

MATLAB TOOLBOX FOR CAD OF SELF-TUNING CONTROL OF TIME-DELAY PROCESSES

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ABSTRACT

The proposed Toolbox is dedicated to design of algorithms for self-tuning digital control of processes with time-delay. The algorithms are based on modifications of the Smith Predictor (SP). The first modification of the SP is based on the digital PID controller and the second is based on polynomial approach (pole assignment method). The identification and controller algorithms were created in the MATLAB/SIMULINK environment. The designed algorithms are also suitable for implementation in real time conditions. The verification of the designed Toolbox is demonstrated on a control of laboratory heat exchanger in simulation conditions.

Keywords: time-delay process, self-tuning control, Smith predictor, MATLAB/SIMULINK

1. INTRODUCTION

Time - delays appear in many processes in industry and other fields, including economical and biological systems (Normy-Rico and Camacho 2007). They are caused by some of the following phenomena:

- the time needed to transport mass, energy or information,
- the accumulation of time lags in a great numbers of low order systems connected in series,
- the required processing time for sensors, such as analyzers; controllers that need some time to implement a complicated control algorithms or process.

Consider a continuous-time dynamical linear SISO (single input $u(t)$ – single output $y(t)$) system with time-delay T_d . The transfer function of a pure transportation lag is $e^{-T_d s}$, where s is a complex variable. Overall transfer function with time-delay is in the form

$$G_p(s) = G_m(s) e^{-T_d s} \quad (1)$$

where $G_m(s)$ is the transfer function without time-delay. Processes with significant time-delay are difficult to control using standard feedback controllers. When a

high performance of the control process is desired or the relative time-delay is very large, a predictive control strategy must be used. The predictive control strategy includes a model of the process in the structure of the controller. The first time-delay compensation algorithm was proposed by (Smith 1957). This control algorithm known as the Smith predictor (SP) contained a dynamic model of the time-delay process and it can be considered as the first model predictive algorithm.

Although time-delay compensators appeared in the mid 1950s, their implementation with analog technique was very difficult and these were not used in industry. Since 1980s digital time-delay compensators can be implemented. In spite of the fact that all these algorithms are implemented on digital platforms, most works analyze only the continuous case. The digital time-delay compensators are presented e.g. in (Vogel and Edgar 1980; Palmor and Halevi 1990).

The majority of processes in the industrial practice have stochastic characteristics and eventually they exhibit nonlinear behaviour. Traditional controllers with fixed parameters are often unsuitable for such processes because their parameters change. One possible alternative for improving the quality of control for such processes is the use of adaptive control systems. Different approaches were proposed and utilized. One of the successful approaches is self-tuning control (STC). Two STC modifications of the digital Smith Predictors (STCSP) are designed and implemented into MATLAB/SIMULINK Toolbox ((Bobál, Chalupa, and Novák 2011b).

The paper is organized in the following way. The problem of a control of the time-delay systems is described in Section 1. The principle of the digital Smith Predictor is described in Section 2. Section 3 contains brief description of the recursive identification procedure. Two modifications of digital controllers that are used for self-tuning versions of SPs are proposed in Section 4. The designed Toolbox is briefly described in Section 5. Section 6 contains an example of the simulation control of the laboratory heat exchanger. Section 7 concludes the paper.

2. DIGITAL SMITH PREDICTORS

The discrete versions of the SP and their modifications are suitable for time-delay compensation in industrial

practice. Most of authors designed the digital SP using discrete PID controllers with fixed parameters. However, the SP is more sensitive to process parameter variations and therefore requires an auto-tuning or adaptive approach in many practical applications.

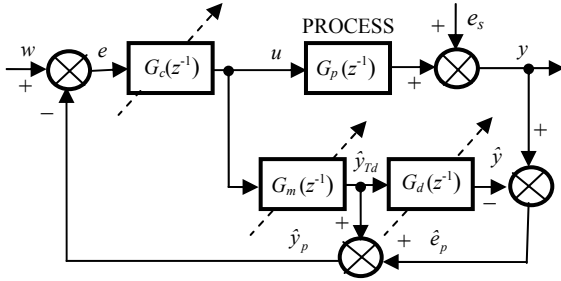


Fig. 1: Block diagram of a digital Smith Predictor with tuning

The block diagram of a digital SP (see Hang, Lim, and Chong 1989; Hang, Tong, and Weng 1993) is shown in Fig. 1. The function of the digital version is similar to the classical analog version. The block $G_m(z^{-1})$ represents process dynamics without the time-delay and is used to compute an open-loop prediction. The difference between the output of the process y and the model including time delay \hat{y} is the predicted error \hat{e}_p as shown in Fig. 1, whereas e and e_s are the error and the noise, respectively and w is the reference signal. If there are no modelling errors or disturbances, the error between the current process output y and the model output \hat{y} will be null. Then the predictor output signal \hat{y}_p will be the time-delay-free output of the process. Under these conditions, the controller $G_c(z^{-1})$ can be tuned, at least in the nominal case, as if the process had no time-delay. The primary (main) controller $G_c(z^{-1})$ can be designed by different approaches (for example digital PID control or methods based on algebraic approach). The outward feedback-loop through the block $G_d(z^{-1})$ in Fig. 1 is used to compensate for load disturbances and modelling errors. The dash arrows indicate the self-tuned parts of the Smith Predictor.

Most industrial processes can be approximated by a reduced order model with a pure time-delay. Consider the following second order linear model with a time-delay

$$G(z^{-1}) = \frac{B(z^{-1})}{A(z^{-1})} z^{-d} = \frac{b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}} z^{-d} \quad (2)$$

for demonstration of some approaches to the design of the adaptive Smith Predictor. The term z^{-d} represents the pure discrete time-delay. The time-delay is equal to dT_0

where T_0 is the sampling period. Model (2) is used in control algorithms of the designed Toolbox.

3. IDENTIFICATION PROCEDURE

3.1. Identification of Time-delay

In this paper, the time-delay is assumed to be known approximately or possible to be obtained separately from an off-line identification using the least squares method

$$\hat{\theta} = (F^T F)^{-1} F^T y \quad (3)$$

where the matrix F has dimension $(N-n-d, 2n)$, the vector y $(N-n-d)$ and the vector of parameter model estimates $\hat{\theta}$ $(2n)$. N is the number of samples of measured input and output data, n is the model order. Equation (4) serves for a one-off calculation of the vector of parameter estimates $\hat{\theta}$ using N samples of measured data. The individual vectors and matrix in equation (3) have the form

$$y^T = [y(n+d+1) \quad y(n+d+2) \quad \cdots \quad y(N)] \quad (4)$$

$$\hat{\theta}^T = [\hat{a}_1 \quad \hat{a}_2 \quad \cdots \quad \hat{a}_n \quad \hat{b}_1 \quad \hat{b}_2 \quad \cdots \quad \hat{b}_n] \quad (5)$$

$$F = \begin{bmatrix} -y(n+d) & -y(n+d-1) & \cdots & -y(d+1) \\ -y(n+d+1) & -y(n+d) & \cdots & -y(d+2) \\ \vdots & \vdots & \cdots & \vdots \\ -y(N-1) & -y(N-2) & \cdots & -y(N-n) \end{bmatrix} \quad (6)$$

$$\begin{bmatrix} u(n) & u(n-1) & \cdots & u(1) \\ u(n+1) & u(n) & \cdots & u(2) \\ \vdots & \vdots & \cdots & \vdots \\ u(N-d-1) & u(N-d-2) & \cdots & u(N-d-n) \end{bmatrix}$$

3.2. Recursive Identification Algorithm

The regression (ARX) model of the following form

$$y(k) = \theta^T(k) \Phi(k) + e_s(k) \quad (7)$$

is used in the identification part of the designed controller algorithms, where

$$\theta^T(k) = [a_1 \quad a_2 \quad b_1 \quad b_2] \quad (8)$$

is the vector of model parameters and

$$\Phi^T(k-1) = [-y(k-1) \quad -y(k-2) \quad u(k-d-1) \quad u(k-d-2)] \quad (9)$$

is the regression vector. The non-measurable random component $e_s(k)$ is assumed to have zero mean value

$E[e_s(k)] = 0$ and constant covariance (dispersion) $R = E[e_s^2(k)]$.

All digital adaptive SP controllers use the algorithm of identification based on the Recursive Least Squares Method (RLSM) extended to include the technique of directional (adaptive) forgetting. Numerical stability is improved by means of the LD decomposition (Kulhavý 1987; (Bobál, Böhm, Fessl, and Macháček 2005)). This method is based on the idea of changing the influence of input-output data pairs to the current estimates. The weights are assigned according to amount of information carried by the data.

When using the self-tuning principle, the model parameter estimates must approach the true values right from the start of the control. This means that as the self-tuning algorithm begins to operate, identification must be run from suitable conditions – the result of the possible *a priori* information. The role of suitable initial conditions in recursive identification is often underestimated.

4. CONTROLLER ALGORITHMS

4.1. Digital PID Smith Predictor

Hang *et al.* (1989, 1993) used the Dahlin PID algorithm (Dahlin 1968) for the design of the main controller $G_c(z^{-1})$. This algorithm is based on the desired close-loop transfer function in the form

$$G_e(z^{-1}) = \frac{1 - e^{-\alpha}}{1 - z^{-1}}; \quad \alpha = \frac{T_0}{T_m} \quad (10)$$

where T_m is a desired time constant of the first order closed-loop response. It is not practical to set T_m to be small since it will demand a large control signal $u(k)$ which may easily exceed the saturation limit of the actuator. Then the individual parts of the controller are described by the transfer functions

$$G_c(z^{-1}) = \frac{(1 - e^{-\alpha}) \hat{A}(z^{-1})}{(1 - z^{-1}) \hat{B}(1)}; \quad G_m(z^{-1}) = \frac{z^{-1} \hat{B}(1)}{\hat{A}(z^{-1})}$$

$$G_d(z^{-1}) = \frac{z^{-d} \hat{B}(z^{-1})}{z^{-1} \hat{B}(1)} \quad (11)$$

where $B(1) = \hat{B}(z^{-1})|_{z=1} = \hat{b}_1 + \hat{b}_2$.

Since $G_m(z^{-1})$ is the second order transfer function, the main controller $G_c(z^{-1})$ becomes a digital PID controller having the following form:

$$G_c(z^{-1}) = \frac{U(z)}{E(z)} = \frac{q_0 + q_1 z^{-1} + q_2 z^{-2}}{1 - z^{-1}} \quad (12)$$

where $q_0 = \gamma$, $q_1 = \hat{a}_1 \gamma$, $q_2 = \hat{a}_2 \gamma$ using by the substitution $\gamma = (1 - e^{-\alpha}) / \hat{B}(1)$. The PID controller output is given by

$$u(k) = q_0 e(k) + q_1 e(k-1) + q_2 e(k-2) + u(k-1) \quad (13)$$

Some simulation experiments using this digital SP are presented in (Bobál, Chalupa, Dostál, and Kubalčík 2011a).

4.2. Digital Pole Assignment Smith Predictor

The digital pole assignment SP was designed using a polynomial approach in (Bobál, Chalupa, Dostál, and Kubalčík 2011a). Polynomial control theory is based on the apparatus and methods of linear algebra (see e.g. Kučera 1993). The design of the controller algorithm is based on the general block scheme of a closed-loop with two degrees of freedom (2DOF) according to Fig. 2.

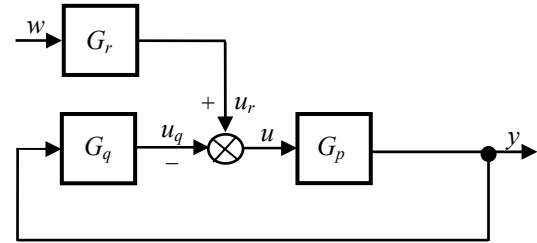


Fig. 2: Block Diagram of a Closed Loop 2DOF Control System

The controlled process is given by the transfer function in the form

$$G_p(z^{-1}) = \frac{Y(z)}{U(z)} = \frac{B(z^{-1})}{A(z^{-1})} \quad (14)$$

where A and B are the second order polynomials. The controller contains the feedback part G_q and the feedforward part G_r . Then the digital controllers can be expressed in the form of discrete transfer functions

$$G_r(z^{-1}) = \frac{R(z^{-1})}{P(z^{-1})} = \frac{r_0}{1 + p_1 z^{-1}} \quad (15)$$

$$G_q(z^{-1}) = \frac{Q(z^{-1})}{P(z^{-1})} = \frac{q_0 + q_1 z^{-1} + q_2 z^{-2}}{(1 + p_1 z^{-1})(1 - z^{-1})} \quad (16)$$

According to the scheme presented in Fig. 3 and Equations (14) – (16) it is possible to derive the characteristic polynomial

$$A(z^{-1})P(z^{-1}) + B(z^{-1})Q(z^{-1}) = D(z^{-1}) \quad (17)$$

where

$$D(z^{-1}) = 1 + d_1 z^{-1} + d_2 z^{-2} + d_3 z^{-3} + d_4 z^{-4} \quad (18)$$

The feedback part of the controller is given by solution of the polynomial Diophantine equation (18). The procedure leading to determination of controller parameters in polynomials Q , R and P (15) and (16) is in (Bobál, Böhm, Fessl, and Macháček 2005). The asymptotic tracking is provided by the feedforward part of the controller given by solution of the polynomial Diophantine equation

$$S(z^{-1})D_w(z^{-1}) + B(z^{-1})R(z^{-1}) = D(z^{-1}) \quad (19)$$

For a step-changing reference signal value $D_w(z^{-1}) = 1 - z^{-1}$ holds and S is an auxiliary polynomial which does not enter into controller design and it is possible to solve Equation (19) by substituting $z = 1$

$$R(z^{-1}) = r_0 = \frac{D(1)}{B(1)} = \frac{1 + d_1 + d_2 + d_3 + d_4}{b_1 + b_2} \quad (20)$$

The 2DOF controller output is given by

$$u(k) = r_0 w(k) - q_0 y(k) - q_1 y(k-1) - q_2 y(k-2) + (1 + p_1)u(k-1) + p_1 u(k-2) \quad (21)$$

The control quality is very dependent on the pole assignment of the characteristic polynomial

$$D(z) = z^4 + d_1 z^3 + d_2 z^2 + d_3 z + d_4 \quad (22)$$

inside the unit circle. The simple method for choice of individual poles is based on the following approach. Consider 1DOF control loop where controlled process (14) with second-order polynomials A and B is controlled using PID controller which is given by transfer function

$$G_q(z^{-1}) = \frac{Q(z^{-1})}{P(z^{-1})} = \frac{q_0(1 + a_1 z^{-1} + a_2 z^{-2})}{(1 - z^{-1})} \quad (23)$$

Substitution of polynomials A , B , Q , P into Equation (17) yields the following relation

$$\begin{aligned} \hat{A}(z^{-1})(1 - z^{-1}) + \hat{B}(z^{-1})q_0 \hat{A}(z^{-1}) = \\ = \hat{A}(z^{-1})[(1 - z^{-1}) + \hat{B}(z^{-1})q_0] = D(z^{-1}) \end{aligned} \quad (24)$$

where

$$\hat{A}(z^{-1}) = 1 + \hat{a}_1 z^{-1} + \hat{a}_2 z^{-2}; \quad \hat{B}(z^{-1}) = \hat{b}_1 z^{-1} + \hat{b}_2 z^{-2} \quad (25)$$

are polynomials with model parameter estimates. From Equation (24) it is obvious that polynomial

$$A(z) = z^2 + a_1 z + a_2 \quad (26)$$

which have two different real poles α, β , is included in polynomial $D(z)$ (22). Its parameter estimates are known from process identification. Two possibilities is possible to solve using the Time-delay Toolbox.

Pole assignment with user-defined multiple pole (PAMP) method:

Polynomial (22) has two different real poles α, β and user-defined multiple pole γ . Then polynomial (22) has the form $D(z) = (z - \alpha)(z - \beta)(z - \gamma)^2$ and it is possible to express its individual parameters as:

$$\begin{aligned} d_1 &= -(2\gamma + \alpha + \beta) \\ d_2 &= 2\gamma(\alpha + \beta) + \alpha\beta + \gamma^2 \\ d_3 &= -(2\alpha\beta\gamma + \gamma^2(\alpha + \beta)) \\ d_4 &= \alpha\beta\gamma^2 \end{aligned} \quad (27)$$

Pole assignment with user-defined different real poles (PADP) method:

Polynomial (22) has two different real poles α, β and user-defined real poles γ, δ . Then polynomial (22) has the form $D(z) = (z - \alpha)(z - \beta)(z - \gamma)(z - \delta)$ and it is possible to express its individual parameters as:

$$\begin{aligned} d_1 &= -(\alpha + \beta + \gamma + \delta) \\ d_2 &= \alpha\beta + \gamma\delta + (\alpha + \beta)(\gamma + \delta) \\ d_3 &= -[(\alpha + \beta)\gamma\delta + (\gamma + \delta)\alpha\beta] \\ d_4 &= \alpha\beta\gamma\delta \end{aligned} \quad (28)$$

5. TOOLBOX FUNCTIONS

The Toolbox (Bobál, Chalupa, and Novák 2011b) contains three main scripts (**start_PAMP.m**, **start_PADP.m** and **start_PID.m**) and other programs (functions, models and scripts) that are called by these main scripts. These scripts perform similar sequence of operations:

- definition of the controlled system (transfer function, time delay), sample time and controller parameters,
- off-line identification of the controlled system,
- pole assignment control or PID control of the system.

Toolbox files are summarized in Table 1. The detailed instructions for use of the Toolbox are introduced in the User's Guide (Bobál, Chalupa, and Novák 2011b).

Table 1: Toolbox Files

File	Description
start_PAMP.m	top-level script for pole assignment control (multiple pole γ)
start_PADP.m	top-level script for pole assignment control (poles γ, δ)
start_PID.m	top-level script for PID control
LSM_2or2td.m	off-line identification
Sm_adapt_pp2i.m	computation of control value in pole assignment control scheme SmP_ad_PA.mdl.
sid.m	on-line identification s-function used by both control schemes (SmP_ad_PA.mdl and SmP_ad_PID.mdl)
Ident_c_LSM.mdl	Simulink scheme used to collect data for off-line identification
SmP_ad_PA.mdl	Simulink control scheme of pole assignment control
SmP_ad_PID.mdl	Simulink control scheme of PID control

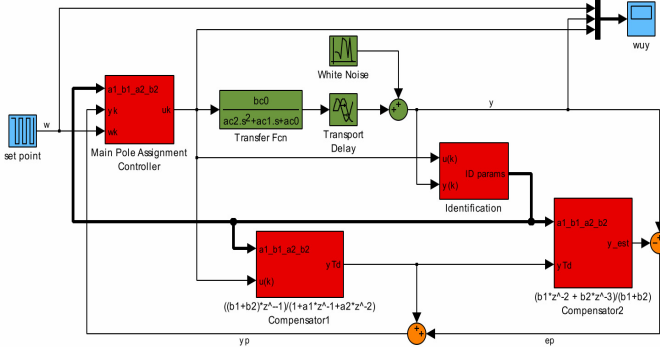


Fig. 3: Simulink control scheme

A typical control scheme used is depicted in Fig. 3. This scheme is used for systems with time-delay of two sample steps. Individual blocks of the SIMULINK scheme correspond to blocks of the general control scheme presented in Fig. 1. The green blocks represent the controlled system. Constants $bc0, ac2, ac1,$ and $ac0$ are parameters of continuous-time system. Blocks Compensator 1 and Compensator 2 are parts of the Smith Predictor and they correspond to $G_m(z^{-1})$ and $G_d(z^{-1})$ blocks of Fig. 2 respectively. The control algorithm is encapsulated in Main Pole Assignment

Controller which corresponds to $G_c(z^{-1})$ Fig. 2 block.

The Identification block performs the on-line identification of controlled system and outputs the estimates of the 2nd order ARX model ($a1, b1, a2, b2$) parameter.

6. SIMULATION RESULTS

The use of the Time-delay Toolbox is demonstrated on a control of laboratory heat exchanger in simulation conditions. The laboratory heat exchanger (see Pekař et al. 2009) is based on the principle of transferring heat from a source through a piping system using a heat transferring media to a heat-consuming appliance. A scheme of the laboratory heat exchanger is depicted in Fig. 4.

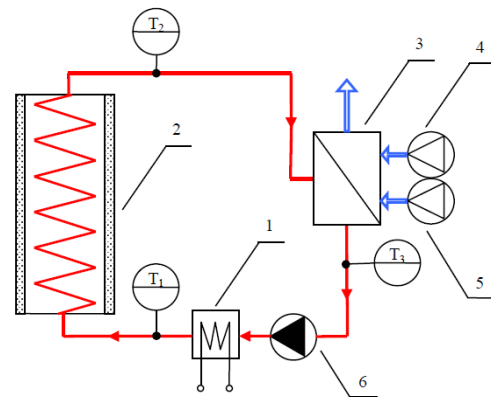


Fig. 4: Scheme of laboratory heat exchanger

The heat transferring fluid (e. g. water) is transported using a continuously controllable DC pump (6) into a flow heater (1) with max. power of 750 W. The temperature of a fluid at the heater output T_1 is measured by a platinum thermometer. Warmed liquid then goes through a 15 meters long insulated coiled pipeline (2) which causes the significant delay in the system. The air-water heat exchanger (3) with two cooling fans (4, 5) represents a heat-consuming appliance. The speed of the first fan can be continuously adjusted, whereas the second one is of on/off type. Input and output temperatures of the cooler are measured again by platinum thermometers as T_2 , resp. T_3 . The laboratory heat exchanger is connected to a standard PC via technological multifunction I/O card. For all monitoring and control functions the MATLAB/SIMULINK environment with Real Time Toolbox.

The dynamic model of the laboratory heat exchanger was obtained from processed input (the power of a flow heater P [W]) and output (the temperature of a T_2 [deg C] of the cooler) data. The input signal $u(k)$ was generated using a Random Gaussian Signal (RGS). The MATLAB code

$$u = \text{idinput}(N, 'rgs', [0 \ B], [Umin, Umax])$$

generates an RGS of the length N , where $[0 \ B]$ determines the frequency passband. $Umin, Umax$ defines the minimum and maximum values of u . The

signal level is such that U_{min} is the mean value of the signal, minus one standard deviation, while U_{max} is the mean value plus one standard deviation. Using the optimization MATLAB function „fminsearch“, the followed discrete transfer function for sampling period $T_0 = 100$ s was identified

$$G(z^{-1}) = \frac{0.1494z^{-1} + 0.028z^{-2}}{1 - 0.6376z^{-1} - 0.1407z^{-2}} z^{-2} \quad (29)$$

The simulation verification of the control model (30) using the PAMP controller is shown in Fig. 5 and the simulation conditions are following:

Sampling period $T_0 = 100$ s, time delay $T_d = 200$ s, white noise with mean value $\mu=0$ and variance $\sigma^2=0.01$ was used as a random signal and characteristic polynomial (22) was set to:

$$D(z) = z^4 - 0.8876z^3 + 0.0337z^2 + 0.0256z - 0.0021$$

with poles $\alpha=0.8111$; $\beta=-0.1735$; $\gamma=0.1$; $\delta=-0.0021$.

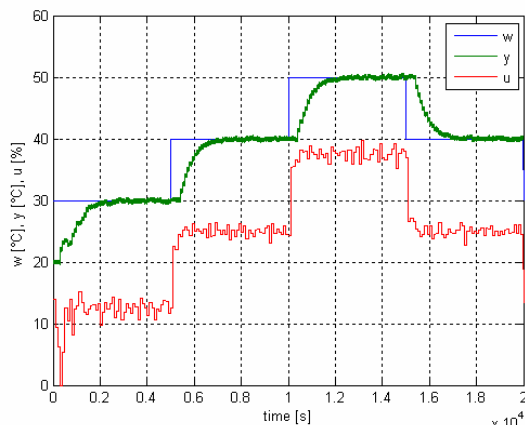


Fig. 5: Simulation Verification of PAMP Controller

From Fig. 5 it is obvious that after start-up phase the control quality is very good, including of intervals, during the transient response.

7. CONCLUSIONS

The Toolbox for CAD and verification of digital adaptive control of time-delay systems was created in the MATLAB/SIMULINK environment. The purpose of this Toolbox is to create an environment suitable for the design and testing of adaptive control of time-delay systems. The Toolbox consists of some .m and .mdl files for identification and adaptive control blocks. The algorithms are based on some modifications of the Smith Predictor using PID and pole assignment control strategy. The controllers were derived intentionally by an analytical way (without utilization of numerical methods) to obtain algorithm with easy implementability in industrial practice. The Toolbox can be applied for off-line identification of time-delay processes and for design of adaptive controllers. Individual controllers were successfully verified not only by simulation but also in real-time laboratory conditions for control of the heat exchanger. This Toolbox is available free of charge from

the Tomas Bata University Zlín Internet site (Bobál, Chalupa, and Novák 2011b).

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