DATA DRIVEN ADAPTIVE MODEL PREDICTIVE CONTROL WITH CONSTRAINTS

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ABSTRACT
This paper proposes a Direct Adaptive Model Predictive Controller (DAMPC) with constraints that employs subspace identification techniques to directly identify and implement the controller. The direct identification of controller parameters is desired to reduce the design effort and computational load. The DAMPC method requires a single QR-decomposition for obtaining the controller parameters and uses a receding horizon approach to collect input-output data needed for the controller identification. The paper studies the effect of different horizon schemes, the stability robustness and compares the performance of the proposed control scheme when applied to a nonlinear process with that of a linear model predictive control scheme.

Keywords: Subspace identification, Model predictive control, Adaptive control, Activated sludge process

1. INTRODUCTION

Adaptive controllers are traditionally derived from polynomial transfer function models. This paper demonstrates the use of subspace techniques to provide a state space based adaptive control technique for Model Based Predictive Control (MBPC) design.

Subspace identification techniques have emerged as one of the more popular identification methods for the estimation of state space models from measurement data. Using these techniques, subspace matrices can be constructed and used to obtain prediction of the process outputs. These predictions can subsequently serve as a basis for model predictive controller design. By continuously updating these predictions models an adaptive predictive control method can be obtained.

As an alternative to the two-step adaptive predictive control method that results when a model is explicitly estimated as shown in Fig.1, it is also possible to estimate the control parameters directly from the measurements. This direct adaptive control method was introduced by the adaptive control community in the early 70s (Åström and Wittenmark 1995) and has been widely deployed. Such algorithm combines system identification and control design simultaneously (see Fig.2).

Some previous work has been reported on the design of MPC using subspace matrices such as model-free LQG and subspace predictive controller (Favoreel et al. 1998; Favoreel et al. 1999; Kadali et al. 2003; H.Yang et al. 2005), or using the state space model identified through subspace approach (Ruscio 1997b, c; X.Wang et al. 2007). The main result of (Favoreel et al. 1998, 1999) is that the system identification and the calculation of controller parameters are replaced by a single QR decomposition. Although the idea of combination of subspace methods and MPC has been around for few years, designing an adaptive subspace MPC is still open to discussion. Previous development in subspace based constrained model predictive controllers (Kadali et al. 2002). In H.Yang et al. 2005, for example, considers an adaptive predictive control with sliding window, but does not include the constraints.

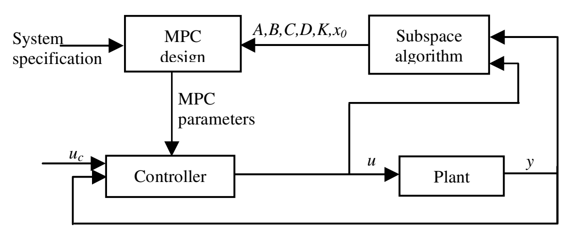

Figure 1: Indirect Adaptive Control

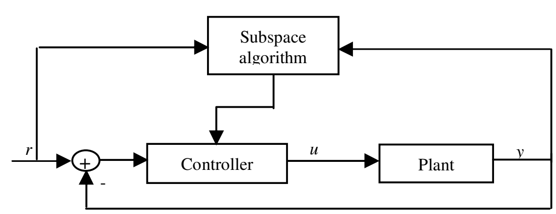

Figure 2: Direct Adaptive Control

Therefore, the objective of this paper is to develop subspace-based adaptive MPC with inclusion of constraint that can cope with mildly nonlinear processes. Other approaches for dealing with these types of processes include linear MPC, nonlinear MPC and neural network based MPC approaches. In practice, however, linear MPC approaches tend to be favoured. Linear MPC approaches include linearization approaches, where a nonlinear model is linearized at each sampling instance (e.g. Krishnan and Kosanovich 1998), and multiple model based approaches (for example see Narenda and Xiang 2000). Previous efforts in the area of adaptive predictive control have also seen the application of neural networks (Wang and J.Huang 2002), however, due to the complexity and computational load typically associated with these methods they have made few inroads in practice.

The proposed adaptive linear MPC method can offer an attractive alternative to existing predictive control methods for mildly nonlinear systems. The proposed method combines the simplicity of linear model predictive control with the power of a self-tuning. The main advantages of the proposed approach are that the usually tedious and time-consuming modelling task can be eliminated and that the controller can adapt to changing process conditions while the physical constraints are satisfied.

The paper is organized as follows: In Section 2 we briefly recapitulate the main concepts of subspace identification and QR-decomposition. The proposed constrained subspace-based MPC approach is developed in Section 3. Section 4 introduces the adaptive MPC. Section 5 describes the application to a wastewater system. The comparison of different control strategies are also presented. The report ends with some conclusion.

2. THE IDENTIFICATION METHOD

A linear discrete time-invariant state space system can be represented as,

$$x(k+1) = Ax(k) + B\Delta u(k) + Ke(k) \quad (1)$$

$$y(k) = Cx(k) + D\Delta u(k) + e(k) \quad (2)$$

where $\Delta u(k)$, $y(k)$ and $x(k)$ are the incremental inputs, outputs and states respectively and where $e(k)$ is a white noise sequence with variance $E[e_p e_q^T] = S\delta_{pq}$. The following matrix input-output equations (De Moor 1988) play an important role in the problem treated in linear subspace identification:

$$Y_f = \Gamma_i X_f + H_i U_f \quad (3)$$

where data block Hankel matrices for $u(k)$ represented as U_p and U_f are defined as:

$$U_p = \begin{pmatrix} u_0 & u_1 & \cdots & u_{j-1} \\ u_1 & u_2 & \cdots & u_j \\ \vdots & \vdots & \ddots & \vdots \\ u_{i-1} & u_i & \cdots & u_{i+j-2} \end{pmatrix} \quad (4)$$

$$U_f = \begin{pmatrix} u_i & u_{i+1} & \cdots & u_{i+j-1} \\ u_{i+1} & u_{i+2} & \cdots & u_{i+j} \\ \vdots & \vdots & \ddots & \vdots \\ u_{2i-1} & u_{2i} & \cdots & u_{2i+j-2} \end{pmatrix} \quad (5)$$

where the subscripts p and f represent 'past' and 'future' time. The same way, the outputs block Hankel matrices Y_p and Y_f can be defined. i is the prediction (i=H_p) and j is receding window size, n respectively. The extended observability matrix, Γ_i and the lower block triangular Toeplitz matrix, H_i are defined as:

$$\Gamma_i = \begin{pmatrix} C \\ CA \\ \vdots \\ CA^i \end{pmatrix}, H = \begin{pmatrix} CB & 0 & \cdots & 0 \\ CAB & CB & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ CA^{i-1}B & CA^{i-2}B & \cdots & CB \end{pmatrix} \quad (6)$$

The linear predictor equation can now be defined as:

$$\hat{Y}_f = L_w W_p + L_u U_f \quad (7)$$

where, given of past inputs and outputs W_p and future inputs U_f, the problem of subspace identification can be expressed as the solution to the following squares minimisation problem:

$$\min_{Lw,Lu} \left\| Y_f - (L_w, L_u) \begin{pmatrix} W_p \\ U_f \end{pmatrix} \right\|_F^2 \quad (8)$$

The solution to (8) can be found by applying an orthogonal projection of the row space of Y_f into the row space spanned by W_p and U_f as:

$$\hat{Y}_f = Y_f \Big/ \begin{pmatrix} W_p \\ \hline U_f \end{pmatrix} = \underbrace{Y_f /_{U_f} W_p}_{L_w W_p} + \underbrace{Y_f /_{W_p} U_f}_{L_u U_f} \quad (9)$$

The most efficient way to obtain this projection is by applying a QR-decomposition to (9):

$$\begin{pmatrix} W_p \\ U_f \\ Y_f \end{pmatrix} = \begin{pmatrix} R_{11} & 0 & 0 \\ R_{21} & R_{22} & 0 \\ R_{31} & R_{32} & R_{33} \end{pmatrix} \begin{pmatrix} Q_1^T \\ Q_2^T \\ Q_3^T \end{pmatrix} \quad (10)$$

By posing:

$$L = \begin{pmatrix} R_{31} & R_{32} \end{pmatrix} \begin{pmatrix} R_{11} & 0 \\ R_{21} & R_{22} \end{pmatrix}^+ \quad (11)$$

where + denotes the Penrose-Moore pseudo-inverse, equation (9) can be written as:

$$Y_f \Big/ \begin{pmatrix} W_p \\ \hline U_f \end{pmatrix} = L \begin{pmatrix} W_p \\ U_f \end{pmatrix} \quad (12)$$

Then, L_w and L_u can be found from L using (written in Matlab notation):

$$L_w = L(:,1:i(N_u + N_y)) \tag{13}$$

$$L_u = L(:,i(N_u + N_y)+1:i(2*N_u + N_y)) \tag{14}$$

where N_u and N_y denote the number of input and output, respectively.

3. MODEL PREDICTIVE CONTROL METHOD

This section describes the combination of subspace identification and model predictive control. Note that the steps of identification and control design can be carried out simultaneously by applying a single QR-decomposition to the input-output data. This stands in contrast to the design of conventional MPC controllers, where modelling and control design is usually distinctly separated tasks.

The model predictive control problem can in mathematical terms be expressed as the minimization of

$$J = \sum_{i=1}^{H_p} \left(\hat{y}(k+i) - r(k+i)\right)^T Q\left(\hat{y}(k+i) - r(k+i)\right) + \sum_{i=0}^{H_c-1} \Delta u(k+i)^T R \Delta u(k+i) \tag{15}$$

where H_p and H_c denote the prediction and control horizons, respectively. The output and input weighting matrices Q and R are assumed positive definite. By using the linear predictor in equation (7), rewrite the output sequence to include integral action in the predictor:

$$\Delta\hat{y}_f = \tilde{L}_w \Delta w_p + \tilde{L}_u \Delta u_f \tag{16}$$

where $\tilde{L}_w$ and $\tilde{L}_u$ are obtained directly from the previous identification of L_w and L_u while $\Delta w_p = \begin{bmatrix}\Delta y_p{}^T & \Delta u_p{}^T\end{bmatrix}^T$ and $\Delta\hat{y}_f = \begin{bmatrix}\Delta\hat{y}_1, & \cdots, & \Delta\hat{y}_i\end{bmatrix}^T$. Thus, for a k-step ahead predictor as:

$$\hat{y}_f = y_t + \tilde{L}_w \Delta w_p + \tilde{L}_u \Delta u_f \tag{17}$$

and the current output is:

$$y_t = \begin{bmatrix} y_t & y_t & \cdots & y_t \end{bmatrix}^T \tag{18}$$

By substitution of the new integrated linear predictor in equation (17) into the cost function J, differentiate it with respect to Δu_f and equating it to zero gives the control law:

$$\Delta u_f = \left(\tilde{L}_u{}^T Q \tilde{L}_u + R\right)^{-1} \tilde{L}_u{}^T Q\left(r_f - y_t - \tilde{L}_w \Delta w_p\right) \tag{19}$$

Since only the first Δu_f (1) is implemented and the calculation is repeated at each time instant t, therefore given the input $u(t)$ as:

$$u(t) = u(t-1) + \Delta u(t) \tag{20}$$

3.1. Constraints

The following constraints are considered,

$$u_k^{\min} \le u_k \le u_k^{\max}, \; \Delta u_k^{\min} \le \Delta u_k \le \Delta u_k^{\max} \tag{21}$$

Let $u_k = \Delta u_k + u_{k-1}$, then from eq.(21) gives:

$$\begin{aligned} & u_k^{\min} \le \Delta u_k + u_{k-1} \le u_k^{\max} \\ & R\Delta u_k \le u_k^{\max} - u_{k-1} \\ & -R\Delta u_k \le -u_k^{\min} + u_{k-1} \end{aligned} \tag{22}$$

where R is a lower triangular unity matrix. Rewrite the constraint in the incremental inputs as:

$$\Delta u_k \le \Delta u_k^{\max} \qquad -\Delta u_k \le -\Delta u_k^{\min} \tag{23}$$

and then combine those constraints into a single linear inequality:

$$W\Delta u_f \le b \tag{24}$$

where

$$W = \begin{bmatrix} -R & -I & R & I \end{bmatrix}^T$$

$$b = \begin{bmatrix} -\left(u_k^{\min} - u_{k-1}\right) & -\Delta u_k^{\min} & u_k^{\max} - u_{k-1} & \Delta u_k^{\max} \end{bmatrix}$$

This solution uses a standard QP optimization problem. QP is applied at every instant such that:

$$\begin{aligned} \min_{\Delta u} J &= \left(r_f - \hat{y}_f\right)^T Q\left(r_f - \hat{y}_f\right) + \Delta u_f{}^T R \Delta u_f \\ &= \left(r_f - F - \tilde{L}_u \Delta u_f\right)^T Q\left(r_f - F - \tilde{L}_u \Delta u_f\right) + \Delta u_f{}^T R \Delta u_f \\ &= \Delta u_f{}^T\left(\tilde{L}_u{}^T Q \tilde{L}_u + R\right)\Delta u_f - 2\tilde{L}_u{}^T Q\left(r_f - F\right)\Delta u_f \\ &= \frac{1}{2}\Delta u_f{}^T \Phi \Delta u_f - \phi \Delta u_f \end{aligned} \tag{25}$$

where $\Phi = \left(\tilde{L}_u{}^T Q \tilde{L}_u + R\right)$ and $\phi = -2\tilde{L}_u{}^T Q\left(r_f - F\right)$

4. DIRECT ADAPTIVE MODEL PREDICTIVE CONTROL

This section considers the online implementation of the subspace based model predictive controller. Two types of controllers are considered; one in which the subspace identification data is collected over a sliding (receding) window and one where the identification data is collected in batches.

4.1. Sliding (receding) window

The procedure of using a sliding window for identification is illustrated in Fig.3. The main advantage of this approach is that the controller parameters are updated each sample, which usually means a quicker response to process changes. The main drawback of this method is that a QR-decomposition needs to be computed each sample instance which increases the computation load.

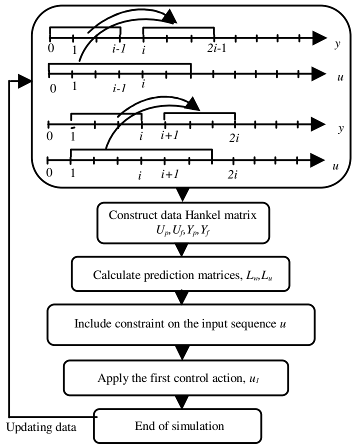

Figure 3: Sliding window for *i=4*

4.2. Batch window

In this windowing approach the control parameters are updated every n^{th} sample, where n is size of the identification window. Since a QR-decomposition is only needed every n^{th} sample the computational load requirements is lower for this method. The main drawback is that the method generally will respond slower to changing process conditions.

5. SIMULATION STUDY

To benchmark the proposed direct adaptive MPC control technique, it has been applied to an activated sludge processes. This process is comprised of an aerator and a settler as shown in Fig.4. The bioreactor includes a secondary clarifier that serves to retain the biomass in the system while producing a high quality effluent. Part of the settled biomass is recycled to allow the right concentration of micro-organisms in the aerated tank. A component mass balance that yields the following set of nonlinear differential equations was previously derived (Takács, I. et al. 1999)

$$\dot{X}(t)=\mu(t)X(t)-D(t)(1+r)X(t)+rD(t)X_r(t) \quad (26)$$

$$\dot{S}(t)=-\frac{\mu(t)}{Y}X(t)-D(t)(1+r)S(t)+D(t)S_{in} \quad (27)$$

$$\dot{C}(t)=-\frac{K_O\mu(t)}{Y}X(t)-D(t)(1+r)C(t)+K_{La}(C_S-C(t))+D(t)C_{in} \quad (28)$$

$$\dot{X}_r(t)=D(t)(1+r)X(t)-D(t)(\beta+r)X_r(t) \quad (29)$$

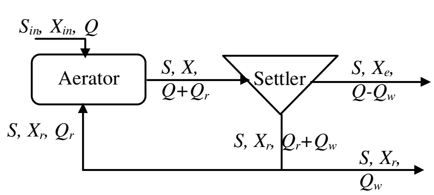

Figure 4: Activated Sludge Reactor

where the state variables, $X(t)$, $S(t)$, $C(t)$ and $X_r(t)$ represent the concentrations of biomass, substrate, dissolved oxygen (DO) and recycled biomass respectively. $D(t)$ is the dilution rate, while S_{in} and C_{in} correspond to the substrate and DO concentrations of influent stream. The parameters r and β represents the ratio of recycled and waste flow to the influent flow rate, respectively. The kinetics of the cell mass production is defined in terms of the specific growth rate μ and the yield of cell mass Y. The term K_o is a constant. C_s and K_{La} denote the maximum dissolved oxygen concentration and the oxygen mass transfer coefficient, respectively. The Monod equation gives the growth rate related to the maximum growth rate, to the substrate concentration, and to DO concentration:

$$\mu(t)=\mu_{\max}\frac{S(t)}{K_s+S(t)}\frac{C(t)}{K_c+C(t)} \quad (30)$$

where μ_{max} is the maximum specific growth rate, K_s is the affinity constant and K_c is the saturation constant. In this simulation, two controlled outputs substrate (S) and DO and two manipulated inputs dilution rate (D) and airflow rate (W) are considered.

5.1. The prediction horizons

The prediction horizon and control horizons that have been employed in the simulations are H_p=35 and H_c=5. The prediction horizon weakly related to the length of the identification window, which in turn is directly related to the computational load of the controller (size of the QR decomposition). The computation time for batch window is 15 minutes, which is 5 minutes faster than sliding window. This is reasonable since that of sliding window updates the controller at each sample time, whilst batch updates at every n^{th} sample. A trade-off between the length of the prediction horizon and the computational load of the controller must therefore be employed. Moreover, the accuracy of predictor depends on the choice of prediction horizon length. In this instance the 'best' prediction horizon length was found by fixing the length of the identification window to n=400. Then several different prediction horizons where benchmarked and it was eventually found that H_p= 35 provided for the best performance. The weighting matrices were tuned using a trial and error approach, and eventually chosen as $Q=diag\{10,1000\}$ and $R=diag\{1,1\}$.

Simulations were carried out for three different control strategies. The first two methods are those proposed in this paper, using a sliding identification window and batch identification window. The third strategy employs a non-adaptive linear MPC controller.

The simulation ran from a steady-state operating point at outputs S=41.23mg/l, D=6.11mg/l and inputs D=0.08 1/h and W=90m^3/h. The set point given for the outputs were allowed to vary approximately 10% around the system's steady state condition. The constraint on the input was given as $0.02\le u_1\le 0.15$ and $0\le u_2\le 300$ whilst constraint on the input changes were allowed to $-0.01 \le \Delta u_1 \le 0.01$ and $-5 \le \Delta u_2 \le 5$. Fig.5 shows the setpoint tracking for sliding window approach under different constraints on the input change, Δu. For the sliding window$_1$ is $|\Delta u_1| \le 0.01; |\Delta u_2| \le 5$, whilst sliding window$_2$ is $|\Delta u_1| \le 0.001; |\Delta u_2| \le 3$. It can be seen that smaller the magnitude of the maximum allowed input changes, more sluggish is the controller response to setpoint changes. Fig.6 compares the setpoint tracking performance of the three control strategies with the input constraints denoted above and constraints on the input changes, ($|\Delta u_1| \le 0.01; |\Delta u_2| \le 5$). The sliding window approach converges quickly whilst the batch approach takes somewhat longer to converge. The proposed sliding window algorithm also demonstrates less interaction than the other two and good tracking properties. The linear MPC shows an overshoot to setpoint change.

Fig.7 shows the performance of control when the measurements of substrate and DO are corrupted with step input (Amp=0.001) disturbance at t=1750. It can be seen that the performances deteriorate then able to track back to the setpoint quickly. The linear MPC shows large peak of disturbance on substrate measurement compare to the subspace MPC. Though batch window present the lowest interaction on substrate output due to disturbance, it converge slowly compare to sliding one. The measurement of DO given by sliding window and linear MPC was slightly same compared to batch window which is much slower. The stability test has been performed by simulation. The open loop poles are 0.1360;0.9924;0.8180;0.7727. Table 1 evaluated the closed loop poles for different levels of setpoint in substrate measurement. (see Fig.7)

Table 1: Closed loop poles for a different setpoint

SP A	0.1769	0.4898	0.9885	0.8297
SP B	0.2217	0.3087	0.985	0.8297
SP C	0.1552	0.5273	0.9884	0.8290

It can be seen that the closed loop system remains stable for the setpoint changes. Similar conclusions can be obtained as in the second output DO.

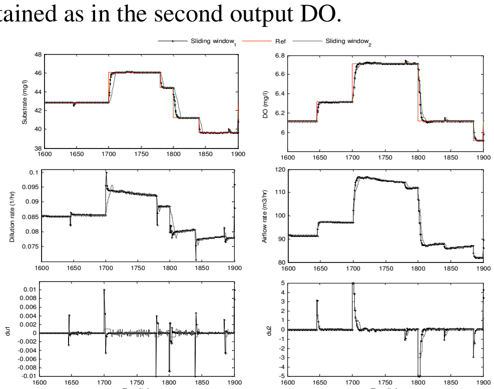

Figure 5: Different constraint case on input changes

6. CONCLUSION

In this paper, the design of model predictive controller from subspace matrices in a framework of adaptive controller is addressed and successfully applied to an activated sludge process. The subspace based model predictive controller using sliding window approach is shown to be more efficient and robust for both outputs than that of batch approach and a linear MPC controller when applied to a mildly nonlinear process.

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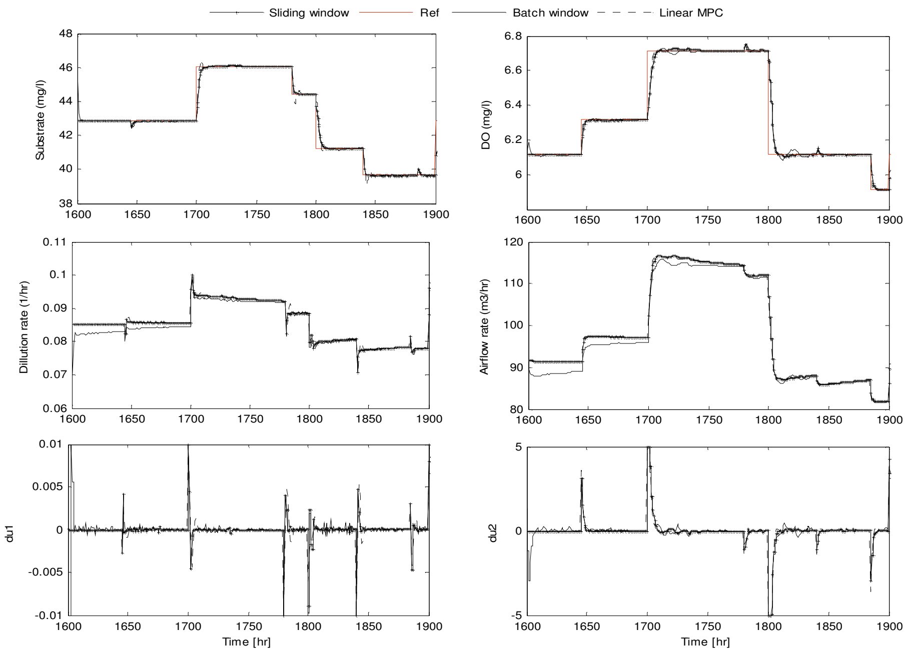

Figure 6: The comparison of control performance (set point tracking)

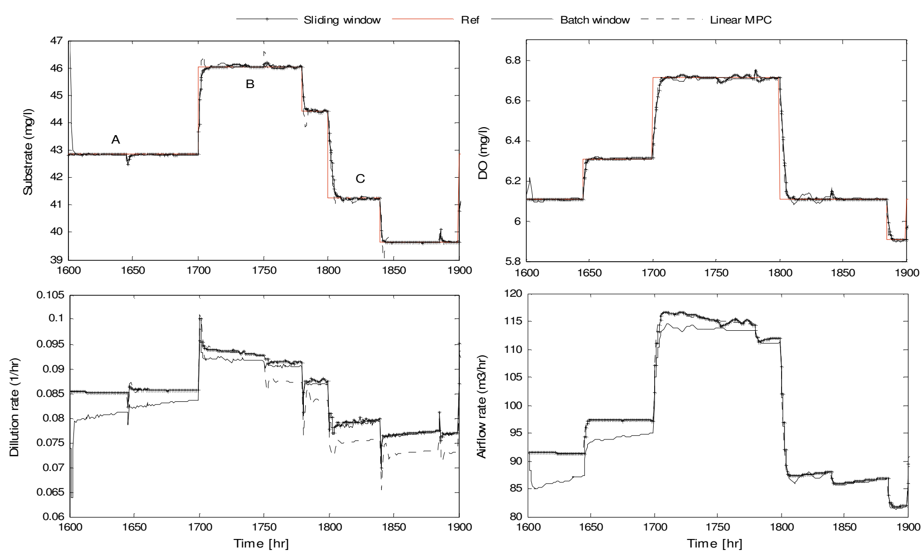

Figure 7: The comparison of control performance with input disturbance at t=1750