{O_{i,i+1}(z)}{I_0(z)} = \frac{K_p z^{L-1}(z-1)}{z^{L+T_o} - z^{(L+T_o)-1} + K_p}$$

(8)

Similarly we obtain the transfers function that relates the physical inventory with the inventory desired.

$$G_I(z) = \frac{I_i(z)}{I_0(z)} = \frac{K_p(z-1)}{z^{L+T_o} - z^{(L+T_o)-1} + K_p} \tag{9}$$

Both relations contain the same characteristic equation, hence it is possible to do stability analysis for both transfer functions using this equation.
The characteristic equation is given below:

$$z^{L+T_o} - z^{(L+T_o-1)} + K_p = 0 \tag{10}$$

3.2 Stability Analysis

The objective of this section is to examine some cases on supply chain operations with a proportional control of inventory levels. A system is stable if all the roots of the characteristic equation lie within the unit circle.

$$\begin{aligned} &|z_i| < 1 \text{ Stable.} \\ &|z_i| = 1 \text{ Marginally stable.} \\ &|z_i| > 1 \text{ Unstable} \end{aligned} \tag{11}$$

The Jury stability criterion Ogata (1996), is essentially the discrete time analogue of the continuous time Routh stability criterion. It is a technique for verifying the stability of a linear discrete time system described in the z-domain. This is used directly to characteristic equation without evaluating the roots.

The Jury criterion is to check that four conditions are met.

Condition 1: If F(z) is a function of integer n grade $F(z) = a_0 z^n + a_1 z^1 \ldots\ldots a_n z^n$, which represents the characteristic equation of the transfer function, which must be fulfilled:

$$F(1) > 0 \tag{12}$$

Applying this condition in the characteristic equation (10):

$$(1)^{T_o+L} - (1)^{T_o+L-1} + K_p > 0 \tag{13}$$

As a result:

$$K_p > 0 \tag{14}$$

Equation (14) tells us that the values of K_p must by positive.

Condition 2: When the value (L+ To) is even number F(-1)>0, the result of applying this approach will be:

$$(-1)^{T_o+L} - (-1)^{T_o+L-1} + K_p > 0 \tag{15}$$

It follows that:

$$K_p > -2 \tag{16}$$

Condition 3: When the value (L+ To) be odd the function F(-1)<0, the result of applying this approach will be:

$$(-1)^{T_o+L} - (-1)^{T_o+L-1} + K_p < 0 \tag{19}$$

That is same as:

$$K_p < 2 \tag{20}$$

Condition 4: this result is immediate

$$|a_n| < a_0 \tag{21}$$

Hence the result is show in equation (22).

$$K_p < 1 \tag{22}$$

This value is more restrictive than the obtained in equation (20).

Using the conditions 1 and 4 we can find the range of values of K_p in which the system is stable. This range is given by:

$$0 < K_p < 1 \tag{23}$$

In this case for our simulations we chose a set point of 10000 units, with values T_0=1 sample and L=2 samples. Figure 4 shows the orders placed in the upstream for the *i* node $O_{i,i+1}(t)$ and the inventory level of the echelon of chain $I_i(k)$ for different values of K_p. We can see that the orders and inventory signals are stable for $K_p < 1$ and unstable for $K_p \geq 1$. With the proportional controller is possible to stabilize the system whit certain values of K_p but a phenomenon (offset) that is a mistake of steady state is inevitable because if the error is constant the control action is constant too.

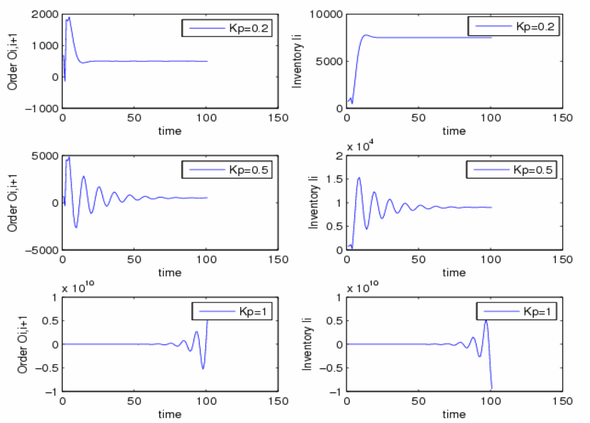

Figure 4. Simulation Results Of A Supply Chain Unit With A P Controller Using Different Values Of K_p And Stochastic Demand From Downstream.

3.4 Transfer Function With PI Actions

The integral action has some characteristics that improve the response of the system in close loop. One of these properties is that it removes the offset because the control increases although the error remains constant (integrates the error), hence we analyze the behaviour in a supply chain. In this section we show the transfer functions of the system in close loop with the proportional and integral action introducing the PI controller.

The discrete transfer function of PI controller with sample period T=1 sample is given by:

$$C(z) = K_p + \frac{K_i}{z-1} \tag{24}$$

Where K_p is the proportional constant and K_i is the integral constant.

The transfer function in closed loop of the system which relates to orders delivered by upstream is:

$$\frac{O_{i,i+1}(z)}{I_0} = \frac{K_p z^L + (K_i - K_p) z^{L-1}}{z^{L+T_o+1} - 2z^{L+T_o} + z^{L+T_o-1} + K_p z - K_p + K_i} \tag{25}$$

Similarly, the transfer function in closed loop of the system which relates the inventory balance and the desired inventory is:

$$\frac{I_i(z)}{I_0} = \frac{K_p z - K_p + K_i}{z^{L+T_o+1} - 2z^{L+T_o} + z^{L+T_o-1} + K_p z - K_p + K_i} \tag{26}$$

The orders of both functions depend of lead time and we can see it is possible to tune the PI parameters in order to stabilize the system.

The characteristic equation is given by:

$$z^{L+T_o+1} - 2z^{L+T_o} + z^{L+T_o-1} + K_p z - K_p + K_i = 0 \tag{27}$$

The basic block diagram is shown in figure 5.

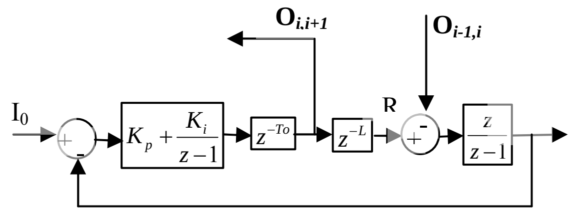

Figure 5. The Block Diagram Of Node *I* Of A Supply Chain With Pi-Controller.

The goal of this section is to examine some asymptotic cases of the supply chain operations with a proportional integral control PI of the inventory levels. Although the lead time is a parameter that can vary depending on the characteristic of each chain, we can through Jury criterion estimate a range of K_p and K_i around which the system is stable. $0 < K_p < 1, K_i < 0.35K_p$.This asymptotic analysis provides useful insights.

Using matlab we simulate different values of K_i in order to observer the influence over the stability of the system. Using the order policy defined by Eqs. (25) And (26) with values K_p=0.2, T_0=1, and L=2.

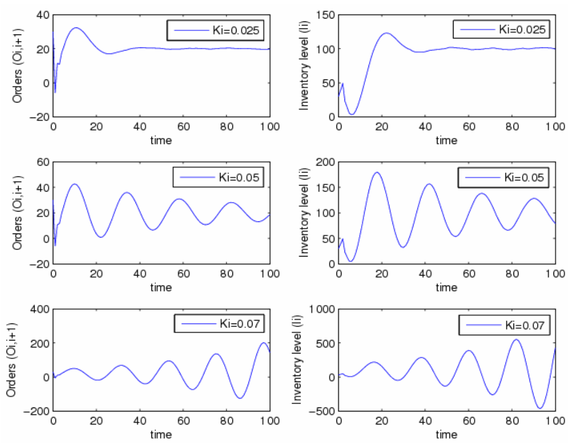

Figure 6. Simulation Results Of A Supply Chain Unit With A PI Controller Using Different Values Of Ki And Stochastic Demand From Downstream.

This allows us to conclude that it is possible through tuning PI to stabilize the pattern of the orders and thus avoid the bullwhip effect. It eliminates offset by which allows us to control inventory, which translates into a major supply chain management.

4. BULLWHIP REDUCTION

We are interested in the ratio of amplitude of the generate orders ($O_{i,i+1}$) over the amplitude of demand ($O_{i-1,i}$) that is know as ratio amplitude (AR) and this is a measurement of bullwhip Dejonckheere, Disney, Lambrecht and Towill (2002). Some new metrics for the bullwhip effect are introduced specifically based on the Frequency Response (FR). The Fourier transform, is an algebraic method of decomposing any time series into a set of pure sine waves of different frequencies, with a particular amplitude and phase angle associated with each frequency therefore through of FFT over (AR) we can measure the bullwhip effect. In this case we use a step signal plus white noise (σ =0.1) added to represent the demand. The FR is given in figure 7 for the order policy defined by Eqs. (10) And (11) with values T_0=1 sample, and L=2 samples.

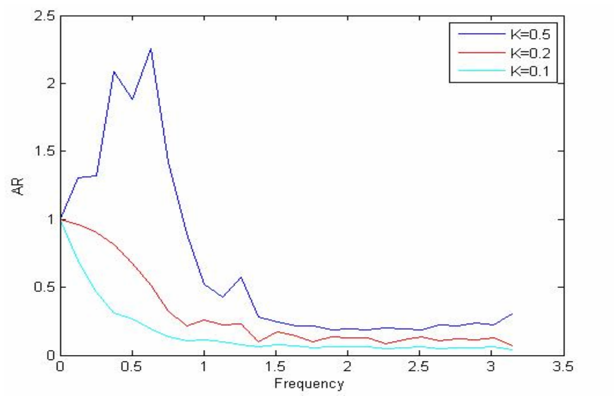

Figure 7. FR For The P Controller As Replenishment Rule.

We can see that exist a reduction of bullwhip using K_p<0.5.

The figure below shows the amplitude ratio (RA) where it is observed that for values of K_p = 0.2 and K_i = 0.02 presents a demand smoothing and thus reduction of Bullwhip effect.

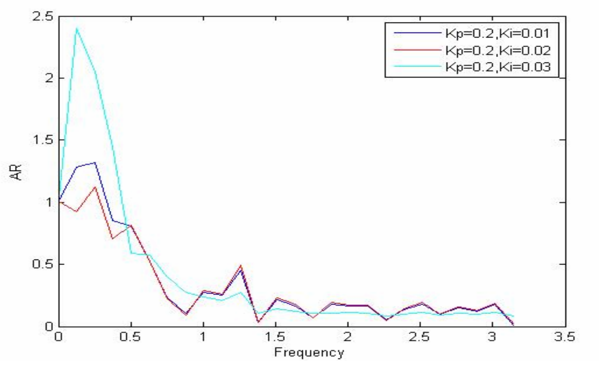

Figure .8 FR For The Pi Controller As Replenishment Rule.

5. REPLENISHMENT POLICIES VS PI CONTROL ACTIONS

In this section we will observe the mathematics structure of replenishment strategies approach by other authors Marko and Rusjan (2008), in order to classify them according to the laws of control, and the type of feedback that is used.

The policies analysed basically consists of three major components: exponentials smoothing of the demand (feedforward), order smoothing feedback loop and inventory position smoothing feedback loop.

These strategies are based on the demand forecasting and also on the feedback of signals as the level of inventory (I) and the previous orders $O_{i,i+1}$ (k-1). Our research highlights the most used strategies:

Strategy 1: We observe that usually other authors use the demand forecasting from downstream echelons through an exponential smoothing to estimate demand for the next period, that is:

$$O_{i,i+1}(k)=\hat{O}_{i-1,i}(k)=\hat{O}_{i-1,i}(k-1)+\alpha(O_{i-1,i}-\hat{O}_{i-1,i}(k-1)) \quad (28)$$

This policy reduces the bullwhip effect but does not ensure a safety inventory level; therefore this does not ensure the satisfaction of demand.

Observe that with the notation used, $O_{i-1,i}(k)$ represents the observed downstream orders (Demand) from the previous period, which we tried to predict by the demand forecast made in the previous period $(k-1)$, $\hat{O}_{i-1,i}(k-1)$.

The transfer function is give by:

$$\frac{O_{i,i+1}(z)}{O_{i-1,i}(z)}=\frac{\alpha z}{z+\alpha-1} \quad (29)$$

Strategy 2: A rule with a similar form that the exponential smoothing where order quantity $O_{i,i+1}(k-1)$ plays the role of orders from downstream (demand) forecasting $\hat{O}_{i-1,i}(k-1)$ is derived from the equation (16) which is given by:

$$O_{i,i+1}(z)=O_{i,i+1}(k-1)+\gamma\left(\hat{O}_{i-1,i}(k)-O_{i,i+1}(k-1)\right) \quad (30)$$

In this rule there exists an order smoothing feedback loop where the parameter γ has the same role as constant α in the equation (16) with $0<(\gamma)<1$.

Strategy 3: This rule is somewhat more complicated than the previous two, due to introduction concepts such as the inventory position, lead time and safety inventory

$$O_{i,i+1}(z)=\hat{O}_{i-1,i}(k)+\beta\left(I_0(z)-I_i\right) \quad (31)$$

This strategy is analogue to the approach in this paper since there exists a feedback level of inventory by a proportional factor 0<β<1 which coincides with our stability analysis. This policy allows control of inventory levels and reduces considerably the bullwhip effect.

We can see it is feasible to use different combinations in order to improve the reduction of bullwhip effect.

6. CONCLUSIONS

The discrete model for an echelon of the beer game has been derived using the z-transform. Some alternative ordering policies were formulated as P and PI control schemes. We obtain the characteristic equations of the closed loop and the stability of the system for asymptotic values has been investigated. The bullwhip effect is also analyzed through FFT over (AR), and we can conclude that using P and PI controllers the bullwhip effect of a supply chain unit can be suppressed. Finally we can see that the mathematical structure of replenishment strategies approach by other authors can be view as laws of control. We can therefore conclude that control theory is applicable to analysis of supply chains and that is would be possible to improve results using more efficient controllers as PI and PID.

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