

MODELLING AND ROBUST CONTROL DESIGN OF A SEMI-BATCH REACTOR

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ABSTRACT

The paper uses modelling and simulation means to design a convenient control system for an exothermic semi-batch reactor used for tanning waste recovery. It is shown that this highly nonlinear system can be successfully controlled by a relatively simple fixed controller designed in a robust way. The systematic polynomial approach with some results from the robust control theory are fruitfully exploited for the goal in this paper. First, a simplified mathematical model of the process with all physical parameters and technological limits is presented. Further, a complete procedure of control system design including system identification, controller design, robustness analysis and simulation verification are described in detail.

Keywords: semi-batch reactor, modelling, robust control, simulation

1. INTRODUCTION

Modelling and simulation tools play an important role in many fields of our lives nowadays and their significance grows with the accessibility and potential of computer technology. As it is often simpler, cheaper, safer and less time consuming than real-time experiments the modelling and simulation methodologies are widely used also in industry. The exploitation can range from, e.g. analyses of system behaviour in the pre-production and production phase, examination of different (often dangerous) conditions of the process to e.g. operators training. The basics of process modelling and simulation can be found in e.g. books (Wellstead 1979; Ljung and Torkel 1994; Ogunnaike and Ray 1994 Severance 2001). Chemical reactors are essential parts of many industrial processes and their analysis and design is therefore of special interest, e.g. (Froment, Bischoff, and Wilde 2010). As experiments with chemical reactors are expensive, time-consuming and even dangerous the modelling and simulation tools are widely used also in this field (Ingham et al. 2000; Dimian 2003).

In this paper modelling and simulation tools are fruitfully utilised to design a suitable control system for a semi-batch reactor used for the tanning waste recovery (Macků 2004). The waste comes from the tannery industry where the process of leather-to-hide conversion

takes place. It contains chromium chemicals with negative impact on the environment problematic to recycle. One approach to deal with this tanning waste is the enzymatic hydrolysis which separates the chrome from protein in the form of the chromium filter cake (Kolomazník et al. 1996). Control oriented simulation analysis of the semi-batch reactor used for this process can be found in e.g. (Gazdoš and Macků 2009). This reactor is successfully controlled by means of predictive and adaptive control in works (Sámek and Macků 2008; Novosad and Macků 2011). In this contribution it is shown that the process can be controlled by a relatively simple fixed-parameters controller designed in a robust way. The controller is designed using the polynomial approach, e.g. (Kučera 1993; Hunt 1993; Grimble and Kučera 1996) and some results from the robust control theory (Zhou et al. 1995; Bhattacharyya, Chapellat, and Keel 1995; Morari and Zafirov 1989; Grimble 2006). Most of the steps are performed with the help of modelling and simulation tools, namely the MATLAB/Simulink environment.

2. PROCESS DESCRIPTION

The chromium sludge from the tannery industry is processed in a semi-batch chemical reactor sketched in Fig.1 by an exothermic chemical reaction with chrome sulphate acid. All the variables and symbols appearing in this figure and further in the paper are clearly defined in Table 1.

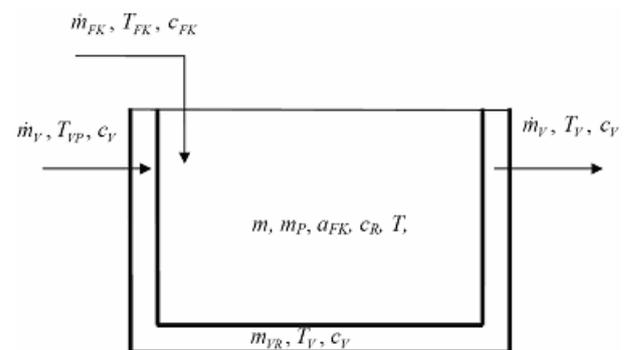


Figure 1: Chemical Reactor Scheme

During the reaction a considerable quantity of heat is developing so that reaction control is necessary. In the beginning the reactor contains initial filling

m_p [kg] given by the solution of chemicals without the chromium sludge (filter cake). This is fed into the reactor by \dot{m}_{FK} [kg/s] to control the developing heat since the temperature has to stay under a certain critical level ($T(t) < 100^\circ\text{C}$) otherwise the reactor could be destroyed. The reaction is cooled by the water flow \dot{m}_v inside the reactor jacket. The goal is to utilise the maximum capacity of the reactor to process the maximum amount of waste in the shortest possible time (higher temperature is desirable). Therefore an optimal control strategy has to find a reasonable trade-off between these opposite requirements on the operation temperature.

Table 1: Reactor Variables

Symbol	Meaning	Value [unit]
A	Pre-exponential factor	219.6 [s ⁻¹]
a_{FK}	Chromium sludge mass concentration	[-]
c_{FK}	Chromium sludge specific heat capacity	4400 [J.kg ⁻¹ .K ⁻¹]
c_R	Specific heat capacity of the reactor content	4500 [J.kg ⁻¹ .K ⁻¹]
c_v	Coolant specific heat capacity	4118 [J.kg ⁻¹ .K ⁻¹]
E	Activation energy	29968 [J.mol ⁻¹]
ΔH_r	Reaction heat	1392350 [J.kg ⁻¹]
k	Reaction rate constant	[s ⁻¹]
K	Conduction coefficient	200 [J.m ⁻² .K ⁻¹ .s ⁻¹]
m	Total mass in the reactor	[kg]
m_p	Initial filling	1810 [kg]
\dot{m}_v	Coolant mass flow	1 [kg.s ⁻¹]
\dot{m}_{FK}	Chromium sludge mass flow	0÷3 [kg.s ⁻¹]
m_{vR}	Coolant mass	220 [kg]
R	Gas constant	8.314 [J.mol ⁻¹ .K ⁻¹]
S	Heat transfer surface	7.36 [m ²]
T	Temperature in the reactor	[K]
T_v	Coolant temperature	[K]
T_{FK}	Chromium sludge temperature	293.15 [K] = 20 [°C]
T_{vp}	Input coolant temperature	288.15 [K] = 15 [°C]

Constants presented in the table were obtained analytically, experimentally, estimated or taken from the literature; for details see (Macků 2004).

2.1. Mathematical Model

A simplified mathematical model of the reactor was suggested in (Macků 2004). It was further refined and analysed for control purposes in e.g. (Gazdoš and Macků 2009). The model is described by the following four nonlinear ordinary differential equations:

$$\dot{m}_{FK} = \frac{d}{dt}m(t) \quad (1)$$

$$\dot{m}_{FK} = km(t)a_{FK}(t) + \frac{d}{dt}[m(t)a_{FK}(t)]$$

$$\dot{m}_{FK}c_{FK}T_{FK} + \Delta H_r km(t)a_{FK}(t) =$$

$$= KS[T(t) - T_v(t)] + \frac{d}{dt}[m(t)c_R T(t)]$$

$$\dot{m}_v c_v T_{vp} + KS[T(t) - T_v(t)] = \dot{m}_v c_v T_v(t) + m_{vR} c_v \frac{d}{dt}T_v(t)$$

where all the used variables and symbols are clearly defined in Table 1.

The first equation in the model expresses the total mass balance of the chemical solution in the reactor.

The second equation represents the chromium sludge mass balance where the expression $km(t)a_{FK}(t)$ defines the chromium sludge extinction by the chemical reaction. Here, k is the reaction rate constant expressed by the Arrhenius equation (2):

$$k = A e^{-\frac{E}{RT(t)}} \quad (2)$$

The third equation describes the enthalpy balance. The input heat entering the reactor in the form of the chromium sludge is expressed by the term $\dot{m}_{FK}c_{FK}T_{FK}$, the heat arising from the chemical reaction is given by the expression $\Delta H_r km(t)a_{FK}(t)$ and the heat transmission through the reactor wall is expressed by the formula $KS[T(t) - T_v(t)]$.

The last equation describes coolant heat balance. The input heat is given by $\dot{m}_v c_v T_{vp}$, the heat entering the coolant by the reactor wall is expressed by $KS[T(t) - T_v(t)]$, the heat going out with the coolant is described as $\dot{m}_v c_v T_v(t)$ and the heat accumulated in the double wall describes the last term $m_{vR} c_v T_v'(t)$.

Variables $\dot{m}_{FK}, \dot{m}_v, T_{FK}, T_{vp}$ are manipulated signals, however, practically only \dot{m}_{FK} and \dot{m}_v are usable. The temperature change of T_{FK} or T_{vp} is inconvenient due to the economic reasons (great energy demands).

2.2. Initial Conditions and Technological Limits

Initial conditions for the reactor model are defined as:

$$m(0) = m_p = 1810 \text{ kg} \quad (\text{initial reactor filling}),$$

$a_{FK}(0) = 0$ (initial mass concentration of the chromium sludge in the reactor), $T(0) = 323.15 \text{ K} = 50^\circ\text{C}$ (initial temperature of the reactor filling) and initial coolant temperature $T_v(0) = 293.15 \text{ K} = 20^\circ\text{C}$.

Maximum filling of the reactor is limited by its volume to the value of $m < 2450 \text{ kg}$ approximately. Then the process of feeding by the chromium sludge \dot{m}_{FK} has to be stopped. The feeding can be practically realized in the range $\dot{m}_{FK} \in \langle 0; 3 \rangle \text{ kg}\cdot\text{s}^{-1}$. As stated in the process description, the temperature cannot exceed the limit $T(t) < 100^\circ\text{C}$ which holds also for the coolant (water).

3. CONTROL SYSTEM DESIGN

Present control strategy of the introduced semi-batch reactor uses only the chromium sludge mass flow \dot{m}_{FK} as the main manipulated variable to control the process dynamics. As explained in the process description section, the goal is to process the maximum amount of the waste in the shortest possible time while maintaining the process quantities within the defined limits. Main critical variable is the temperature in the reactor which has to stay under the limit of 100°C during the whole process. Therefore the controlled variable is the temperature inside the reactor $T(t)$ and the manipulated variable is the chromium sludge mass flow \dot{m}_{FK} . Consequently, from the control theory point of view the process can be seen as a single input – single output (SISO) system.

3.1. Approximate Linear Model

Based on the works (Gazdoš and Macků 2008; Gazdoš and Macků 2009) it is possible to approximate the nonlinear process by the following linear transfer function:

$$G(s) = \frac{k_s(\tau s + 1)}{(T_1 s + 1)(T_2 s + 1)} = \frac{b_1 s + b_0}{s^2 + a_1 s + a_0}, \quad (3)$$

where k_s is the process gain, τ , T_1 , T_2 are time-constants of the numerator and denominator and “ s ” is the complex Laplace variable. Then among the transfer function coefficients the following relations hold:

$$\begin{aligned} b_1 &= k_s \tau / (T_1 T_2); & b_0 &= k_s / (T_1 T_2); \\ a_1 &= (T_1 + T_2) / (T_1 T_2); & a_0 &= 1 / (T_1 T_2); \end{aligned} \quad (4)$$

This model arises from the linearization of the nonlinear model (1) in a general operating point and neglecting the minor terms. Parameters k_s , τ , T_1 , T_2 and consequently also coefficients $\{b_i, a_i\}_{i=0,1}$ of this model are changing in time and are functions of the reactor operating conditions. In the time-domain, the

approximate model (3) can be expressed using the step-function as:

$$h(t) = \frac{k_s}{T_1 - T_2} \left[T_1 - T_2 + (\tau - T_1) e^{-\frac{t}{T_1}} + (T_2 - \tau) e^{-\frac{t}{T_2}} \right]. \quad (5)$$

3.2. Identification of the Approximate Linear Model

Using the simulation means, as a response to different operating conditions of the reactor (changes in the manipulated variable \dot{m}_{FK}) the following possible intervals of the model parameters were obtained:

$$\begin{aligned} k_s &\in \langle 82; 540 \rangle [^\circ\text{C} \cdot \text{s} / \text{kg}]; & \tau &\in \langle -650; 8293 \rangle [\text{s}]; \\ T_1 &\in \langle 202; 9536 \rangle [\text{s}]; & T_2 &\in \langle 734; 6601 \rangle [\text{s}]; \end{aligned} \quad (6)$$

or for the coefficients $\{b_i, a_i\}_{i=0,1}$:

$$\begin{aligned} b_1 &\in \langle -0.1715; 0.1219 \rangle \\ b_0 &\in \langle 1.069 \times 10^{-5}; 2.637 \times 10^{-4} \rangle \\ a_1 &\in \langle 2.937 \times 10^{-4}; 5.10 \times 10^{-3} \rangle \\ a_0 &\in \langle 1.980 \times 10^{-8}; 7.499 \times 10^{-7} \rangle. \end{aligned} \quad (7)$$

These intervals were obtained using the identification of the step-responses of the nonlinear model (1) to different changes of the input signal \dot{m}_{FK} . For the identification the approximate linear model described by (3), (5) was employed and standard MATLAB functions for nonlinear regression were applied. It is necessary to say that all the simulated step-responses were well-fitted by this approximate model. Several records of the simulated step-responses of the nonlinear reactor model (1) are presented in Fig. 2 to show the complex dynamics of the process. Here the simulations were performed only until the limits of the reactor were reached (maximum capacity or temperature, see section 2.2 for details).

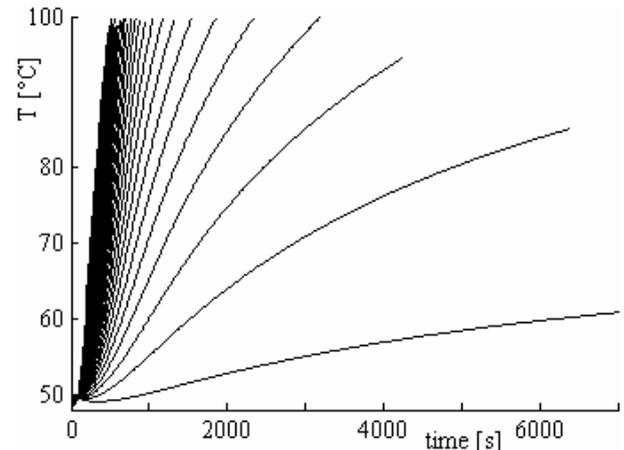


Figure 2: Limited Step-responses of the Reactor Model

From the results it is obvious that the system is highly nonlinear changing its dynamics significantly with gain from tens to hundreds and time-constants from several minutes to several hours. In addition the process can behave as a non-minimum phase system in some conditions (time-constant τ and consequently the coefficient b_1 can become negative). These properties class the process generally as difficult to control by conventional fixed parameters controllers (e.g. the wide-spread PI or PID regulators).

A nominal process model used for the further control system design is based on the middle values of the uncertainty intervals of its parameters (6)-(7). Therefore it takes the form:

$$G(s) = \frac{b_1 s + b_0}{s^2 + a_1 s + a_0} = \frac{-2.479 \times 10^{-2} s + 1.372 \times 10^{-4}}{s^2 + 2.698 \times 10^{-3} s + 3.849 \times 10^{-7}} \quad (8)$$

This model is used for the further controller design, however, the resultant control system has to work properly for the whole range of the parameters uncertainty intervals (6)-(7). This is ensured by useful tools from the robust control theory (Morari and Zafiroiu 1989; Zhou et al. 1995) presented further in this work.

3.3. Controller Design – Theoretical Framework

For the control system design the classical control set-up of Fig. 3 is considered where G denotes the controlled process – the reactor in our case, C stands for the designed controller and the signals w , e , u , y describe the reference (set-point), control error, control input and controlled variable respectively. Signals v_u and v_y represent general disturbances.

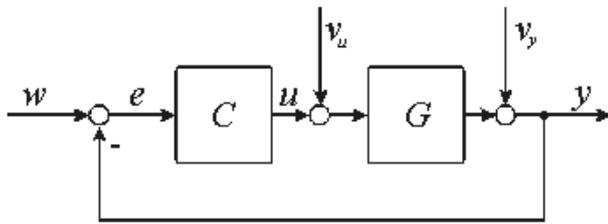


Figure 3: Control System Configuration

The process can be approximated by the transfer function (3) with the nominal values (8) as described in the previous section:

$$G(s) = \frac{b(s)}{a(s)} = \frac{b_1 s + b_0}{s^2 + a_1 s + a_0} = \frac{-2.479 \times 10^{-2} s + 1.372 \times 10^{-4}}{s^2 + 2.698 \times 10^{-3} s + 3.849 \times 10^{-7}} \quad (9)$$

Further, the controller C can be also described by a transfer function (10) with $q(s)$, $p(s)$ coprime polynomials satisfying (11).

$$C(s) = \frac{q(s)}{p(s)} \quad (10)$$

$$\deg p(s) \geq \deg q(s) \quad (11)$$

Requirements for the control system are formulated as stability, asymptotic tracking of the reference signal, disturbances attenuation and inner properness. Besides these the system has to be robust in order to cope with the real nonlinear plant (not only with the adopted nominal linear model) and possible disturbances.

From the scheme of Fig. 3 and assuming (9), (10) it is easy to derive following relationships between the controlled variable y ($Y(s)$ in the complex domain) and input signals w , v_u and v_y ($W(s)$, $V_u(s)$ and $V_y(s)$ similarly); the argument “ s ” is in these formulas omitted somewhere to keep them more compact and readable):

$$\begin{aligned} Y(s) &= \frac{G \cdot C}{1 + G \cdot C} \cdot W(s) + \frac{G}{1 + G \cdot C} \cdot V_u(s) + \\ &+ \frac{1}{1 + G \cdot C} \cdot V_y(s), \\ Y(s) &= \frac{b \cdot q}{a \cdot p + b \cdot q} \cdot W(s) + \frac{b \cdot p}{a \cdot p + b \cdot q} \cdot V_u(s) + \\ &+ \frac{a \cdot p}{a \cdot p + b \cdot q} \cdot V_y(s), \\ Y(s) &= \frac{b \cdot q}{d} \cdot W(s) + \frac{b \cdot p}{d} \cdot V_u(s) + \frac{a \cdot p}{d} \cdot V_y(s), \\ Y(s) &= T \cdot W(s) + S_u \cdot V_u(s) + S \cdot V_y(s). \end{aligned} \quad (12)$$

Here, the symbol d defines the characteristic polynomial of the closed-loop given generally as:

$$a \cdot p + b \cdot q = d. \quad (13)$$

Symbols S , T , S_u denote important transfer functions of the loop known as the sensitivity function, complementary sensitivity function, and input sensitivity function respectively. The sensitivity function S further helps to make the designed control system robust.

Similarly, it is straightforward to derive the formula (14) for the control error:

$$E(s) = \frac{p}{d} [a \cdot W(s) - b \cdot V_u(s) - a \cdot V_y(s)]. \quad (14)$$

3.3.1. Control System Stability

From (12) it is clear that the control system of Fig. 3 will be stable if the characteristic polynomial $d(s)$ given by (13) is stable. This Diophantine equation, after a proper choice of the stable polynomial $d(s)$, is used to compute unknown controller polynomials $q(s)$, $p(s)$. Sometimes it is useful to require also so called *strong stability* which guarantees also stability of the designed controller, i.e. stability of the polynomial $p(s)$ in (10). As the controlled process is nonlinear with possible non-minimum phase behaviour and the suggested design methodology relies on the approximate linear model only, the strong stability condition is also considered in this work for safety reasons.

3.3.2. Asymptotic Tracking of the Reference Signal and Disturbances Attenuation

Let us assume that the reference signal $w(t)$ is a step function, defined in the complex domain as:

$$W(s) = \frac{w_0}{s}, \quad (15)$$

and, further suppose that disturbances $v_u(t)$, $v_y(t)$ can be also approximated by step-functions:

$$V_u(s) = \frac{v_{u0}}{s}, \quad V_y(s) = \frac{v_{y0}}{s}. \quad (16)$$

Then substituting (15)-(16) into (14) yields:

$$E(s) = \frac{P}{d} \left(a \cdot \frac{w_0}{s} - b \cdot \frac{v_{u0}}{s} - a \cdot \frac{v_{y0}}{s} \right), \quad (17)$$

which shows that in order to guarantee zero-control error in the steady-state, the denominator polynomial of the controller $p(s)$ needs to be divisible by the “ s ”-term. This will be fulfilled for this polynomial in the form:

$$p(s) = s \cdot \tilde{p}(s). \quad (18)$$

Then the controller (10) can be written as:

$$C(s) = \frac{q(s)}{s \cdot \tilde{p}(s)}, \quad (19)$$

and the Diophantine equation (13) defining stability will be:

$$a \cdot s \cdot \tilde{p} + b \cdot q = d. \quad (20)$$

3.3.3. Control System Inner Properness

Inner properness of the control system is satisfied if all its parts (transfer functions) are proper. With regard to the proper approximate transfer function of the process (9), condition (11) and taking into account solvability of (13) it is possible to derive following formulae for degrees of the unknown polynomials q , \tilde{p} and d :

$$\begin{aligned} \deg q(s) &= \deg a(s), \\ \deg \tilde{p}(s) &\geq \deg a(s) - 1, \\ \deg d(s) &\geq 2 \cdot \deg a(s). \end{aligned} \quad (21)$$

Equalities are chosen in the formulas above in order to obtain the simplest controller structure satisfying the given requirements.

3.3.4. Robust Setting of the Designed Loop

In this work the control system design is based on the nominal linear model of the system in the form (8) with uncertainty intervals of its coefficients (7). These uncertainty intervals describe nonlinearities of the original nonlinear process model (1). In order to fulfill the requirements introduced in the previous sections not only for the nominal model but for the whole family of models given by the uncertainty intervals the robust control approach is fruitfully utilized, e.g. (Zhou et al. 1995; Bhattacharyya, Chapellat, and Keel 1995; Morari and Zafrou 1989; Grimble 2006).

A good measure of the control loop robustness is the peak gain of the sensitivity function frequency response (Skogestad and Postlethwaite 2005). The sensitivity function S , see (12), is defined as:

$$\begin{aligned} S(s) &= \frac{Y(s)}{V_y(s)} = \frac{1}{1 + G(s) \cdot C(s)} = \\ &= \frac{a(s) \cdot p(s)}{a(s) \cdot p(s) + b(s) \cdot q(s)} = \frac{a(s) \cdot p(s)}{d(s)}, \end{aligned} \quad (22)$$

and it describes the impact of output disturbance v_y on the process output y ; moreover, it gives the relative sensitivity of the closed-loop transfer function $T(s)$ to the relative plant model error. Therefore it can be utilized to make the control system robust. In this work this is done via tuning some of the closed loop poles – roots of the characteristic polynomial $d(s)$ (13).

In order to ensure that the control system will be stable not only for the nominal model (8) but also for the original nonlinear model (1), i.e. the family of linear models given by uncertainty intervals (7) the concept of robust stabilization is employed. The Kharitonov’s theorem, e.g. (Bhattacharyya, Chapellat, and Keel 1995), is a useful tool for this task. It enables to check stability of interval polynomials relatively simply. Therefore it is used further to check whether the

designed controller stabilizes the whole family of models given by the uncertainty intervals.

3.4. Controller Design – Implementation

Given the nominal model of the process (8) it is easy to derive a suitable controller structure using the formulas (21). The resultant controller has a form:

$$C(s) = \frac{q(s)}{s \cdot \tilde{p}(s)} = \frac{q_2 \cdot s^2 + q_1 \cdot s + q_0}{s \cdot (\tilde{p}_1 \cdot s + \tilde{p}_0)}, \quad (23)$$

hence, it is a real (filtered) PID controller. Its coefficients are obtained by a solution of the polynomial equation (13) for some stable characteristic polynomial $d(s)$. Therefore, the next task is to choose this polynomial which must be, according to (21), of the 4th order. Here it is suggested to have it in this simple form:

$$d(s) = (s + \alpha)^4. \quad (24)$$

Although this simple choice limits possible behaviour of the designed loop it enables to tune the control loop simply using one parameter $\alpha > 0$. Having the prescribed behaviour of the loop given by the characteristic polynomial $d(s)$ (24), from (20) it is possible to derive following relationships between the unknown controller coefficients, known nominal model coefficients and the tuning parameter α :

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ a_1 & 1 & b_1 & 0 & 0 \\ a_0 & a_1 & b_0 & b_1 & 0 \\ 0 & a_0 & 0 & b_0 & b_1 \\ 0 & 0 & 0 & 0 & b_0 \end{bmatrix} \begin{bmatrix} p_1 \\ p_0 \\ q_2 \\ q_1 \\ q_0 \end{bmatrix} = \begin{bmatrix} 1 \\ 4\alpha \\ 6\alpha^2 \\ 4\alpha^3 \\ \alpha^4 \end{bmatrix}. \quad (25)$$

This matrix equation is used to compute the unknown controller coefficients for a chosen parameter $\alpha > 0$.

Dependence of the infinity norm H_∞ (peak value) of the sensitivity function (22) upon the tuning parameter α is depicted in Fig. 4. From the graph it is obvious that smaller values of the parameter α close to zero result in very sensitive control system, i.e. non-robust. A detail of the graph presented in Fig. 5 reveals a minimum point for $\alpha \doteq 0.0008$ leading to the most robust (least sensitive) control loop. Values in the left interval $\alpha \in (0; 0.0008)$, especially those close to the zero value, result in very sensitive, i.e. non-robust control loop. Values higher than the minimum lead to relatively robust control loop, however, higher values of the parameter give rise to higher control action of the controller. Therefore one has to find a reasonable trade-off between the robustness of the loop and limitations

on the control input (manipulated variable \dot{m}_{FK} of the reactor).

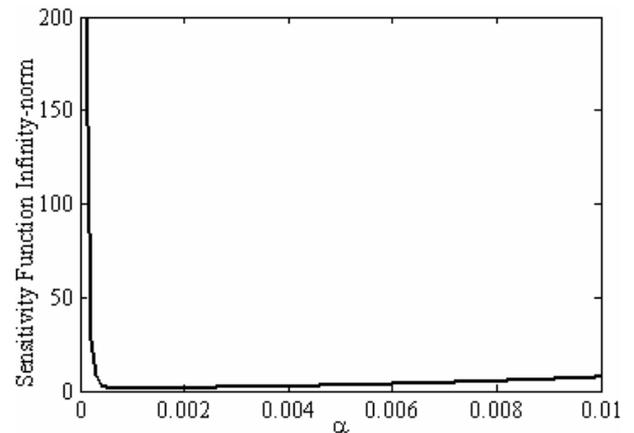


Figure 4: Sensitivity Function Infinity-norm with α

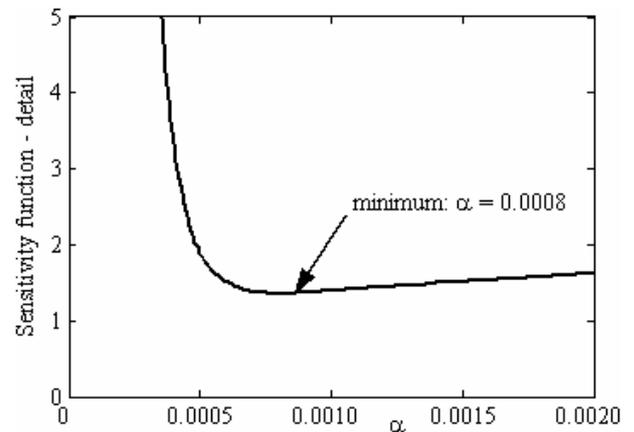


Figure 5: Sensitivity Function Detail

Further computations using the Kharitonov's theorem provide this interval $\alpha \in \langle 0.0005; 0.0017 \rangle$ for the tuning parameter α in which robust stabilization is ensured. In this interval, stabilization of the whole family of models given by the uncertainty intervals (7) is guaranteed, i.e. the designed controller will stabilize not only the nominal model (8) but all the models represented by the uncertainty intervals of its coefficients (7). Consequently it can be supposed that the controller will stabilize also the nonlinear process model (1) and the real plant.

When checking the *strong stability* condition, i.e. also the controller stability, see section 3.3.1, one has to ensure that the controller coefficient p_0 in (23) is positive (since from (25) $p_1 = 1$). Further computations show that this condition is fulfilled for $\alpha \geq 0.0006$. As a result, a “safe” interval for the tuning parameter is:

$$\alpha \in \langle 0.0006; 0.0017 \rangle, \quad (26)$$

where both, robust stabilization and controller stability is ensured.

4. SIMULATION EXPERIMENTS

Simulation experiments with the nonlinear model of the reactor (1) were performed with the help of the MATLAB/Simulink environment which offers both powerful computations and user-friendly simulation interface.

4.1. Open-loop Responses

Open-loop (without control) step-responses of the reactor are presented in the next figures, Fig. 6-9 (Gazdoš and Macků 2009). They reveal complex dynamics of the process for different changes of the manipulated variable - \dot{m}_{FK} (chromium sludge mass flow) in the admissible interval:

$$\dot{m}_{FK} = [0.05 \ 0.1 \ 0.5 \ 1 \ 3] \ [kg \cdot s^{-1}].$$

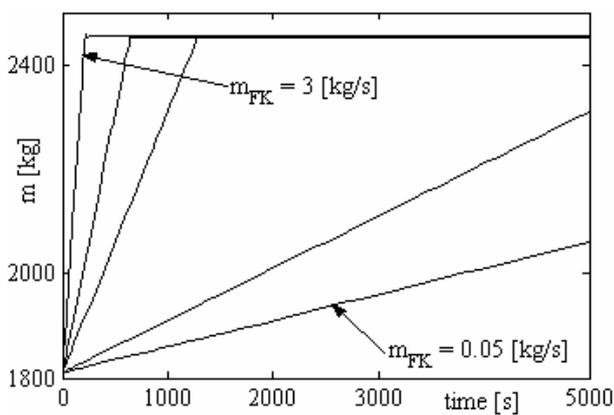


Figure 6: Total Mass Response

The first figure shows increase of the total mass in the reactor for various input flow rates of the chromium sludge. The simulation reveals integrating, astatic behaviour limited to the defined max. capacity of the reactor, see section 2.2 for details.

Next response presented in Fig. 7 reveals derivative behaviour of the chromium sludge mass concentration $a_{FK}(t)$ for various values of \dot{m}_{FK} .

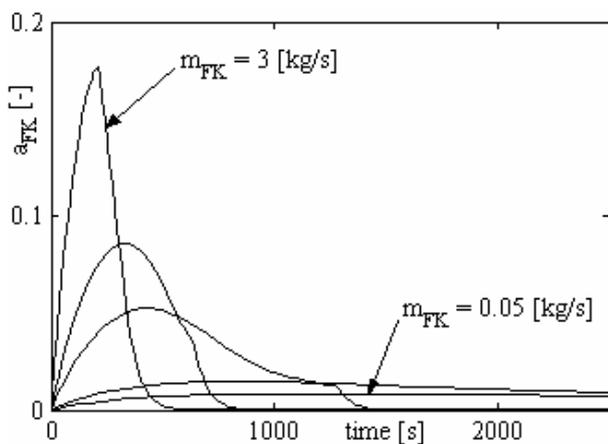


Figure 7: Chromium Sludge Mass Concentration Response

The response of Fig. 8 shows the temperature increase inside the reactor – it can be seen how the temperature rises as a result of the chemical reaction. The faster input flow rate of the chromium sludge, the faster reaction and temperature increase. Then, the next increase is limited by the restriction on the maximum possible mass in the reactor followed by gradual temperature fall. From the graph it is also clear that for the simulated range of \dot{m}_{FK} the temperature goes beyond the allowed limit $T(t) < 100$ [°C] for higher values of \dot{m}_{FK} , therefore the process needs to be controlled properly.

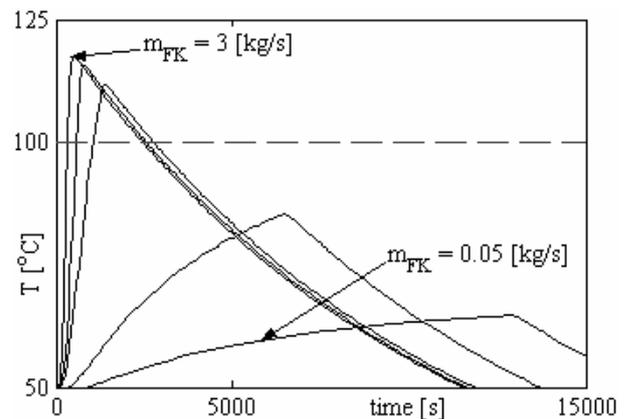


Figure 8: Temperature-in-the-Reactor Response

A record of the coolant temperature is presented in Fig. 9. As can be seen from the graph, the temperature of the media for the whole range of \dot{m}_{FK} is not critical since water is used for the cooling (provided its defined flow rate $\dot{m}_v = 1$ [kg/s] is ensured).

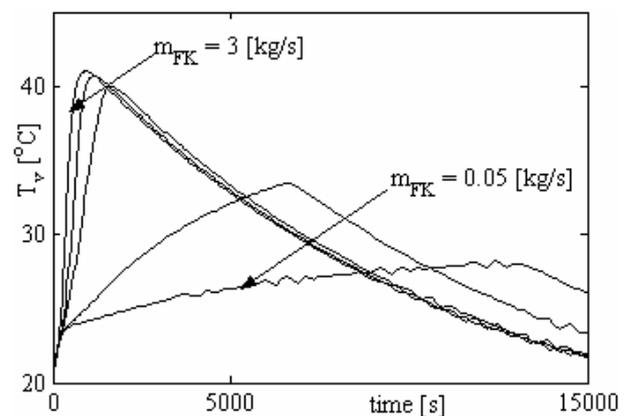


Figure 9: Coolant Temperature Response

4.2. Closed-loop Responses – Simulation of Control

For the simulation of the reactor control, the reference value of the temperature was set as: $w(t) = 98$ [°C] for safety reasons – the temperature in the reactor cannot exceed the limit of 100 [°C] as described in the section 2.2, however higher temperature is desirable in order to process the maximum amount of waste in the shortest possible time-interval.

A resultant controller for a chosen tuning parameter $\alpha = 0.0014$ from the suggested range of robust stability and controller stability (26) has the following form:

$$C(s) = \frac{0.0265s^2 + 7.51 \times 10^{-5}s + 2.8 \times 10^{-8}}{s(s + 0.00356)}, \quad (27)$$

and provides the control response presented in Fig. 10-12.

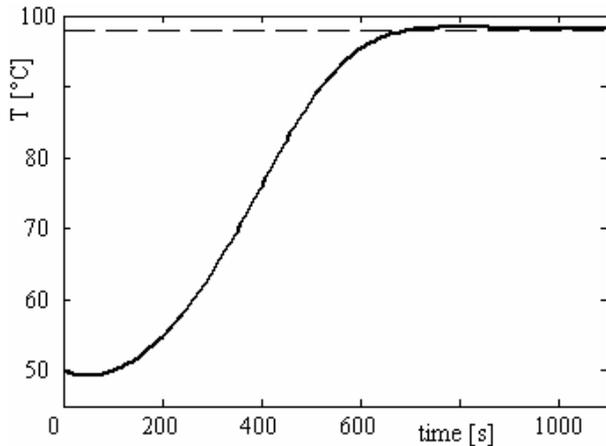


Figure 10: Robust Control Response for $\alpha = 0.0014$

As can be seen from the first graph the control is stable with only a minor overshoot reaching and tracking the desired value relatively quickly. Next figure shows that the control action (manipulated variable \dot{m}_{FK}) is within the allowed limit $0 \div 3 \text{ [kg}\cdot\text{s}^{-1}]$ during the whole process of control. The last graph, Fig. 12 displays increase of the reactor total mass before reaching the maximum allowed capacity $m < 2450 \text{ kg}$. Then the process of feeding by the chromium sludge \dot{m}_{FK} is stopped and after cooling of the reactor it is emptied and the process continues with a next batch.

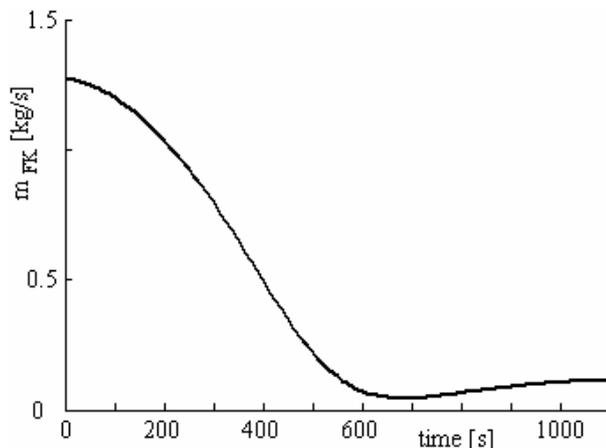


Figure 11: Control Input Response for $\alpha = 0.0014$

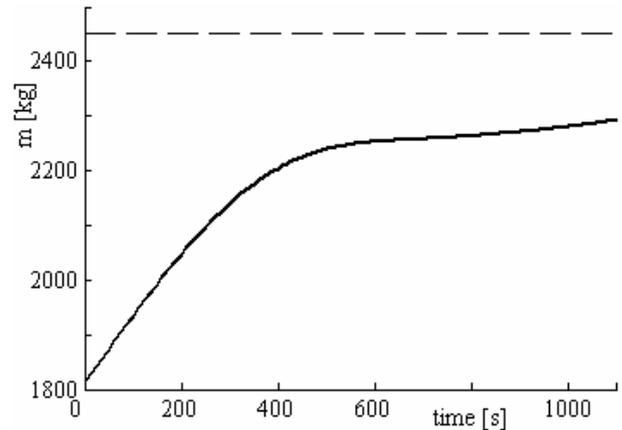


Figure 12: Total Mass Response for $\alpha = 0.0014$

Next two graphs, Fig. 13-14 show the influence of the tuning parameter α on the control process.

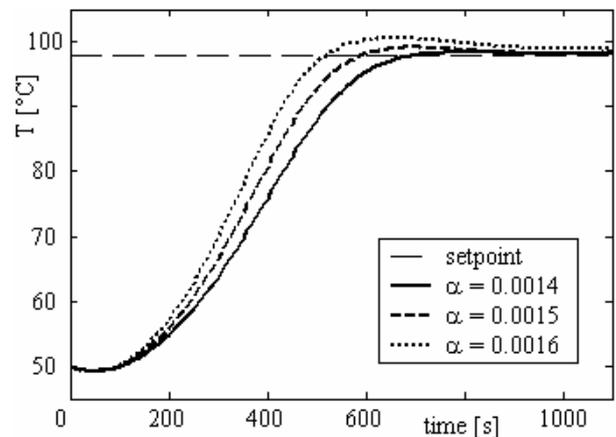


Figure 13: Robust Control Response – Influence of the Tuning Parameter α

It can be seen that its higher values speed-up the control process but result in higher overshoots and wider range of the manipulated variable \dot{m}_{FK} . Therefore, in the real application one has to find a reasonable trade-off between the speed of the control response and particular process limitations.

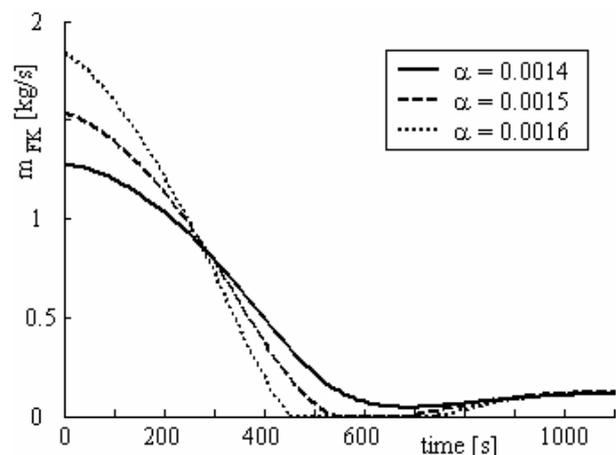


Figure 14: Control Input Response – Influence of the Tuning Parameter α

Last three graphs show non-robust setting of the tuning parameter α – out of the suggested range of the robust and controller stability (26). The first picture reveals poor control response with unacceptable overshoots and tracking. Next figure reveals that limits of the manipulated variable are reached very soon and therefore the control process does not work properly. Last figure shows the total mass increase in the reactor – in the first case ($\alpha = 0.0003$) the reactor is soon filled up without the expected results of proper control. In the second case ($\alpha = 0.0030$) it is not limited so early, however, as a result of control input limitation the control process also does not work properly.

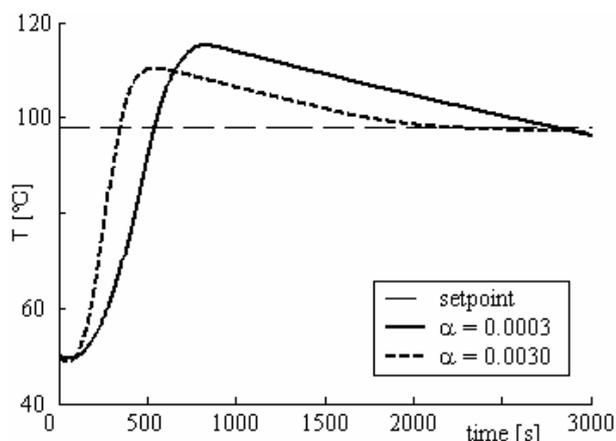


Figure 15: Non-robust Control Response

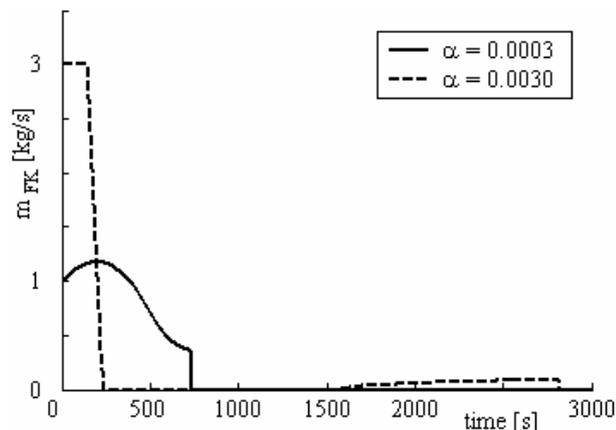


Figure 16: Non-robust Control Input Response

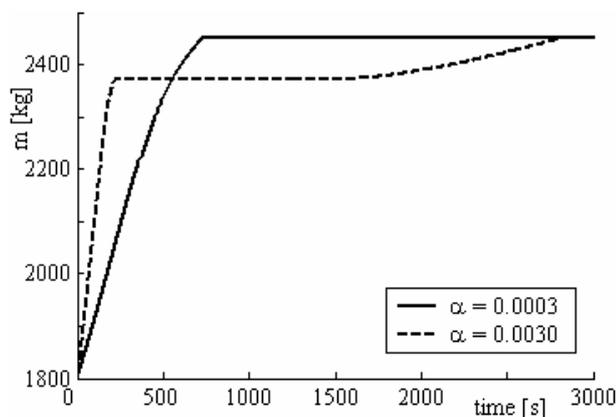


Figure 17: Non-robust Control – Total Mass Response

4.3. Discussion of the Results

Presented results confirm the need for robust control of the reactor when the control system design is based on the linear nominal model only. Another approach is adopted in the works (Sámek and Macků 2008; Novosad and Macků 2011) where predictive and adaptive control were applied to this reactor successfully, however, these control design methodologies are more complex and computationally demanding. The aim of this contribution was to show that this process can be also successfully controlled by a relatively simple (PID) fixed-parameters controller designed in a robust way.

It can be expected that more complex choice of the characteristic polynomial (24) will provide even better responses and more robust control loops, however, for the cost of optimising not only one, but more, up to four poles of the characteristic polynomial. In this limited space it can be just added that two optimised parameters provided more robust control loop, however, the achieved response was very slow and consequently not suitable for the defined limits of this reactor control application.

In this work the classical control set-up of Fig. 3 with one feedback controller was employed. Different control configurations, e.g. with also feedforward part of the controller filtering the reference signal could help to reduce overshoots of the controlled variable and decrease the control action in order to stay in its defined limits.

5. CONCLUSION

This paper presents a simulation study of control system design for a semi-batch reactor used for tanning waste recovery. The complete procedure of control system design is shown in detail, including process modelling, identification, controller design and simulation verification. It is shown that this highly non-linear complex process can be successfully controlled by a relatively simple fixed-parameters controller designed in a robust way. For this purpose the polynomial approach and some useful tools from robust control theory were exploited.

In this work, the process was controlled as a single input – single output (SISO) system only – the temperature in the reactor was controlled by means of the chromium sludge mass flow \dot{m}_{FK} . It would be interesting and practically possible to include also the coolant mass flow \dot{m}_v , as the manipulated variable and treat this system as multi input – multi output (MIMO). It can be supposed that the achieved results could be even better.

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