

A HEURISTICS WITH SIMULATIVE APPROACH FOR THE DETERMINATION OF THE OPTIMAL OFFSETTING REPLENISHMENT CYCLES TO REDUCE THE WAREHOUSE SPACE

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ABSTRACT

An important issue in a supply chain is to manage carefully replenishment cycles of stored items because of its strong impact both on production and stock management costs. The literature offers several papers debating this issue with a recent focus on the possibility to offset the items inventory cycles in order to reduce the maximum required space. A genetic algorithm (GA) is proposed in order to determine the optimal offsetting inventory cycles of items stored in the same warehouse. The heuristics was validated through the comparison with the most adopted and cited methodology, showing the effectiveness of the GA, able to provide better results than those previously presented in the literature. The GA was finally applied to a realistic case of a production system, showing a minimization of the maximum peak in the time horizon without relevant additional costs, together with a higher and more regular saturation of the warehouse.

Keywords: Offsetting, Genetic Algorithm, Inventory Management, Warehouse.

1. INTRODUCTION

The JRP (Joint Replenishment Problem) is one of the fundamental problem in the inventory management and its classic assumptions are similar to the EOQ (Economic Order Quantity) model: the problem includes deterministic and uniform demand, no shortages allowed, no quantity discounts and linear holding costs. The purpose of JRP is to minimize the total costs incurred per unit time: generally the considered cost terms include setup costs and inventory holding costs. A decision maker may ignore the warehouse-space restrictions and then he/she may apply the conventional heuristics developed over the years to solve the JRP, but he/she will often find that the obtained solutions are not applicable in a real case because the maximum required warehouse-space is greater than available. The JRP, in fact, does not evaluate the existence of space constraints, therefore, several researchers have already proposed an extension of the JRP that presented warehouse-space restrictions. It is known in literature also as “staggering problem” or

“inventory cycle offsetting problem (ICOP)” and consists of offsetting the replenishment cycles of a large number of items stored in the same warehouse with the aim to minimize the peak usage of the aforementioned resource. To solve this problem, or at least try to reduce the maximum peak in stock, it is necessary to determine the optimal offsetting of many items’ replenishment cycles, develop methodologies able to do it and, therefore, minimize the maximum volume peak. This issue has assumed considerable importance in research as early as the 80s and Gallego, Shaw and Simchi-Levi (1992) showed that the class of complexity of the problem is NP-complete: it justifies the efforts that have been made to develop appropriate heuristics that can lead to tangible improvements in practical cases.

This research was focused to study how to offset the inventory cycles of many products stored in the warehouse in order to minimize the maximum warehouse-space. In particular, a genetic algorithm (GA) was proposed to search the optimal replenishment schedule.

The paper is organized as follows.

Section 2 presents a review of existing relevant literature. Section 3 shows firstly the description of the problem, secondly the mathematical formulation of the problem and, finally, a short description of the generic genetic algorithms and their characterizing parameters. Section 4 presents the heuristics developed in this paper for the specific issue. Section 5 shows the validation of the genetic algorithm through its application to a benchmark example (Murthy, Benton, and Rubin 2003) and reveals the superiority of the obtained results both compared to Murthy et al.’s procedure and a heuristics later proposed by Moon, Cha, and Kim (2008). Furthermore, the algorithm is applied to a case of a production system, and also in this case, its application leads to a considerable reduction of the maximum peak observed in the time horizon. Finally, section 6 presents the conclusions of the work.

2. LITERATURE REVIEW

The JRP has been studied over thirty years, a lot of heuristics may be used for solving it and many researches addressed their efforts to lot sizing problems

with limited warehouse-space, namely “the staggering problem”: Goyal (1975) introduced the JRP with one resource constraint and developed a heuristic algorithm using the Lagrangian multiplier; Silver (1976) developed a simply methodology to determine the order quantity of each product in the warehouse, whose demand is supposed constant, and discussed the advantage and disadvantage of coordinating replenishments; Kaspi and Rosenblatt (1983) have made an improvement to the algorithm previously developed by Silver (1976), showing that the errors of the algorithm can be reduced, on average, of an order of magnitude; later Kaspi and Rosenblatt (1991) proposed an approach based on trying several values of the basic cycle time between a minimum and a maximum value, then they ran the heuristic of Kaspi and Rosenblatt (1983) for each value of the basic cycle time and they showed that their procedure (called RAND) outperforms all available heuristics; Gallego, Shaw, and Simchi-Levi (1992) showed that “the staggering problem” is NP-complete even if only one item has a different reorder interval and, thus, it is not possible to find an optimal solution but it must use a heuristic technique, then this problem is not solvable by polynomial-time algorithms. Other studies have been made by Hariga and Jackson (1995) and by Hall (1998), that examined the case in which all cycle lengths are equal, and showed that the problem is NP-hard. Khouja, Michalewicz and Satoskar (2000) applied the GA approach to the basic JRP and compared the performance of their GA with Kaspi and Rosenblatt’s heuristic algorithm (1991). Murthy, Benton, and Rubin (2003) considered the presence of space constraints and presented an interesting heuristics for offsetting independent and unrestricted ordering cycles for items on the time axis to minimize their joint storage requirements over an infinite time horizon when warehouse-space is limited. Given that the aforementioned procedure represents the benchmark of many subsequent works in this field, for simplicity, from now on we will call the method as MBRP, stands for Murthy, Benton and Rubin Procedure. Moon and Cha (2006) were focused on the development of two algorithms to solve the JRP with resource restrictions: firstly they modified the existing RAND algorithm, then they developed a GA for the JRP with resource restriction; Yao (2007) conducted a research, focused on JRP with warehouse-space restrictions, establishing the lot size of each item, with the aim to minimize the total cost per unit of time and generate a program supply for many products without exceeding the available space: he has proposed a hybrid genetic algorithm. Yao and Chu (2008) conducted theoretical analysis based on Fourier series and Fourier transforms, proposing a procedure to calculate maximum warehouse space requirement; then, they employed this procedure in a genetic algorithm showing improvements, compared to the MBRP. In the same year Moon, Cha, and Kim (2008) proposed a Mixed Integer Programming (MIP), based on the same assumptions

imposed by MBRP, and a GA, realizing that both led to the same results and, by comparison with the example presented by Murthy, Benton and Rubin (2003), which from now on we will call MBRE, were able to obtain better results; furthermore, they implemented a MIP for a finite time horizon and applied the GA to this case. Then, Boctor (2010) proposed a new formulation of the problem proposed by MBRP, and a heuristic algorithm based on Simulated Annealing through which they achieved the same results as Moon, Cha, and Kim (2008). Successively, Boctor and Bolduc (2012) presented a new mathematical formulation for the “staggering problem”, showed two heuristic approaches and evaluated the obtained performance using their techniques. Finally, Croot and Huang (2013) proposed a series of algorithms, operating randomly, for the determination of offsetting inventory cycles: they studied this problem from the view of probability theory and their algorithm can work when the number of item is large, while the time horizon and unit volumes are not too large.

Despite of the several researches, there is not yet a procedure which leads to the optimal solution or, in other words, to an exact methodology for the resolution of the above mentioned problem. Furthermore, the heuristics developed in the course of the years, hint at a wide margin for improvements and, for this reason, the idea of developing a new algorithm able to bring to most interesting solutions is becoming very relevant.

3. PROBLEM DESCRIPTION AND FORMULATION

As shown in the following example (which reproduces the MBRE), the offsetting of items’ inventory cycles in the warehouse does not alter the stock management costs and reduces the maximum volume peak in storage. Consider the data reported in Table 1 and see the trend of the two items in Figure 1. In particular, the maximum peak without the offsetting is equal to $2Q$ because both the items will be ordered for the first time at the beginning of the time horizon; with the application of MBRP, the order of item 2 slides to the right of P units of time while the item 1 is always ordered for the first time at the initial instant: in this way the maximum peak will be equal to $1.75Q$ (Figure 2) and will occur at P unit of time.

Table 1: MBRE

Item	Storage space per unit time	Time between orders (TBO)	Order quantity
#1	$s=1$	$4T$	Q
#2	$s=1$	$2T$	Q

In conclusion, instead of ordering all EOQs immediately, as predicted by the Wilson model, initial stocks are supposed for satisfying the demand of the early days and, then, the respective EOQ will be ordered only when it is really necessary, namely when these stocks are inferior to the demand. In this way, it is possible to have a better use of warehouse-space. Then,

the objective is the determination of the optimal offsetting for each item with the aim to minimize the maximum peak over time.

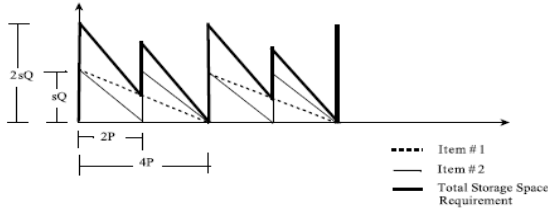


Figure 1: Space Requirement Without Offsetting

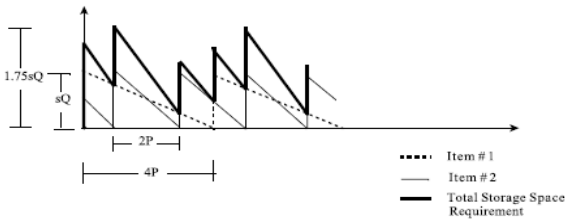


Figure 2: Space Requirement With Offsetting

The problem supposed the same assumptions of MBRP:

- the daily demand rate is deterministic and constant;
- the replenishment is instantaneous;
- the Time Between Order (TBO) is known and constant.

The objective is to minimize the maximum peak in the time horizon.

We introduce the following notation:

- $N \rightarrow$ number of items
- $d_j \rightarrow$ daily demand for the j th item
- $TBO_j \rightarrow$ time between orders for the j th item
- $Q_{j,t} \rightarrow$ replenishment quantity for the j th item at time t
- $y_{j,t} \rightarrow$ occupied quantity from item j in the warehouse at time t
- $q_{iniz,j} \rightarrow$ initial stock of the j th item
- $S_t \rightarrow$ total storage space required for all items at time t
- $s_j \rightarrow$ required space per unit time for the j th item
- $S_{max} \rightarrow$ maximum storage space required for all items in the time horizon T

The mathematical model of the problem is shown below:

$$\min S_{max} \quad (1)$$

subject to:

$$y_{j,t} = q_{iniz,j} \quad t = 0 \quad (2)$$

$$y_{j,t} = y_{j,t-1} + Q_{j,t} - d_j \quad t = 1, \dots, T \quad (3)$$

$$\begin{cases} Q_{j,t} = 0 & \text{if } y_{j,t-1} \geq d_j \\ Q_{j,t} = d_j TBO_j & \text{if } y_{j,t-1} < d_j \end{cases}$$

$$S_t = \sum_{j=1}^N s_j \times y_{j,t} \quad j = 1, \dots, N \text{ and } t = 0, \dots, T \quad (4)$$

$$S_{max} = \max_t \{S_t\} \quad t = 0, \dots, T \quad (5)$$

The objective (1) is to minimize the maximum space required in stock over the considered time horizon. The constraint (2) indicates that the present quantities at $t = 0$ must be equal to the assumed initial quantities for each item; the constraint (3), however, indicates that the present quantities from the second day onwards, until the end of the time horizon T , will be equal to the sum of the present amount on the previous day and the lot Q_j less demand of the j th item. The equation (4) establishes the total space occupied by the items at all time instants, while the constraint (5), finally, defines the peak capacity utilization.

As regards the time horizon, since the total space requirement pattern is periodic, the maximum will occur in the time interval from $t = 0$ to $t = \text{LCM}(TBO_1, \dots, TBO_n)$ where LCM is the acronym of least common multiple. However, in the real cases, the items in the warehouse are numerous and, consequently, the time horizon (the LCM of the TBOs) grows exponentially. But, as also observed by Moon, Cha, and Kim (2008), it is totally unrealistic to think that the daily demand, the price and the management costs of each product do not change during a so long period. For these reasons, we assume a more realistic time horizon which has been prudentially fixed to a working year, namely $T = 220$ days. During such period, the order quantities (EOQs), the daily demands, the purchasing, ordering and holding costs are supposed known and constant.

Moreover, as mentioned above, many studies suggest that the replenishment of each item must be present at the beginning of the considered time horizon, namely at $t = 0$. We relax this hypothesis and, to determine the optimal offsetting in order to minimize the maximum peak in the store, suppose initial quantities for the items that, when satisfying the demand of the first days, delay consequently the time of the first replenishment. In other words, the day in which the satisfaction of the demand is not more possible with the assumed initial quantities, the lot EOQ_j is ordered and that day represents the offsetting of the item j . For example, if in the third day it is not more possible to satisfy the demand of the product j with the assumed initial amount, then $t = 3$ represent the offsetting of the j th item. The optimal offsetting is therefore the combination of the single delays which minimizes the peak of required space in stock.

To obtain the initial quantities that allow to minimize the maximum peak and maximizing the saturation, we performed a series of simulations using the genetic algorithm developed for the specific issue that can establish the best case, among all those assumed, namely the optimal initial quantities that will be needed in the warehouse for a better management of the space and a more regular saturation.

Genetic Algorithms (GAs), widely used in recent decades by various researchers, are techniques based on

the population, have an objective dynamic function, memory and methods are inspired by nature, unlike traditional methods that are based on research of single point, have an objective static function, have no memory and are not inspired by nature.

In particular, GA have demonstrated good performances in lot sizing issues (Macchiaroli and Riemma 2002), also in presence of space and budget constraints (Yang and Wu 2003).

The main parameters of the genetic algorithms are numerous and known from the literature thanks to many articles such as in Dowsland (1996) and to several books as in Mitchell (1998).

The principal steps of the GAs are:

1. The choice of an initial **population**, namely the first set of possible solutions, is generated randomly. Each individual of the population is known as chromosome representing a possible solution to the problem and evolves through the iterations called generations. The size of the population can vary considerably according to the kind of problem.
2. The **fitness function** is used to measure the goodness of the founded solutions at each iteration, the individuals are tested, are given them a value that will be considered in subsequent phases and will allow to the algorithm to move toward the best solution.
3. The **fitness scaling** converts the raw scores returned by the fitness function, in values, in a range that is suitable for selection.
4. The **selection** chooses at each generation the most promising solutions to create the next population of solutions.
5. The **reproduction** determines how the GA creates the children at each generation: from two parents, GA creates a child who will have similar characteristics to the parents and, therefore, later, it is appropriate to apply techniques of crossover and mutation.
6. Through the **mutation** is possible to make small random changes in the individuals of the population in order to have a wider genetic diversity and to allow for a broader search space of the solutions.
7. The **crossover** combines two chromosomes to form a new one for the next generation.
8. The process of generation of populations is repeated until a **stop condition**, set by the user, is reached (for example, it is reached the maximum number of generations or is exceeded the imposed temporal/economical limit, etc.).

4. THE PROPOSED PROCEDURE

In this section we propose a specific genetic algorithm to obtain an optimal solution for the JRP with warehouse-space constraints.

Due to the large number of available techniques for each step described above, several configurations could be used for the genetic algorithm and, for this reason, an experimental analysis, based on the ANOVA technique, was conducted in order to investigate the effect of the

most influencing parameters and to identify the optimal configuration of the proposed procedure: it is necessary to set properly control parameters so that GA is able to search for good solutions.

After this analysis has been performed, the techniques for each step were chosen and the parameters have been established for the best configuration.

1. The **population size**, in this case, was set equal to 50 individuals after a series of tests carried out firstly with 20 individuals and then with 100 and 200, and it was found that if it increases the size of the population, it does not significantly improve the results in a meaningful way, but rather the duration of the algorithm for the research of the optimal solution, grows appreciably; instead, if the population is too small (20 chromosomes) there is little variability of the possible solutions and then there is the risk of not finding the correct solutions for the specific case.
2. The **fitness function**, subsequently implemented in MatLab 7.9 and explained by the flow chart in Figure 3, was built specifically for the JRP with space constraints, assumes the values of initial quantities for each item and searches, in every generation, those that lead to the minimization of the maximum peak.
3. The technique used for the **fitness scaling** is called “Shift Linear”, namely scale raw scores so that the expectation of the stronger individual is equal to a constant multiplied per an average score: during each generation chromosomes are evaluated using the fitness function;
4. The technique used for the **selection** is the so-called “Remainder”, that assigns a parent from the integer part of each individual’s scaled value, while the rest of the parents are chosen stochastically by the fractional part of the scaled value (for example, if the scaled value of an individual is 2.3, that individual is listed twice as a parent, while the probability that other additional parents will be choose for the next generation is proportional to the fractional part, in this specific case to 0.3);
5. As for the **reproduction**, in this case we have established, as suggested by the literature, that 5 individuals of 50 (10%) of the new created population have direct access to the next population (formed at the subsequent generation) because they are considered the best, while it is necessary to apply mutation and crossover techniques to the remaining population;
6. The utilized technique for the **mutation** is the so-called “Uniform”: the value of the gene is replaced with the uniform value between the minimum and the maximum specified for that gene: through mutation, the small changes in individuals are carried out according to a random mutation probability p_m established equal to 0.05;
7. The **crossover** probability p_c was established equal to 0.8. The used technique is called “Scattered”, which involves creation of a random binary vector and the genes of the first parent are inserted in the above-mentioned binary vector if it present the number ‘1’,

otherwise, if there is the number '0', the genes of the second parent are inserted: so, by these two parents, will born the new chromosome;

8. In this case, the established **stopping condition** is the number of generations equal to 300, exhaustive number allowing convergence to optimal solutions.

Figure 3 schematizes the logic of the model created and subsequently implemented in MatLab that represent the fitness function of the proposed GA.

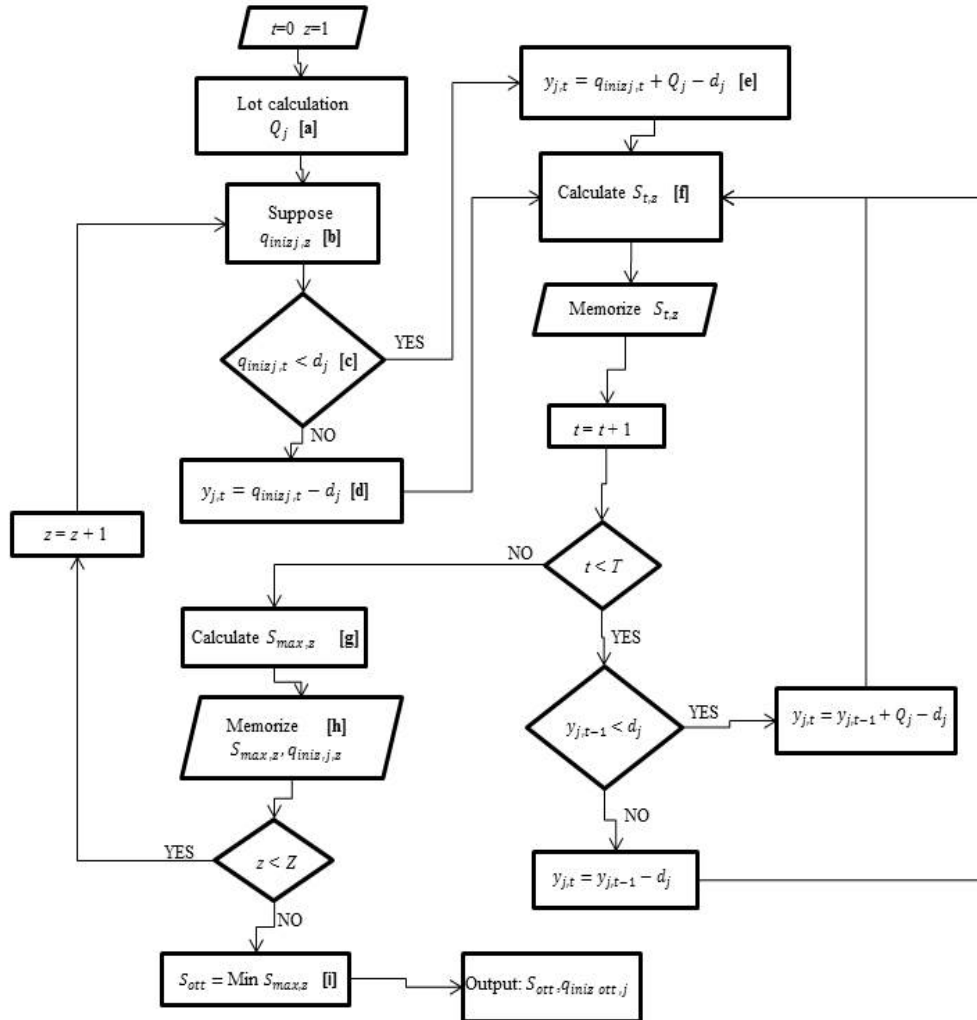


Figure 3 – Flow chart of the proposed fitness function

The model needs in input the number of items, the required unit space for each item, the daily demand, the Time Between Orders (TBO_j) for the j th item and the time horizon T . Once this information is known, it is possible to calculate the EOQ for each item [a] using the following formula:

$$Q_j = d_j TBO_j \quad (6)$$

where Q_j represent the EOQ which must be ordered for the j th item. Consider that z represents the subscript of each GA's generation, and that the total number of generations (Z) is equal to 300, as mentioned before.

The model must hypothesize randomly the values of the initial quantities [b] for the first generation ($z = 1$) and

for each item that are included within the lower and upper limits:

$$0 \leq q_{iniz,j} \leq Q_j \quad (7)$$

Based on the presumed values, it will have a certain inventory trend: as long as the initial stock $q_{iniz,j}$ can satisfy the demand d_j [c], then it is unnecessary to order Q_j , and the present quantity in the second day, $y_{j,t}$, will be equal to $q_{iniz,j} - d_j$ [d]; when the demand of the j th item is greater than the initial quantity, then order Q_j , that will add up to the remaining quantity of the j th item, namely $y_{j,t} = q_{iniz,j} - d_j + Q_j$ [e]; at this point, the algorithm calculates the space occupied by the items [f] according to (4) which will be particularized

considering also the subscript z for the number of generations and, consequently, becomes:

$$S_{t,z} = \sum_{j=1}^N s_j \times y_{j,t} \quad (8)$$

for $t = 0, 1, \dots, T$ $j = 1, 2, \dots, N$ and $z = 1, 2, \dots, Z$

Then the heuristics memorizes this value. From the later instant of time, and until the last day of the required time horizon, the procedure is repeated; when the condition $t > T$ occurs (whereas $T = 220$ days, it will have as a result 220 spaces calculated for each generation), the algorithm calculates the maximum occupied space $[g]$ according to (5) but particularized with the subscript z as follows:

$$S_{max,z} = \max S_{t,z} \quad (9)$$

The algorithm stores both the peak $S_{max,z}$ and the value of the initial quantities that have brought such peak $[h]$. Until $z \leq Z$ the entire procedure is repeated and new initial quantities are assumed, according to (7) and to the established parameters for the GA configuration with the aim to hypothesize, at each generation, new initial quantities that lead to a maximum peak $S_{max,z}$ equal or lower than previous one ($S_{max,z-1}$). When the condition $z > Z$ occurs, $[i]$ the algorithm shows its best solution corresponding to the minimum required space S_{ott} given by:

$$S_{ott} = \min S_{max,z} \quad \text{for } z = 1, 2, \dots, Z \quad (10)$$

and the corresponding initial quantities that led to this value: then, as the algorithm is structured, S_{ott} will coincide to S_{max} of the last generation.

5. GA VALIDATION AND APPLICATION

5.1. Validation

Until this time, many heuristics or procedures have been tested on the MBRE (whose data are shown in Table 2),

achieving large improvements: for example, the algorithm developed by Moon, Cha, and Kim (2008), implemented on the MBRE, shows a reduction of the maximum peak equal to 24.06% compared to the case without offsetting while the MBRP applied to the MBRE has led to a reduction of 15.46% compared to the same case without offsetting. In the same way, the algorithm implemented in this paper was applied to the MBRE, obtaining a huge reduction of the maximum peak in the time horizon.

Table 3 reports the results obtained in the case without offsetting, with the application of MBRP, Moon et al.'s procedure and, finally, the proposed algorithm, showing that in the first three cases the maximum peak in the time horizon always occurs at the initial instant while with the application of the developed GA, the maximum peak occurs on day 42 and is equal to 700m³ with a reduction of 32.37% (compared to the case without offsetting). This percentage is obtained thanks to the values of the initial quantities, indicated in Table 3, leading, consequently, to the shown values of $y_{j,42}$, and then to the maximum peak of 700m³.

Figures 4, 5, 6 and 7 show respectively: the trend of the total warehouse required by the items without offsetting (the classic EOQ model), in the case of application of MBRP, Moon et al.'s procedure and, finally, with the application of the GA developed in this paper. In the last case it is interesting to note the significant reduction of the maximum peak in the stock and the evident increase of the regularity of the saturation. Moreover, it can be noted that in these figures only the trends for the first 50 days are reported because these are considered sufficient for the understanding of the trends which, after that moment, present a similar behaviour.

Table 2 – Data of MBRE

Item j	1	2	3	4	5	6	7	8	9
$s_j Q_j$	100	200	81	144	150	160	90	60	50
TBO_j	4	5	9	12	15	8	6	12	2

Table 3 – Comparison with literature

	Without Offsetting	MBRP	Moon et al.'s Procedure	The Proposed Procedure	
Item	$q_{iniz,j} = \text{max peak}$	$q_{iniz,j} = \text{max peak}$	$q_{iniz,j} = \text{max peak}$	$q_{iniz,j}$	$y_{j,42} = \text{max peak}$
1	100	100	100	50	25
2	200	160	200	40	200
3	81	81	72	63	18
4	144	144	84	36	120
5	150	150	120	82	122
6	160	120	120	80	60
7	90	75	60	75	90
8	60	20	5	40	15
9	50	25	25	25	50
Sum	1035	875	786	491	700
Reduction	-	15.46%	24.06%		32.37%

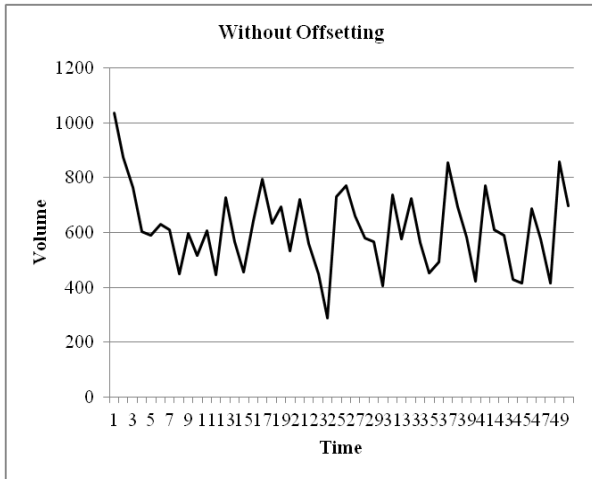


Figure 4 – Trend without offsetting.

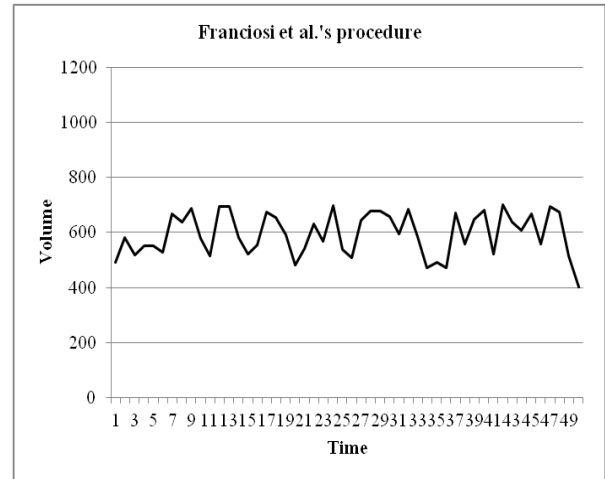


Figure 7 – Trend with the proposed procedure.

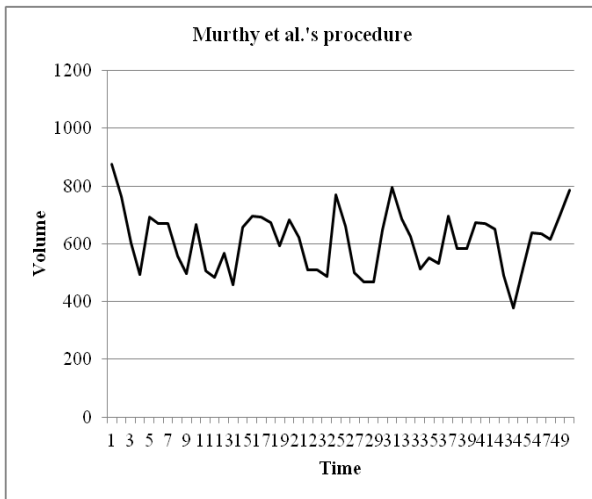


Figure 5 – Trend with Murthy et al.'s procedure.

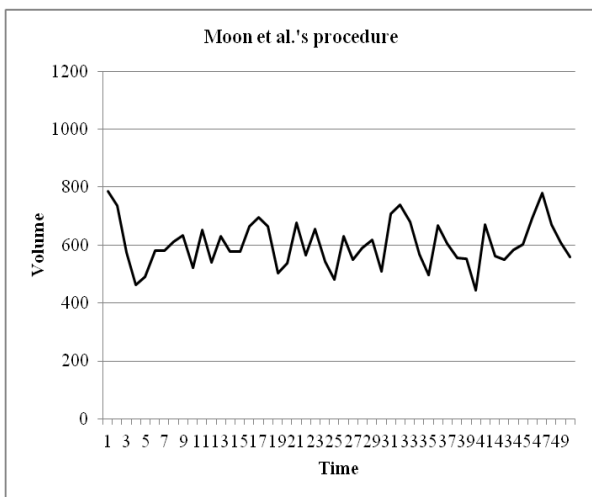


Figure 6 – Trend with Moon et al.'s procedure

5.2. Application

After its validation, the proposed heuristics was applied to a case of a production system operating in the engineering sector which manages 200 items in warehouse that has a volume capacity of 21.000 m³ and, then, this value will represent the space constraint. Table 4 indicates the range of values in which the characteristics of the items are included.

Table 4 - Characteristics of items

Characteristic	Measure Unit	Range
Daily Demand (d_j)	units/day	1÷100
Ordering Cost (C_l)	€	50÷100
Holding Cost (k_j)	€/unit*day	0,001÷0,05
Specific Volume (v_j)	m ³ /unit	0,001÷1
Purchasing Cost (p_j)	€/unit	1÷100

Initially, the EOQ model is applied to this case, the individual cost of the items, the total cost and the maximum peak are calculated (see Table 5), noting that the peak occurs at the initial instant, as shown in Figure 8, and the space constraint is violated.

Table 5 - EOQ model's costs and max volume

Cost Type	Value
Purchasing Cost (€)	113.840.980
Ordering Cost (€)	274.974
Holding Cost (€)	274.990
Total Cost (€)	114.390.944
Max Volume (m ³)	32.507

With the application of the GA, on the contrary, the costs remain almost constant (consider only an extra ordering cost for the initial hypothesized quantities that is negligible compared to the total cost), while the maximum peak is significantly reduced (see Table 6).

Table 6 - GA's costs and max volume

Cost Type	Value
Purchasing Cost (€)	113.840.980
Ordering Cost (€)	274.974
Extra Ordering Cost (€)	3.253
Holding Cost (€)	274.990
Total Cost (€)	114.394.197
Max Volume (m ³)	20.020

Such peak does not occur at the first day, as indicated in Figure 8: in this way the space constraint is respected. From the Figure 8, that shows the trend of the total

warehouse required by the items over the time with the application of the traditional EOQ model, the MBRP and finally the GA, it may be noted, moreover, that in the second case the initial peak decreases significantly but the saturation of the warehouse is not much constant because of the initial peak. In the third case, however, the saturation is more regular than the two previous cases, the peak is strongly lower, does not occur at the initial time and allows to respect the space constraint. Moreover, since there will be approximately 1.000 m³ on average that can be used for possible safety stocks, a better management of the space is achievable.

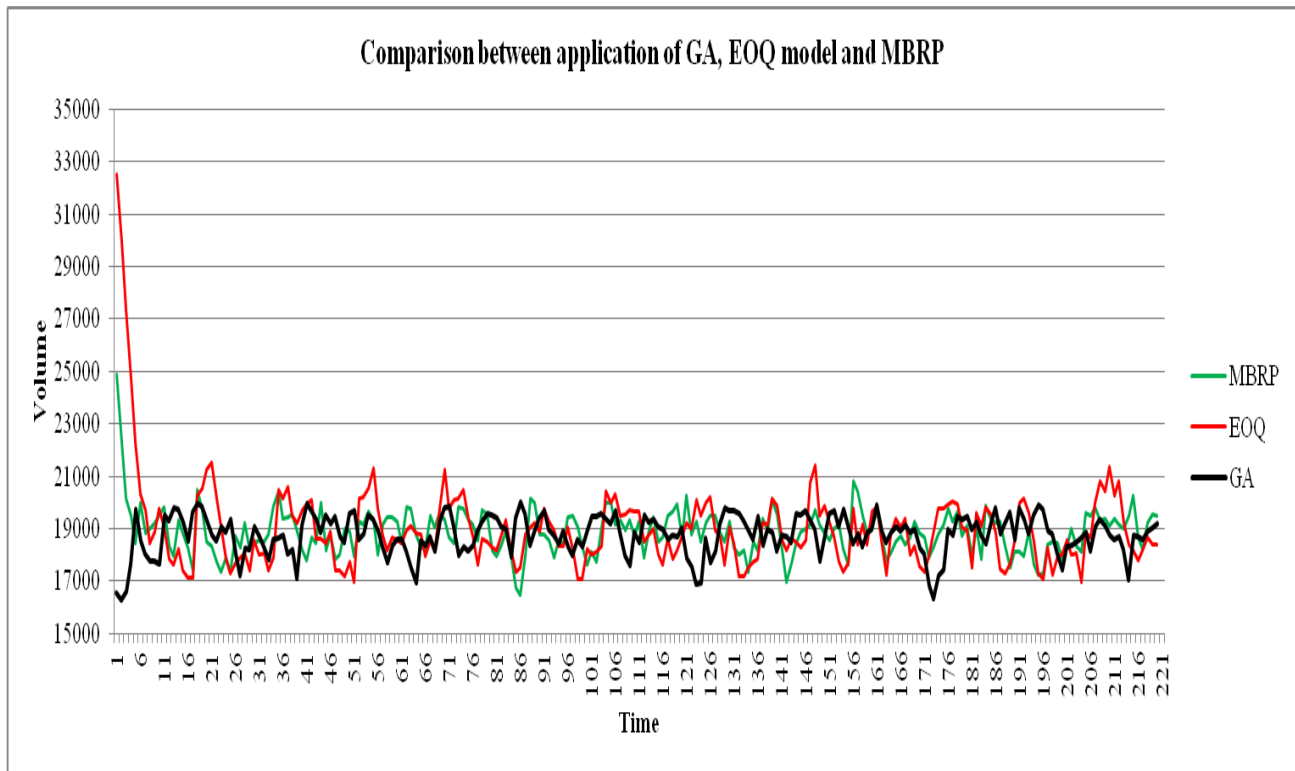


Figure 8 – Comparison between application of GA, EOQ model and MBRP

6. CONCLUSIONS

This paper deals with the joint replenishment problem subject to space constraints assuming constant and deterministic demand rates, instantaneous replenishments, known and constant time between orders for all items and a finite time horizon, which is more realistic than the infinite time horizon.

A genetic algorithm, which allows to calculate the optimal initial quantities required to minimize the maximum peak in the warehouse over the time horizon, has been implemented for the above mentioned problem.

Using the proposed algorithm it is possible to improve the process of inventory management by reducing the maximum peak and maximizing the saturation. GA is able to reduce inefficiencies because the warehouse is better used for the majority of its time, but without excessive peaks and, consequently, very often it can respect the space constraints.

The heuristics has been tested and compared with some procedures previously made available in the literature that consider an infinite time horizon (Murthy, Benton and Rubin 2003; Moon, Cha and Kim 2008), showing better results as regards the maximum peak, which is much lower and not necessarily occurs at $t = 0$.

The algorithm can be used as a tool to support business decisions as it increases the ability to handle multiple items, avoiding the need to rent additional space. Furthermore, it can be considered as an interesting contribution to the study of the heuristic techniques applied to the stock management issue.

This work represents the starting point of possible future developments: firstly a Design of Experiment with ANOVA will be implemented with the purpose of finding a better configuration of the genetic algorithm and an optimal setting of heuristics' parameters; secondly the idea of applying other heuristic techniques to the offsetting problem with the aim of comparing the

results achieved by the GA could be evaluated. Another possible future development could be the comparison of the proposed GA with procedures that consider a finite time horizon, with the aim of making improvements as regards both the computational time and maximum peak in storage, also in presence of high number of items. Moreover it is necessary to highlight that the proposed procedure leads to a considerable reduction of the maximum peak in the warehouse, but does not guarantee the respect of the imposed space constraints. For this reason, a further interesting possible future development could be the implementation of a procedure able to minimize the maximum peak in the stock, maximize the saturation and, at the same time, respect the space constraints.

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