ENVIRONMENTAL PERFORMANCE OF GLOBAL SUPPLY CHAINS OBSERVED THROUGH EXTENDED MATERIAL REQUIREMENTS PLANNING SIMULATION MODEL

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ABSTRACT
Every economy in the globalized world consists of a network of different supply chains. Supply chain is a complex system of various activity cells. Some of them correspond to production and others to distribution and consumption activities. The loop is closed when consumption and production subsystems are connected – used items enter a new production cycle as recycled/reused inputs. Outputs of such system are waste products and pollution which is a result of any of the activities inside the supply chain. Extended Material Requirements Planning (EMRP) Theory provides a powerful tool capable of describing all of the relations in such complex systems. Theory uses input-output analysis for characterization of the structures; Laplace transforms for proper incorporation of lead times; and Net Present Value (NPV) calculation for proper evaluation of supply chain's economic performance. This paper presents an analytical tool for simulations, which are essential to a deeper understanding of global supply chains, with emphasis on the environmental component.

Keywords: Extended Material Requirements Planning (EMRP) Theory, environmental factors, energy, input-output analysis.

1. INTRODUCTION

Material Requirements Planning (MRP) Theory rapidly developed during the last two decades of the 20th century as a scientific answer to practical success of MRP in multi-stage, multi-level production systems (Orlicky 1975). The theory is inspired by many famous scientific approaches, such as input-output analysis (Leontief 1970), activity analysis (Koopmans 1951), scientific programming (Vazsonyi 1958) and Laplace transforms. It proves that using a set of input-output matrices **H** and **G** is the most convenient method for mathematical description of products' complex structures. In practice, structures are usually presented schematically in the Bill of Materials (BOM). Lead times are assigned to these structures using Laplace transforms. This combination gives us a sufficient basis for calculation of Net Present Value (NPV), and thus allows for economic analysis of complex production/inventory systems. A good overview of the evolution of MRP Theory can be found in Grubbström and Tang (2000). The first practical application of Theory's scientific approach to the MRP production systems was successfully done in the case of Paper mill company (Grubbström 1990).

MRP Theory has been recently recognized as an extremely useful method for in-depth studies of not only production systems but also complete supply chains. The so-called Extended Material Requirements Planning (EMRP) Theory extends the model to distribution and consumption activities and to reverse logistics at the end of the chain. Integration of additional subsystems forms a closed loop where some elements are circulating inside the grid, and the rest are lost (usually as waste or any kind of pollution). These lost elements have to be replaced with additional purchases if we want to complete the next production cycle and maintain the system closed.

Production is usually treated as an assembly system where components on a particular level are assembled using elements from a previous assembly level, according to the BOM. On the other hand, products are distributed from one location to several distribution centres and end consumers. This type of system can be presented using reverse BOM and is called arborescent system. Bogataj, Grubbström, and Bogataj (2011) introduce lead times as an integral part of a system at any activity cell, belonging either to assembly or arborescent subsystem. A generalized output matrix is introduced to cover arborescent activities, such as distribution or disassembly/recycling (Bogataj and Grubbström 2012).

Such complete examination of supply chains allows us to integrate different environmental factors as essential parts of the system. Long-term general welfare of the human race cannot be measured only in terms of economic benefits of production and consumption activities. It is also significantly affected by limited natural resources, sources of energy and pollution caused by production, transportation and consumption activities. Bogataj and Grubbström (2013) model the cyclical system consisting of 4 subsystems: production, distribution, consumption and recycling. Cyclical flow of elements in the system between any pair of system's activity cells is captured using input-output matrices **H** and **G**. Special attention is given to the recycling subsystem, where:

- Consumption subsystem appears as an output of used final products. Final products at the end of their lifecycle are therefore inputs into recycling subsystem.
- Reusable outputs of the recycling subsystem are successfully recycled items. They are distributed to production activity cells at the start of the next production cycle.
- Unusable outputs of the recycling subsystem are waste elements. They have to be disposed of at landfills. In order to start the next production cycle, these missing lots of elements have to be replaced with additional purchases on the market.
- Lead times appear inside every activity cell (i.e. production or recycling lead times) or between a pair of activity cells (transportation lead times).
- NPV of such a system can be calculated. This allows us to observe the effect of different parameters on overall economic performance of the system, such as environmental taxes, transportation distances between the nodes, etc.

Such a cyclical system is illustrated in Figure 1. Detailed structure of input-output matrices **H** and **G** and relations between the nodes have been analysed by Kovačić and Bogataj (2011). They show that every activity cell of the system can serve as an input into the recycling subsystem and that every element of the system can be traced over several cycles. Further, Kovačić and Bogataj (2013) show how optimal location of recycling activity cell can be determined using EMRP Theory. Among other environmental factors, they point out availability and prices of energy, which can, in some cases, also act as an usable output of the system.

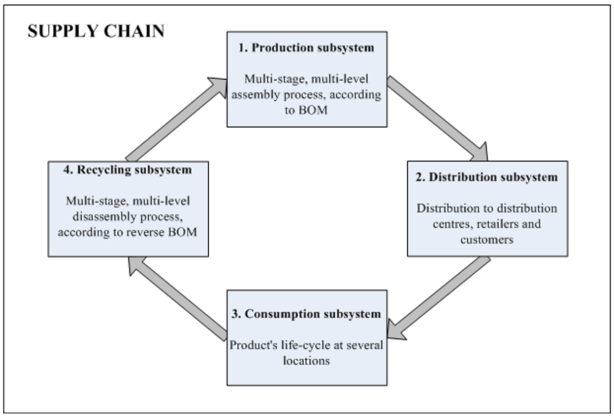

Figure 1: Supply Chain consisting of 4 Subsystems

An environmentally inappropriate structure of a supply chain can lead to a long-term negative overall NPV of the system. Investments into infrastructure which reduces pollution and/or energy consumption or, on the other hand, decreases the volume of scrap and waste, can radically change the overall performance of a supply chain. The integrity, complexity and richness of information in the EMRP Theory make the model an extremely powerful tool for various simulations (Kovačić and Bogataj 2012). The idea of simulating the performance of a supply chain using EMRP Theory stems from Köchel's works on simulating production and inventory systems (Köchel 2009). Simulations based on EMRP Theory can deepen our understanding of supply chains. We can simulate macroeconomic factors and try to find optimal decisions for global, regional or local ecological balance: i.e. optimal environmental taxation can be determined for a specific geographical area. On the other hand, simulations can also be a powerful and useful tool to assist managers in the decision making process. A practical example of usability in the industry can be found in Kovačić, Hontoria, Bogataj, and Ros (2012), where authors use EMRP Theory for economic-ecologic optimization of production processes in a Spanish baby food company. Using simulations, they show how investments into optimisation of the production process, which result in lower volumes of scrap materials, affect long-term NPV. The result is a set of possible solutions, not all of which are technologically feasible. Optimal investment decisions should be made according to feasible solutions found in the set of possible solutions. The solution with the highest long-term NPV is the optimal one.

The purpose of this paper is to present some of the possible simulations of complete supply chains. We will show that EMRP Theory provides an excellent basis for such in-depth research and also decision making. The main advantage of EMRP Theory approach is that research is not isolated only to a part of the problem, but takes into consideration the entire supply chain with all activity cells. This enables us to observe the entire system when any of the input parameters is changing.

2. METHODOLOGY

The structure of global supply chains can be roughly described through 4 main subsystems: 1 - Production, 2 - Distribution, 3 - Consumption and 4 - Recycling, according to Figure 1. The input requirements of the system ($\mathbf{x}$) can then be written as:

$$\mathbf{x} = \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \mathbf{x}_3 \\ \mathbf{x}_4 \end{bmatrix} = \begin{bmatrix} \mathbf{H}_{11}\mathbf{H}_{12}\mathbf{H}_{13}\mathbf{H}_{14} \\ \mathbf{H}_{21}\mathbf{H}_{22}\mathbf{H}_{23}\mathbf{H}_{24} \\ \mathbf{H}_{31}\mathbf{H}_{32}\mathbf{H}_{33}\mathbf{H}_{34} \\ \mathbf{H}_{41}\mathbf{H}_{42}\mathbf{H}_{43}\mathbf{H}_{44} \end{bmatrix} \begin{bmatrix} \mathbf{P}_1 \\ \mathbf{P}_2 \\ \mathbf{P}_3 \\ \mathbf{P}_4 \end{bmatrix} = \mathbf{HP} \qquad (1)$$

In the above notation, input matrix **H** consists of several sub-matrices $\mathbf{H}_{ij}$, each of them representing item requirements of subsystem *i* from running processes in subsystem *j*. Each sub-matrix $\mathbf{H}_{ij}$ consists of several technological coefficients h_{kl}^{ij}, describing input requirements for each activity cell in the system. Detailed analysis of the sub-matrices' complex structures can be found in Kovačić and Bogataj (2011). Further, **P** is activity vector. Its sub-vectors $\mathbf{P}_1$, $\mathbf{P}_2$, $\mathbf{P}_3$ and $\mathbf{P}_4$ belong to each of the 4 subsystems and represent constants: i.e. they describe the total volumes of elements at each activity level of the system during each cycle. Similar, $\mathbf{x}_i$ can be interpreted as input requirements of subsystem *i*, where *i* = 1, 2, 3, 4. When the *l*-th process is run on activity level P_l^j inside subsystem *j*, the amount of required inputs (x_k^i) of item *k* is $h_{kl}^{ij} P_l^j$.

We can present outputs of the system (**y**) in a similar manner:

$$\mathbf{y} = \begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \\ \mathbf{y}_3 \\ \mathbf{y}_4 \end{bmatrix} = \begin{bmatrix} \mathbf{G}_{11}\mathbf{G}_{12}\mathbf{G}_{13}\mathbf{G}_{14} \\ \mathbf{G}_{21}\mathbf{G}_{22}\mathbf{G}_{23}\mathbf{G}_{24} \\ \mathbf{G}_{31}\mathbf{G}_{32}\mathbf{G}_{33}\mathbf{G}_{34} \\ \mathbf{G}_{41}\mathbf{G}_{42}\mathbf{G}_{43}\mathbf{G}_{44} \end{bmatrix} \begin{bmatrix} \mathbf{P}_1 \\ \mathbf{P}_2 \\ \mathbf{P}_3 \\ \mathbf{P}_4 \end{bmatrix} = \mathbf{GP} \qquad (2)$$

Output matrix **G** coincides in dimension with input matrix **H**, and its sub-matrices $\mathbf{G}_{ij}$ represent outputs of subsystem *j* because of activities in subsystem *i*. The amount of produced outputs (y_l^i) of item *k* is $g_{kl}^{ij} P_l^j$.

Overall net production (**y**) of such system is then:

$$\mathbf{z} = \begin{bmatrix} \mathbf{z}_1 \\ \mathbf{z}_2 \\ \mathbf{z}_3 \\ \mathbf{z}_4 \end{bmatrix} = \mathbf{y} - \mathbf{x} = \begin{bmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \\ \mathbf{y}_3 \\ \mathbf{y}_4 \end{bmatrix} - \begin{bmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \\ \mathbf{x}_3 \\ \mathbf{x}_4 \end{bmatrix} = (\mathbf{G} - \mathbf{H})\mathbf{P} \qquad (3)$$

where $\mathbf{z}_i$ is net production of subsystem *i* = 1, 2, 3, 4.

To facilitate understanding, the presented model can be illustrated as numerical example. Figure 2 shows a closed cyclical system where the final product A is assembled according to the BOM: 2 units of elements B and E are used to assemble 1 unit of A; 4 units of element C and 2 units of element E are used to assemble 1 unit of B; 2 units of element D are used to assemble 1 unit of C. Elements D and E can be interpreted as raw materials. They can be extracted during recycling activities and reused in the next production cycle or bought on the market. We assume that 20% of lots of any produced component (B, C) or final product (A) never reach higher assembly or distribution levels. These are scraps from production, components or products of insufficient quality, damaged components or products, lost production, etc. These elements all enter the recycling subsystem from where they can be either recycled or treated as. Remaining 80% of produced final amount of product A enters distribution subsystem: we assume that 75% is distributed to location 1 and another 25% to location 2. Again, 20% is lost during the distribution and enters the recycling subsystem. 80% of distributed products reach end customers. From the consumption subsystem, all used products are sent to the recycling subsystem at the end of their lifecycle. Recycling is a disassembly process where components and final products are disassembled according to reverse BOM. In this numerical example, we suppose that all elements are totally disassembled, therefore outputs of recycling are primary raw materials which can be reused in the next production cycle. The missing amount of raw materials has to be replaced with additional purchases on the market. We assume that 14.2% of all elements E, which are used directly in the assembly of the final product A, are successfully recycled. Further, 19.6% of element E, which is used in assembly of component B, is successfully recycled. We can also reuse 55.6% of element D.

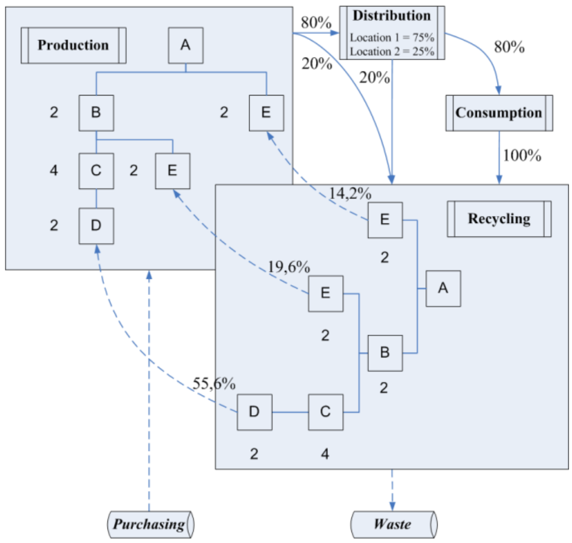

Figure 2: Numerical Example of Production, Distribution, Consumption and Recycling of Product A

The system shown in Figure 2 can now conveniently be written in mathematical form of input and output matrices **H** and **G**:

$$\mathbf{H} = \left[\begin{array}{ccc|c|c|ccc} & & & 0.8 & & & & \\ & & & & & 0.2 & & \\ 2 & & & & & & & \\ & & & & & & 0.2 & \\ & 4 & & & & & & \\ & & & & & & & 0.2 \\ & & 2 & & & & & \\ 2 & 2 & & & & & & \\ \hline & & & & 0.8*0.8*0.75 & & & \\ & & & & 0.8*0.8*0.25 & & & \\ & & & & & 0.8*0.2 & & \\ \hline & & & & & 0.8*0.8 & & \\ \hline & & & & & & & \end{array}\right] \qquad (4)$$

$$\mathbf{G} = \left[\begin{array}{ccc|c|c|ccc}
0.8 & & & & & & & \\
0.2 & & & & & & & \\
 & 0.8 & & & & & & \\
 & 0.2 & & & & & & \\
 & & 0.8 & & & & & \\
 & & 0.2 & & & & & \\
 & & & & & & & 2*0.142 \\
 & & & & & 2*0.556 & 2*0.196 & \\
\hline
 & & & 0.8*0.8*0.75 & & & & \\
 & & & 0.8*0.8*0.25 & & & & \\
 & & & 0.8*0.2 & & & & \\
\hline
 & & & & 0.8*0.8 & & & \\
\hline
 & & & & & & & 2*0.858) \\
 & & & & & 2*0.444 & 2*0.804 &
\end{array}\right] \tag{5}$$

In each production cycle, 1000 units of the final product A are assembled, which requires 2500 units of component B and 12500 units of component C. Activity vector **P** can therefore be written as:

$$\mathbf{P} = \left[\begin{array}{c}
1000 \\
2500 \\
12500 \\
\hline
1000 \\
\hline
1000 \\
\hline
1000 \\
2500 \\
12500
\end{array}\right] \tag{6}$$

Another main pillar of the EMRP Theory is lead times. Proper incorporation of lead times into the model can be done with transformations of time functions into Laplace transforms in the frequency domain. Time function *f*(*t*) can only exist in non-negative time, and its Laplace transform is defined as:

$$\tilde{f}(s) = £\left\{ f(t) \right\} = \int_{t=0}^{\infty} f(t) e^{-st} dt \tag{7}$$

where s is the complex frequency. Function is transformed back to time domain with inverse Laplace transformation:

$$f(t) = £^{-1}\left\{ \tilde{f}(s) \right\} \tag{8}$$

In MRP as well as EMRP Theory we usually use the following theorems in main analyses:

- Time integration theorem:

$$£\left\{ \int_0^t f(\beta) d\beta \right\} = £\left\{ f(t) \right\} / s \tag{9}$$

- Time average theorem:

$$\lim_{t \to \infty} \frac{1}{t} \int_0^t x(\tau) d\tau = \lim_{s \to 0} s\tilde{x}(s) \tag{10}$$

- Net present value theorem, where complex frequency s is replaced with continuous interest rate ρ:

$$\text{NPV} = \int_0^{\infty} a(t) e^{-\rho t} dt = \tilde{a}(\rho) \tag{11}$$

- Time translation theorem, where function is translated uniformly forward in time:

$$£\left\{ f(t-\tau) \right\} = £\left\{ f(t) \right\} e^{-s\tau} = \tilde{f}(s) e^{-s\tau} \tag{12}$$

- Time translation theorem, where function is translated uniformly backward in time:

$$£\left\{ f(t+\tau) \right\} = £\left\{ f(t) \right\} e^{s\tau} = \tilde{f}(s) e^{s\tau} \tag{13}$$

We denote τ_j as a lead time before the start of production *t*, when $h_{ij} P_j(t)$ units of elements *i* are needed for production of a batch $P_j(t)$ of elements *j*. Production lead time matrix can be written as:

$$\tilde{\boldsymbol{\tau}}(s) = \begin{bmatrix} e^{s\tau_1} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & e^{s\tau_m} \end{bmatrix} \tag{14}$$

and the generalized input matrix consisting of both product structures and production lead times as:

$$\tilde{\mathbf{H}}(s) = \mathbf{H}\tilde{\boldsymbol{\tau}}(s) \tag{15}$$

In global supply chains, not only production but also other types of lead times make a significant contribution to the overall performance of a system. Especially noteworthy are transportation lead times which can differ between any pair of neighbouring activity cells. In this case, further generalisation of the input matrix is required:

$$\breve{\mathbf{H}}(s) = \tilde{\mathbf{H}}^{tr}(s) \tilde{\boldsymbol{\tau}}^{pr}(s) \tag{16}$$

Individual lead times can be assigned to each technical coefficient h_{ij} inside $\tilde{\mathbf{H}}^{tr}$: $h_{ij} e^{s(\tau_{ij})^{tr}}$. Similar assumptions can be made concerning generalized output matrix $\breve{\mathbf{G}}(s)$.

Generalized input and output matrices thus consist of all physical structures and transitions together with all lead times in the system, which appear at any of its stages (production, distribution and recycling lead times, lead times in energy distribution, lead times of waste decomposition, etc.):

$$\breve{\mathbf{H}}(s)=\begin{bmatrix}\breve{\mathbf{H}}_{11}(s) & \breve{\mathbf{H}}_{12}(s) & 0 & \breve{\mathbf{H}}_{14}(s)\\ 0 & 0 & \breve{\mathbf{H}}_{23}(s) & \breve{\mathbf{H}}_{24}(s)\\ 0 & 0 & 0 & \breve{\mathbf{H}}_{34}(s)\\ 0 & 0 & 0 & 0\end{bmatrix} \quad (17)$$

$$\breve{\mathbf{G}}(s)=\begin{bmatrix}\breve{\mathbf{G}}_{11}(s) & 0 & 0 & \breve{\mathbf{G}}_{14}(s)\\ 0 & \breve{\mathbf{G}}_{22}(s) & 0 & 0\\ 0 & 0 & \breve{\mathbf{G}}_{33}(s) & 0\\ 0 & 0 & 0 & \breve{\mathbf{G}}_{44}(s)\end{bmatrix} \quad (18)$$

Note that not all of the flows inside and between subsystems are physically feasible, so many of sub-matrices $\mathbf{H}_{ij}$ and $\mathbf{G}_{ij}$, as denoted in (1) and (2), are zero in real life.

Input and output matrices (4) and (5) from the numerical example can now be extended with lead times:

$$\breve{\mathbf{H}}(\rho)=\tilde{\mathbf{H}}^{tr}(\rho)\tilde{\boldsymbol{\tau}}^{pr}(\rho)=$$

$$\left[\begin{array}{ccc|c|c|ccc}
 & & & 0.8e^{2\rho} & & & & \\
 & & & & & 0.2e^{5\rho} & & \\
2e^{(3+3)\rho} & & & & & & & \\
 & & & & & & 0.2e^{3\rho} & \\
 & 4e^{(1+2)\rho} & & & & & & \\
 & & & & & & & 0.2e^{4\rho}\\
 & & 2e^{(5+1)\rho} & & & & & \\
2e^{(1+3)\rho} & 2e^{(3+2)\rho} & & & & & & \\ \hline
 & & & & 0.48e^{3\rho} & & & \\
 & & & & 0.16e^{5\rho} & & & \\
 & & & & & 0.16e^{6\rho} & & \\ \hline
 & & & & & 0.64e^{7\rho} & & \\ \hline
 & & & & & & &
\end{array}\right] \quad (19)$$

$$\breve{\mathbf{G}}(\rho)=\tilde{\boldsymbol{\Delta}}^{pr}(\rho)\tilde{\mathbf{G}}^{tr}(\rho)=$$

$$\left[\begin{array}{ccc|c|c|ccc}
0.8e^{-5\rho} & & & & & & & \\
0.2e^{-4\rho} & & & & & & & \\
 & 0.8e^{-6\rho} & & & & & & \\
 & 0.2e^{-3\rho} & & & & & & \\
 & & 0.8e^{-6\rho} & & & & & \\
 & & 0.2e^{-4\rho} & & & & & \\
 & & & & & & & 0.284e^{(-5-3)\rho}\\
 & & & & & 1.112e^{(-4-2)\rho} & 0.392e^{(-4-3)\rho} & \\ \hline
 & & & 0.48e^{-2\rho} & & & & \\
 & & & 0.16e^{-2\rho} & & & & \\
 & & & 0.16e^{-2\rho} & & & & \\ \hline
 & & & & 0.64e^{-5\rho} & & & \\ \hline
 & & & & & & & 1.716e^{(-4-8)\rho}\\
 & & & & & 0.888e^{(-3-7)\rho} & 1.608e^{(-3-6)\rho} &
\end{array}\right] \quad (20)$$

For example, coefficient $1.112e^{(-4-2)\rho}$, which appears inside sub-matrix $\breve{\mathbf{G}}_{14}(\rho)$, describes successfully recycled element D. During the recycling process, two types of lead times are present: 4 time units are needed for disassembly process and another 2 time units for distribution back to the production subsystem. On the other hand, only 3 time units are needed when disassembly process of element D is not successful, and, due to the considerable distance to a landfill, another 7 time units are needed for distribution (coefficient $0.888e^{(-3-7)\rho}$ inside sub-matrix $\breve{\mathbf{G}}_{44}(\rho)$). Similar assumptions can be made for all other activity cells inside the input-output matrices, but most of them contain only distribution lead times.

Overall NPV of the supply chain can be calculated using equation:

$$\begin{aligned}\mathrm{NPV}=\;&\mathbf{p}\left(\breve{\mathbf{G}}(\rho)-\breve{\mathbf{H}}(\rho)\right)\tilde{\mathbf{P}}(\rho)-\hat{\mathbf{K}}\tilde{\mathbf{v}}(\rho)-\\ &-\mathbf{U}^{\mathrm{T}}\left(\tilde{\boldsymbol{\Pi}}_{\mathrm{G}}(\rho)+\tilde{\boldsymbol{\Pi}}_{\mathrm{H}}(\rho)\right)\tilde{\mathbf{P}}(\rho)-\\ &-\mathbf{c}_{\mathrm{L}}\tilde{\mathbf{L}}(\rho)-\mathbf{U}^{\mathrm{T}}\left(\breve{\mathbf{E}}_{H}(\rho)-\breve{\mathbf{E}}_{G}(\rho)\right)\tilde{\mathbf{P}}(\rho)\end{aligned} \quad (21)$$

Calculation consists of all revenues from produced elements $\mathbf{p}\left(\breve{\mathbf{G}}(\rho)-\breve{\mathbf{H}}(\rho)\right)\tilde{\mathbf{P}}(\rho)$; setup costs for each production cycle $\hat{\mathbf{K}}\tilde{\mathbf{v}}(\rho)$; transportation costs $\mathbf{U}^{\mathrm{T}}\left(\tilde{\boldsymbol{\Pi}}_{\mathrm{G}}(\rho)+\tilde{\boldsymbol{\Pi}}_{\mathrm{H}}(\rho)\right)\tilde{\mathbf{P}}(\rho)$; labour cost $\mathbf{c}_{\mathrm{L}}\tilde{\mathbf{L}}(\rho)$; and energy costs $\mathbf{U}^{\mathrm{T}}\left(\breve{\mathbf{E}}_{H}(\rho)-\breve{\mathbf{E}}_{G}(\rho)\right)\tilde{\mathbf{P}}(\rho)$, where:

- $\mathbf{p}$ is price vector. p_j^i is the price of the element at stage j of subsystem i. p_1^3 is the average price of used item at the end of its life cycle and can be either positive or negative. p_j^4 can be interpreted as environmental taxes for unrecycled elements, which have to be disposed of (negative price).

$$\mathbf{p}=\left[\,p_1^1\,p_2^1\cdots p_m^1\,\middle|\,p_1^2\,p_2^2\cdots p_{(r+1)}^2\,\middle|\,p_1^3\,\middle|\,p_1^4\,p_2^4\cdots p_{(m-2n)}^4\right] \quad (22)$$

- $\tilde{\mathbf{v}}(\rho)$ are given timings, calculated from initial times $\tilde{\mathbf{t}}(\rho)$ and cycle lengths $\tilde{\mathbf{T}}(\rho)$:

$$\tilde{\mathbf{v}}(\rho)=\tilde{\mathbf{t}}(\rho)\tilde{\mathbf{T}}(\rho) \quad (23)$$

- $\tilde{\mathbf{P}}(\rho)$ is activity vector with given timings (cycles):

$$\tilde{\mathbf{P}}(\rho)=\tilde{\mathbf{v}}(\rho)\mathbf{P} \quad (24)$$

- $\hat{\mathbf{K}}$ is a vector of setup costs.
- $\mathbf{U}^{\mathrm{T}}$ is a unit vector.
- $\tilde{\boldsymbol{\Pi}}_{\mathrm{H}}(\rho)$ and $\tilde{\boldsymbol{\Pi}}_{\mathrm{G}}(\rho)$ are matrices consisting of transportation costs for each activity cell. They coincide in dimension with input and output matrices $\mathbf{H}$ and $\mathbf{G}$.

- $\mathbf{c}_{\mathrm{L}}$ is a vector of manpower costs per unit for each activity cell.
- $\tilde{\mathbf{L}}(\rho) = \hat{\mathbf{L}}\tilde{\mathbf{v}}(\rho)$ where $\hat{\mathbf{L}}$ is a vector of needed workforce for each activity cell.
- $\breve{\mathbf{E}}_H(\rho)$ and $\breve{\mathbf{E}}_G(\rho)$ are generalized energy input and output matrices, respectively. They coincide in dimension with input-output matrices **H** and **G**, and consist of both technical coefficients and lead times. Note that the model allows energy to appear as an output of recycling activities, which is extremely important from the environmental point of view and for long-term sustainability of supply chains.

Using above notation, initial values for numerical example are:

$$\mathbf{p} = [2500 \;\; 800 \;\; 150 \;\; 100 \;\; 30 \;\; 20 \;\; 10 \;\; 5 \,|\, 2700 \;\; 2600 \;\; 250 \,|\, 20 \,|\, -5 \;\; -4] \tag{25}$$

$$\hat{\mathbf{K}} = [5600 \;\; 3000 \;\; 2500 \,|\, 1100 \,|\, 1150 \,|\, 800 \;\; 6700 \;\; 5800] \tag{26}$$

$$\tilde{\mathbf{v}}(\rho) = [6956 \;\; 4034 \;\; 2397 \,|\, 3369 \,|\, 2535 \,|\, 5310 \;\; 3940 \;\; 1316]^T \tag{27}$$

$$\mathbf{c}_{\mathrm{L}} = [0{,}03 \;\; 0{,}04 \;\; 0{,}05 \;\; 0{,}07 \;\; 0{,}08 \;\; 0{,}01 \;\; 0{,}02 \;\; 0{,}01] \tag{28}$$

$$\tilde{\mathbf{\Pi}}_H(\rho) == \left[\begin{array}{ccc|c|c|ccc}
 & & & 16.000 & & & & \\
 & & & & & 12.000 & & \\
26.260 & & & & & & & \\
 & & & & & & 4.200 & \\
 & 0.000 & & & & & & \\
 & & & & & & & 8.000 \\
 & & 10.305 & & & & & \\
6.565 & 12.742 & & & & & & \\
\hline
 & & & & 0.000 & & & \\
 & & & & 0.000 & & & \\
 & & & & & 0.000 & & \\
\hline
 & & & & & 0.000 & & \\
\hline
 & & & & & & & \\
\end{array}\right] \tag{29}$$

$$\tilde{\mathbf{\Pi}}_G(\rho) = \left[\begin{array}{ccc|c|c|ccc}
0.000 & & & & & & & \\
0.000 & & & & & & & \\
 & 0.000 & & & & & & \\
 & 0.000 & & & & & & \\
 & & 0.000 & & & & & \\
 & & 0.000 & & & & & \\
 & & & & & & & 8.800 \\
 & & & & & 3.945 & 2.086 & \\
\hline
 & & & 7.680 & & & & \\
 & & & 2.240 & & & & \\
 & & & 1.280 & & & & \\
\hline
 & & & & 19.200 & & & \\
\hline
 & & & & & & & 24.351 \\
 & & & & & 28.405 & 44.088 & \\
\end{array}\right] \tag{30}$$

$$\breve{\mathbf{E}}^H(\rho) = \left[\begin{array}{ccc|c|c|ccc}
 & & & & & 3.7*0.2e^{5\rho} & & \\
1.2*2e^{(0+3)\rho} & & & & & & & \\
 & & & & & & 2.9*0.2e^{3\rho} & \\
 & 1.3*4e^{2\rho} & & & & & & \\
 & & & & & & & 2.7*0.2e^{4\rho} \\
 & & 1.1*2e^{1\rho} & & & & & \\
1.4*2e^{3\rho} & 1.7*2e^{2\rho} & & & & & & \\
\hline
 & & & & & 2.2*0.16e^{6\rho} & & \\
\hline
 & & & & & 2.3*0.64e^{7\rho} & & \\
\hline
 & & & & & & & \\
\end{array}\right] \tag{31}$$

$$\breve{\mathbf{E}}^G(\rho) = \left[\begin{array}{c|c|c|ccc}
 & & & & & 3.9*0.284e^{(-5-3)\rho} \\
 & & & 3.9*1.112e^{(-4-2)\rho} & 4.5*0.392e^{(-4-3)\rho} & \\
\hline
 & & & & & \\
\hline
\hline
 & & & & & 0.9*1.716e^{(-4-8)\rho}e^{-3\rho} \\
 & & & 0.7*0.888e^{(-3-7)\rho}e^{-2\rho} & 0.5*1.608e^{(-3-6)\rho}e^{-4\rho} & \\
\end{array}\right] \tag{32}$$

If continuous interest rate is set to ρ = 0.025%, NPV of the system is positive and economically viable:

$$\begin{aligned}
\mathrm{NPV} &= \mathbf{p}\left(\breve{\mathbf{G}}(\rho) - \breve{\mathbf{H}}(\rho)\right)\tilde{\mathbf{P}}(\rho) - \hat{\mathbf{K}}\tilde{\mathbf{v}}(\rho) - \\
&- \mathbf{U}^{\mathrm{T}}\left(\tilde{\mathbf{\Pi}}_G(\rho) + \tilde{\mathbf{\Pi}}_H(\rho)\right)\tilde{\mathbf{P}}(\rho) - \\
&- \mathbf{c}_{\mathrm{L}}\hat{\mathbf{L}}\tilde{\mathbf{v}}(\rho) - \mathbf{U}^{\mathrm{T}}\left(\breve{\mathbf{E}}_H(\rho) - \breve{\mathbf{E}}_G(\rho)\right)\tilde{\mathbf{P}}(\rho) = \\
&= 3288815.23 - 128072.94 - 2730782.89 - \\
&- 238660.31 - 164930.15 = 26368.94
\end{aligned} \tag{33}$$

Using the above NPV equation, we can evaluate economic performance of the whole system. Wide aspect of input parameters is covered in the model, which gives us opportunity for detailed research on supply chains and their sensitivity to common economic-environmental issues. The large number of details included in the model makes the matrix system complex, thus it is difficult to perform accurate numerical analysis. This fact makes EMRP Theory the perfect basis for conducting in-depth research with the use of simulations.

3. SIMULATIONS

We can show the usability of simulation approach in analysing environmental impacts on a supply chain using the above numerical example. Below we present some of the possible environmentally oriented analyses, but we are not limited to them. Wider aspect of research can be done in practical applications.

First, we simulate isolated impact of changes in the return rates and environmental taxation on the NPV of the system. We distinguish between two different situations:

- Recycling on a particular level is not economically viable. In the numerical example, such situation appears when element D is recycled (Figure 3).
- Recycling on a particular level is economically viable. In the numerical example, such situation appears when element E is recycled from the final product A (Figure 4), and element E is recycled from component B (Figure 5).

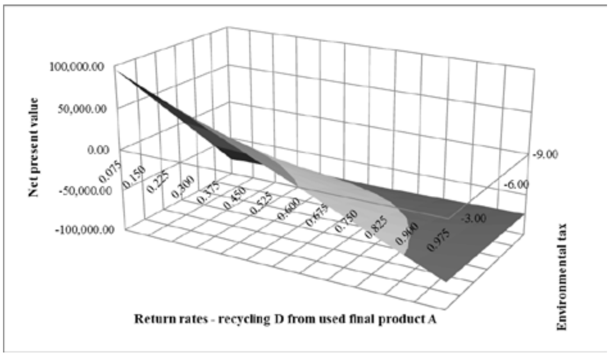

Figure 3: Recycling Element D from A is not Economically Viable

When recycling on a specific micro level is not economically viable, actors in the supply chain will have little incentive to increase recycling activities. Supply chain in such situations creates maximum profit when there are no recycling activities at all, which results in poor long-term environmental balance. Increase in recycling activities leads to a reduction of the NPV of the system. In other words, such a situation means that additional external impulses are required for a shift to more environmentally sustainable behaviour of that particular activity cell. In cases when return rates are low, any environmental tax increase will move the entire supply chain towards negative NPV. Negative overall NPV will encourage actors to change the structure of their supply chains towards more ecologically acceptable behaviour: they will either attempt to substitute element D with a more rational element; invest into research and development of technology with higher recycling efficiency; or geographically reallocate activity cells. On the other hand, if return rates in such situations are already on a high level, changes in environmental taxation will not result in a significant decrease of the overall NPV of the supply chain. Most likely, actors in a supply chain would try to cut return rates in order to generate higher profits, which would bring about negative side effect on the environment. Policy makers can prevent such negative trends or even encourage an increase of return rates with relevant legislation prescribing minimum return rates (an example is the EU Directive 2000/53/EC which prescribes minimum return rates in the automobile industry). In such situations, conflict of interest can be expected:

- Managers of activity cells will try to maximize profits (which is reflected in the maximisation of NPV of supply chains), even for a price of poor environmental balance (low return rates).
- There is a public interest in long-term sustainability of supply chains; this is reflected in the minimization of the industry's impact on the environment (low emissions, reuse of natural resources, etc).

Political leaders are responsible for achieving sustainable long-term balance. Simulating supply chains under different decisions can significantly help in finding long-term optimal solutions which will keep economy competitive and ensure nature conservation.

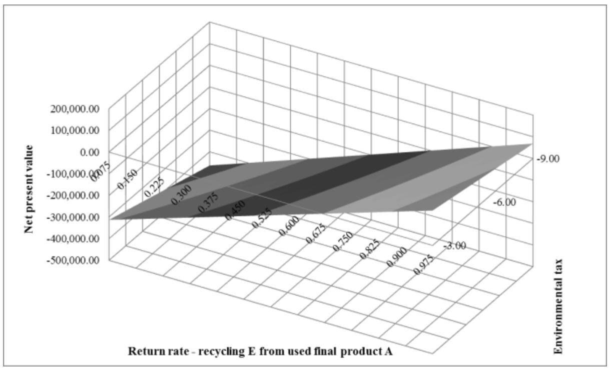

Figure 4: Recycling Element E from A is Economically Viable

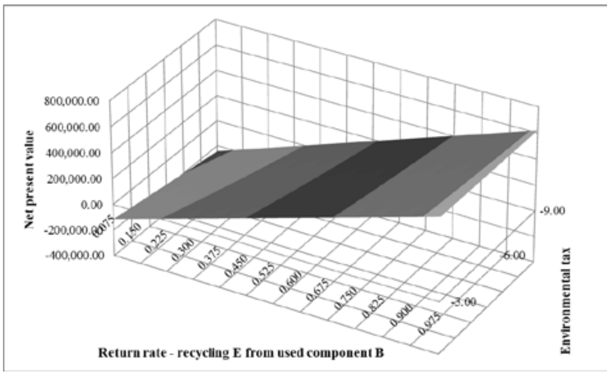

Figure 5: Recycling Element E from B is Economically Viable

A different situation is shown in Figures 4 and 5, where recycling on a specific micro level is economically viable without additional interventions of environmental policy. Economic interest of managers of activity cells coincides with the public interest in long-term sustainability. We can expect such situations to occur in a high-tech environment where recycling activities are already at a high level of development and are highly cost-effective. Similar cases can be expected when input components of the system (raw materials or components) are very rare in nature, or their extraction is difficult and associated with high costs. An environmental tax increase will not encourage recycling activities significantly if return rates are already high. However, if return rates are relatively low despite economic viability of reverse logistics, such tax increase might accelerate the transition of activity cells towards more environmentally friendly business. If we assume that the NPV of a supply chain will remain constant, we can predict achieved return rates after an environmental tax increase if we move along the area with same NPV on the graph (Figures 4 and 5).

In both of the above situations, it is essential that measures used to strengthen economic and environmental balance are proportionate to the level of technological development of the economic environment. Disproportionate measures with which technology is not able to comply can lead to sudden significant reduction in the NPV, which could result in the collapse of the supply chain.

Energy is another crucial environmental factor with a significant impact on the performance of a supply chain. High prices of energy and its irrational consumption can in many cases turn a supply chain from a profitable into an unprofitable one. A lot of energy is stored inside the products, and it can sometimes be restored to some extent. Increases in quantities of recycled energy result in increased NPV of the revenues, which is associated with the sale of that energy on the market (sometimes at a subsidised price). Such increases in recycled quantities usually require investments into appropriate infrastructure. Investments can be studied through setup costs $\hat{\mathbf{K}}$. An investment is economically viable when additional revenues from recycled energy exceed investment costs, which results in an increase of the NPV of the entire system. Figures 6, 7 and 8 show the areas of possible investment decisions for recycling energy during disassembly or incineration of element D; element E, recycled from component B; and element E, recycled from the final product A.

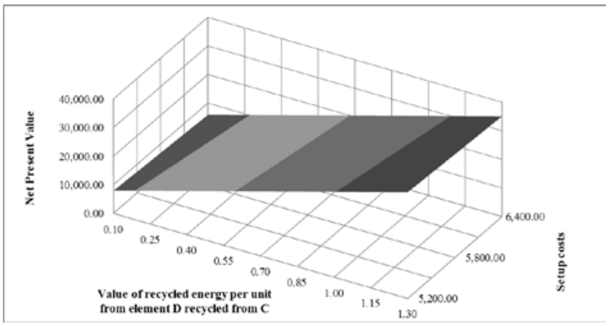

Figure 6: Area of Possible Solutions – Investments into Energy Recycling from Element D

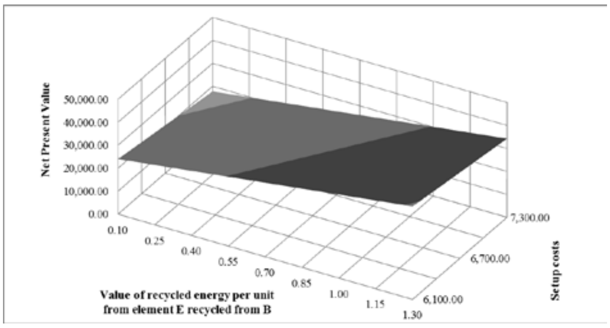

Figure 7: Area of Possible Solutions – Investments into Energy Recycling from Element E, recycled from B

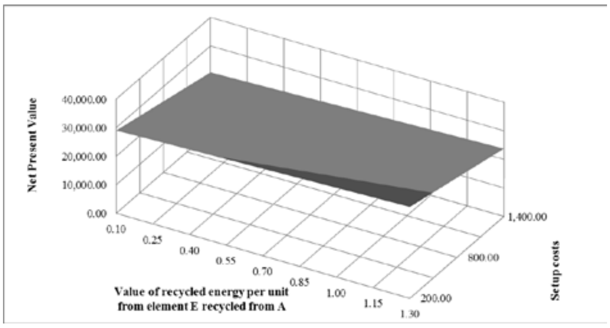

Figure 8: Area of Possible Solutions – Investments into Energy Recycling from Element E, recycled from A

It is clearly evident in the simulation that not all activity cells are equally attractive from an investment perspective. The slope of the graph in Figure 6 is steeper compared to Figure 7 and Figure 8; thus we can expect element D to be more attractive from energy recycling point of view. However, not all solutions on the graphs are feasible, since they are limited by available technology. A comparison of technically feasible solutions between graphs would show the best investment opportunity, which is the one with the highest NPV of the system.

The process of recycling and distribution of successfully recycled elements back to the next production cycle causes time delays. Reduction of lead times has mostly positive effects on the NPV of a system (Figure 9). These positive effects can be interpreted as results of quicker entry of elements into a new production cycle, which reduces the price of capital tied in the stocks of these elements on the way from recycling back to production. For elements that are rare in nature, or whose primary extraction is difficult (therefore their prices on the primary raw materials market are high), we can expect steeper slopes (element D with a lead time Δ_{CD}^{14} in Figure 9). On the other hand, amounts of recycled elements also affect the slope of the curve on the graph. If return rates are higher, contingents of recycled elements are bigger and the decrease of lead times back to production has a greater impact to the NPV of the system.

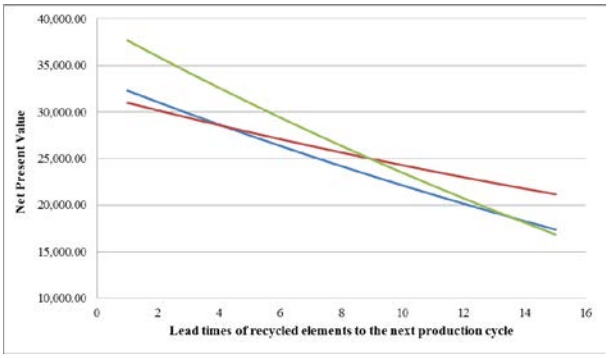

Figure 9: Impact of Lead Times from Recycling to Next Production Cycle

4. CONCLUSION

In this paper, we describe and clearly show that EMRP Theory forms a strong basis for advanced simulations of supply chains. EMRP Theory model has proven to be capable of describing complex structures of supply chains in many previous works. This paper shows that it can also be valuable in practical applications where we can observe the performance of a system by simulating changes in any of the input parameters. Analysis includes environmental factors as an important part of modern supply chains. Such simulations can be helpful, as they can be used in decision making by both managers in the industry and political leaders.

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