ACO TOPOLOGY OPTIMIZATION: THE GEOMETRICAL CONSTRAINT BY LEARNING OVERLAID OPTIMAL ANTS ROUTE

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ABSTRACT
Topology optimization commonly has performed minimization of the mean compliance under a volume constraint. On the other hand, mechanical product designers are considering "a weight minimization under a stress constraint" as an objective and constraints for generating new optimal structure. Moreover, for obtaining this objective, a mechanical structure design has performed to minimize weight of its structure by checking the principal stress vectors as the force's flow, and speculating its desirable structure under maintaining its stiffness, iteratively. These design processes' difference has generated mismatch between actual design practice and the conventional topology optimization theory. Therefore, we have proposed ACO using the principal stress vector for overcoming mismatch of the topology optimization theory. In this paper, ACO Topology Optimization with Geometrical Constraint (ACTO with GC) is proposed to improve unnecessary structures elements problem. Our proposal is new geometrical constraint method which overlays obtained optimal ants route as the shape feature pattern, learns it for next optimization process.

Keywords: Ant Colony Optimization, Topology Optimization, Mechanical Structure Design, Principal Stress Vector

1. INTRODUCTION
Topology optimization has been used for structural optimization, and the various techniques and approaches of topology optimization have been developed and researched since about 1985 (Nishiwaki, Izui and Kikuchi 2012). Topology optimization can change shape, size and number of holes, therefore topology optimization is the most flexible methodology in structural optimization. Topology optimization has generally performed minimization of the mean compliance under a volume constraint until now. First topology optimization’s CAE software, which is named OPTISHAPE, has been sold in Japan from 1989. OPTISHAPE, based on minimization of the mean compliance, has been studied and developed (Bendsoe and Kikuchi 1988, Suzuki and Kikuchi 1991). OPTISHAPE has been utilized in various industries, such as machine, aircraft, building and automobile industries have been used. However, obtained optimal topological structure has complex shapes and layouts. It was difficult to manufacture it efficiently, because it is required that precision technology and great cost should be supplied. Additive Manufacturing (AM), which is a rapidly evolving field, solves the problem between topology optimization and an engineering and manufacturing standpoint. AM via including the 3D printer has changed this situation, because production by AM has flexible and to be able to produce optimal structure introduced by topology optimization. A method of topology optimization focused on AM was suggested in 2011 (Brackett, Ashcroft and Hague 2011). On the other hand, mechanical product designers often consider “a weight minimization under a stress constraint” and the force flow i.e. principal stress vector when design optimal structure. Moreover, for obtaining this objective, a mechanical structure design has performed to minimize weight of its structure by checking the principal stress vectors as the force’s flow, and speculating its desirable structure under maintaining its stiffness, iteratively. These design processes’ difference has generated mismatch between actual design practice and the conventional topology optimization theory. However, topology optimization has generally performed minimization of the mean compliance under a volume constraint. Moreover, topology optimization considering principal stress vector is not much.

Structure optimization using ACO has been suggested by Champ et al. in 2004 (Camp and Bichon 2004), and topology optimization using ACO was applied in 2008 (Kaveh, Hassani, Shojaaee and Tavakkoli 2008). However, these applied optimization only have been introduced a basic principle of ACO theory to for minimization of the mean compliance via a density method (Takada 2006). Subsequently, new topology optimization using ACO, which is called ACO Topology Optimization (ACTO), has been proposed and developed (Guan and Chun 2009, Ito Hoshi and Hasegawa 2016). These methods have versatility of design variable and, optimize in elements of the discretized design domain.
that is ants are moving in design domain. Especially, the
method developed by Ito considered a principal stress
vector as a design variable for filling this mismatch
between the topology optimization theory and
mechanical product designers. However, these
optimization has an improvement problem, and it is that
an optimal topological shape and layout in which many
unnecessary structures elements were included is
obtained. In this paper, ACTO with a geometrical
constraint (ACTO with GC) is proposed to solve the
unnecessary structures element problem. Our proposal
is new geometrical constraint method which overlays
obtained optimal ants route as the shape feature pattern
by ACO, and learns its pattern for next optimization
process. This paper discusses the effect of new proposal
method through a trial of simple cantilever problem.

2. ANT COLONY OPTIMIZATION

ACO algorithm was inspired by a behavior of ant
swarm intelligence. ACO consists of three steps (Figure
1). First step is a setting the initial pheromone of ants on
the route. Next step is to add pheromone on the route
where ants selected. Final step is to update the
pheromone on overall routes. By repeating these steps,
ACO obtains the optimal route by moving of ants which
are guided to its pheromone. This pheromone update
procedure consists of addition and evaporation of
pheromone. The addition of pheromone means that
pheromone is added on the route where ants passed, and
then, the evaporation of pheromone means that a
pheromone evaporates with rain, respectively. Moreover,
ants are able to communicate with a number of ants and
can obtain optimal route through pheromone update.

To compute pheromone updating:

$$\tau(t+1) = \mu \tau(t) + \sum_{i=1}^{m} \Delta \tau_i$$  \hspace{1cm} (1)

\(\mu\): Reduction factor  \(m\): individual Number
\(i\): selected route  \(\Delta \tau_i\):addition pheromone
\(t\): Generation Number

Equation (1) shows update rule of the pheromone.
Where, Reduction factor \(\mu\) become decimal value,
and then the pheromone is reduced via this factor. It means
evaporation of pheromone.

3. ACO TOPOLOGY OPTIMIZATION

First, several ants generate routes in a design region.
The design region is set from Von Mises stress as first
pheromone of first generation. The generation means a
repeat of ACTO. Routes of generated by ants become a
structure one of structures of first generation. This study
regards the structure as an individual. Optimal individual
in first generation is chose by elite strategy when ACTO
satisfy the requirement of individual. ACTO update first
pheromone of next generation base on the optimal
individual of first generation. Moreover, the design space
of a mechanical structure is divided by finite element
where each element shows material or void as Figure 2
(exist: 1, not exist: 0). In addition, ants explore optimal
route in the design space by considering finite elements
as a route. A topology representation is created by an
ant's routes via setting 1 as passed route and 0 as non-
passed of an existence for a finite element. These
elements become design variables of ACTO. The
structure can be described by a discrete function \(p\), as
below:

$$p = \{p_1, p_2, \ldots, p_i\}$$  \hspace{1cm} (2)

$$E_p = \begin{cases} E_1 & \text{if } p_i = 1 \\ E_2 & \text{if } p_i = 0 \\ \rho & \text{in } 1, 0 \end{cases}$$  \hspace{1cm} (3)

\(\rho\): density function  \(i\):element number
\(E_1\): Young’s modulus(material exist)
\(E_2\): Young’s modulus(material don’t exist)

In this case, \(E_1\)is adopt as young’s modulus when the
element has material but \(E_2\) is adopt as young’s modulus
when the element is void. Therefore, the element chose
as route by ants has material, hence optimal structure of
ACO only has 1 or 0 of density (Figure 3).

![Figure 1: Process of ACO](image1)

![Figure 2: Flow chart of ACO topology optimization](image2)
1) Initial Configuration
   The first pheromone is set for each element. Von Mises stress is used to set the first pheromone.

2) Ant of route selection
   The set first pheromone is used by ants to generate routes. This section uses some methods, as follows:
   - Linear rank method
     The linear rank method has been proposed by Baker in 1985 (Baker 1985, Mitchell and Iba 1997). In the linear rank method, its pheromone are ranked normal ascending order from 1st to Nth, redistribute values based on rank order (Figure 4). Moreover, the Max value of the linear rank method used Table 1.

     Equation of linear rank method, as follow:

     \[ \text{Exp Val}(r, t) = \text{Min} + (\text{Max} - \text{Min}) \frac{\text{rank}(r, t) - 1}{N - 1} \]  

     \[ 1 \leq \text{Max} \leq 2 \]  

     \[ \text{Min} = 2 - \text{Max} \]  

     t: time(iteration number) Max: Nth redistribution value Min: 1st redistribution value r: rank

- Roulette wheel selection
   Roulette wheel section (Lipowski and Lipowska 2012) is the probability of selection is proportional to redistribute values of the linear rank method. The better fitted redistribute values of the linear rank method, the larger the probability of selection (Figure 5). This method considers N individuals, each characterized by redistribute values of the linear rank method. Selection of an individual choose randomly. The selection probability of i-th individual \( P_i \) follow as:

   \[ P_i = \frac{\text{Exp Val}(i, t)}{\sum_{i=1}^{N} \text{Exp Val}(i, t)} \quad (i = 1, 2, 3, ..., N) \]  

   \[ \text{Exp Val}(i, t): \text{redistribute value of linear rank method} \]

   \[ i: \text{individual} \quad r: \text{rank of linear rank method} \]

- Consider with the principal stress vector
   The green arrow in Figure 6 is the synthetic vector of the maximum principal stress and the minimum seed stress. The probability of selected the element increase when the element has the principal stress vector (Ito, Hoshi and Hasegawa 2016).

3) Fills population
   Ants repeat route selection in design domain until the set number of individuals is satisfied.
4) Elite preservation strategy

Elite preservation strategy is able to replicate optimal individuals (i.e. optimal structure) in next generation, therefore this method prevents deterioration of volume of optimal structure (Kenneth 1975). Figure 7 shows the flow of Elite preservation strategy.

![Elite preservation strategy](image)

**Figure 7: Elite preservation strategy**

5) Pheromone update

This section executes pheromone update where paths’ ants have walked. Also, existing pheromones decrease because of evaporation of pheromone. Reduction rate of evaporation of pheromone M is often use 0.8 to 0.98.

6) Fill individual

ACTO repeats generating optimal structures until the set number of generations is satisfied.

4. ACO TOPOLOGY OPTIMIZATION WITH GEOMETRICAL CONSTRAINT

An optimal structure is introduced by ACTO, has not been able to generate the intermediate element of density. This is a strong point of ACTO, but many unnecessary structures elements have comprised a large percentage of an optimal structure (Figure 8).

![Optimal structure](image)

(a) Optimal structure of ACTO

![Optimal structure](image)

(b) Optimal structure of other method

**Figure 8: Analysis result of ACTO**

Therefore, we propose new geometrical constraint method (i.e. learning function) by learning overlaid ants route into ACTO, which is named ACTO with GC, to improve unnecessary structures problem. Figure 9 shows the process of ACTO with GC. First step is to perform ACTO of the inner loop using Von Mises stress as the first pheromone value. This is repeated until iteration count in Table 2. Next, the shape feature pattern is made in learning function. Figure 10 shows a way of making the shape feature pattern in learning function. Red part of this figure is necessary structures for optimal structure of ACTO. On the other hand, blue part of this figure is unnecessary structures for optimal structure of ACTO. In this function, optimal ants route is overlaid. After that, elements with small number values change the value to 0 because this elements is unnecessary structures for optimal structure of ACTO. This overlaid route becomes the first shape feature pattern. We repeat ACTO based on the pheromone that was calculated from the first shape feature pattern and its Von Mises stress into outer loop. This outer loop terminates the iteration count of outer loop in Table 2.

**Figure 9: Flowchart of ACTO with GC**

**Table 2: Iteration count of inner and outer loops**

<table>
<thead>
<tr>
<th>Inner loop</th>
<th>Outer loop</th>
</tr>
</thead>
<tbody>
<tr>
<td>3 times</td>
<td>10 times</td>
</tr>
</tbody>
</table>

5. EVALUATION

Initial pheromone distribution of ants is created by Von Mises stress distribution. The optimization definition is described as the following.

\[
f_p(\rho) = V(\rho) + r P(\rho) \rightarrow \text{Min} \tag{5}\]

\[
V(\rho) = \int_{s} \rho \, ds \tag{6}\]

\[
P(\rho) = \begin{cases} 
0, & \text{if } \sigma_{\text{max}} < \sigma_{\text{all}} \\
1, & \text{otherwise} 
\end{cases} \tag{7}\]
Where $f_d(\rho)$, $V(\rho)$, and $P(\rho)$ denote the modified objective function, the volume function for mechanical structure, and the penalty function, respectively. The design variables are defined by density variable $\rho$. The local stress constraint is consisted of maximum stress $\sigma_{\text{max}}$ and allowable stress $\sigma_{\text{all}}$.

6. ANALYSIS SETTING

Figure 11 shows object model in using this study. In addition, the analysis settings of this paper are shown in Table 3. In this study, we analyze 2 approaches to consider the shape feature pattern in ACTO with GC. First, analysis type 1 (no normalization) does not change the shape feature pattern in setting the first pheromone. On the other hand, analysis type 2 (normalization) generate the first pheromone by binarizing the shape feature pattern (Figure 12).

![Figure 11: object model](image)

![Figure 12: Binarizing the shape feature pattern](image)

Table 3: Experiment condition

<table>
<thead>
<tr>
<th>Mesh division Vertical</th>
<th>25</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mesh division Horizontal</td>
<td>50</td>
</tr>
<tr>
<td>The number of generations</td>
<td>10</td>
</tr>
<tr>
<td>Ant to route search per individual</td>
<td>10</td>
</tr>
<tr>
<td>Generation number</td>
<td>200</td>
</tr>
<tr>
<td>Add pheromone value</td>
<td>0.0002</td>
</tr>
<tr>
<td>Pheromone reduction factor</td>
<td>0.99</td>
</tr>
</tbody>
</table>

7. RESULTS AND DISCUSSION

We applied ACTO with GC to the simple cantilever problem. Figure 13 shows the shape feature pattern of cantilever structure in each outer loop of learning function and the optimal structure. Red elements of this figure have strong characteristics of optimal structure, especially these elements are necessary structures for optimal structure of ACTO. On the other hand, yellow and green elements of this figure have weak characteristic of optimal structure, seemingly this part is unnecessary structures for optimal structure of ACTO. In addition, the optimal structure extracted characteristics from the shape feature pattern and made by using modeling tool.

The shape feature pattern of analysis type 1 have red elements all over this shape. Especially, the third shape feature pattern (Figure 13(c)) has its tendency. Hence, ACTO with GC is possible to delete unnecessary structures and generate clearly optimal structure from the shape feature pattern, such as Figure 13(d). However, the optimal structure (Figure 13(d)) have scraggly paths, because the shape feature pattern of analysis type 1 has a tendency that the structure of the upper part of the shape feature pattern is derived thickly. Analysis type 1 need to lessen its tendency.

![Figure 13: Analysis result of ACTO with GC](image)
On the other hand, the shape feature pattern of analysis type 2 has not the tendency that the structure of the upper part of the shape feature pattern is derived thickly. In addition, the shape feature pattern of analysis type 2 has a tendency to generate the internal structure finely. However, the internal shape of the shape feature pattern of analysis type 2 has Red elements less than the shape feature pattern of analysis type 1, because of a global route selection by ants. Therefore, the optimal structure (Figure 13(h)) has not tendency of the internal structure finely. Analysis type 2 need to increase iteration count of inner and outer loops of learning function.

From these facts, the optimal structure is change by a way of setting initial pheromone value of the shape feature pattern in ACTO with GC. Moreover, count of inner and outer loops of learning function is important to improve the optimal structure.

8. CONCLUSION

In this paper, to solve the unnecessary structures problem of ACTO, the geometrical constraint method by learning overlaid optimal ants route have been introduced in ACTO. As the result, we confirmed ACO with GC is able to obtain the optimal structure, and to reduce unnecessary structures elements. However, unnecessary structures elements remain in the shape feature pattern. To remove unnecessary structures elements, it is necessary to increase iteration count of inner and outer loops. In addition, a way of setting initial pheromone value of the shape feature pattern in ACTO with GC is important to improve the optimal structure. In the future work, we plan to reconsider about iteration count of inner and outer loops and way of setting initial pheromone value of the shape feature pattern in learning function. After that, we try to perform ACTO with GC using new iteration count and a way of setting initial pheromone value.

REFERENCES


AUTHORS BIOGRAPHY

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