APPROXIMATE AND EXACT CORRECTIONS OF THE BIAS IN CROSTON'S METHOD WHEN FORECASTING LUMPY DEMAND: EMPIRICAL EVALUATION

Adriano O. Solis[a], Francesco Longo[b], Somnath Mukhopadhyay[c], Letizia Nicoletti[d], Vittoria Brasacchio[e]

[a] School of Administrative Studies, Faculty of Liberal Arts & Professional Studies, York University, Toronto, Canada
[b] [d] [e] Department of Mechanical, Energy and Management Engineering, University of Calabria, Rende, Italy
[c] College of Business Administration, University of Texas at El Paso, El Paso, Texas, USA

[a] asolis@yorku.ca, [b] francesco.longo@unical.it, [c] smukhopadhyhyay@utep.edu, [d] letizia.nicoletti@unical.it,
[e] vittoria.brasacchio@gmail.com

ABSTRACT
Croston's method was developed to forecast intermittent demand, employing separate exponential smoothing estimates of the average demand size and the average interval between demand occurrences. Syntetos and Boylan reported an error in Croston's mathematical derivation of expected demand, and proposed an approximate correction now usually referred to as the Syntetos-Boylan approximation. Subsequently, Shale, Boylan, and Johnston derived the expected bias in Croston's method and proposed an 'exact' correction factor. Both approximate and exact corrections have been derived analytically. In the current study, we empirically investigate whether or not there are actually significant improvements in terms of statistical forecast accuracy as well as inventory control performance obtained by applying the approximate or exact correction.

Keywords: intermittent/lumpy demand forecasting, forecast accuracy, bias correction, inventory control, modeling and simulation

1. INTRODUCTION

Demand for an item of inventory is *intermittent* when there are time intervals in which there are no demand occurrences. Intermittent demand is said to be *lumpy* when there are large variations in the sizes of actual demand occurrences.

Croston (1972) noted that under simple exponential smoothing (SES), which has frequently been used for forecasting demand, a biased estimate arises since forecasts are based on an average of the recent demand occurrences. This bias is greatest immediately following a demand occurrence. Because inventory replenishment decisions are usually taken after a reduction in stock, there can be serious consequences of an upward bias in the demand forecast. To address this upward bias, Croston proposed a method of forecasting intermittent demand using separate exponential smoothing estimates of the average demand size and the average interval between demand occurrences, and combining these to obtain a demand forecast. Leading statistical forecasting software packages include Croston's method (Syntetos and Boylan 2005; Boylan and Syntetos 2007).

While Croston applied a single smoothing constant α, Schultz (1987) proposed the use of separate smoothing constants, α_i and α_s, in updating inter-demand intervals and nonzero demand sizes, respectively. However, Mukhopadhyay, Solis, and Gutierrez (2012) investigated separate smoothing constants, α_i and α_s, in forecasting lumpy demand and reported no substantial improvement in forecast accuracy.

Syntetos and Boylan (2001) established the presence of a positive bias in Croston's method, called an 'inversion bias', arising from an error in Croston's mathematical derivation of expected demand. Syntetos and Boylan (2005) proposed a correction factor of $[1-(\alpha/2)]$ applied to Croston's original estimator of mean demand, where α is the smoothing constant in use for updating the inter-demand intervals. The revised estimator yields an *approximately* unbiased estimator, and is now usually referred to as the Syntetos-Boylan approximation (SBA) in the literature on intermittent demand forecasting (e.g., Gutierrez, Solis, and Mukhopadhyay 2008; Boylan, Syntetos, and Karakostas 2008; Babai, Syntetos, and Teunter 2010; Mukhopadhyay, Solis, and Gutierrez 2012).

Levén and Segerstedt (2004) proposed a modification to Croston's method, which they called a 'modified Croston procedure', involving a new method for estimating the mean and variance of the forecasted demand rate. Boylan and Syntetos (2007), however, found that the smoothing method for estimating the variance is based on an invalid forecast accuracy measure, and that the new method of estimating mean demand produces biased forecasts.

Shale, Boylan, and Johnston (2006) derived the expected bias when the arrival of orders follows a Poisson process, and extended their work to other inter-arrival distributions. They specified $[1-(\alpha/(2-\alpha))]$ as an 'exact' correction factor (hereafter referred to in this paper as SBJ) to remove the inversion bias in Croston's method. However, the SBJ method has not been cited or applied in recent intermittent demand forecasting literature. For instance, Syntetos and Boylan (2010) analyze the "most well-cited intermittent demand estimation procedures" with respect to the variance of their estimates, but do not even mention SBJ. Only the variances of demand estimates using SES, Croston's method, SBA, and an "exactly unbiased modification" of Croston's method (Syntetos 2001) are reported.

A critical issue in forecasting intermittent/lumpy demand is the assumption of a distribution of demand occurrence. Boylan (1997) proposed three criteria for assessing the suitability of demand distributions:

1. *A priori* grounds for modelling demand.
2. The flexibility of the distribution to represent different types of demand.
3. Empirical evidence.

Syntetos and Boylan (2006) argued that compound distributions can represent demand incidence and demand size by separate distributions. Noting that the negative binomial distribution (NBD) is a compound distribution with variance greater than the mean, with "empirical evidence in its support (Kwan 1991)," Syntetos and Boylan declared the NBD to meet all the above three criteria. They accordingly selected the NBD to represent intermittent demand over lead time (plus review period) in their stock control simulation model. Among others, Boylan, Syntetos, and Karakostas (2008) and Syntetos, Babai, Dallery, and Teunter (2009) have also conducted empirical investigations of stock control using the NBD to characterize intermittent demand over the lead time (plus review period). These latter studies have cited Syntetos and Boylan's (2006) declaration that the NBD "satisfies both theoretical and empirical criteria."

Use of the NBD to characterize demand may indeed have been found by previous researchers to apply to intermittent (but not very erratic) demand. Our investigations, however, show that the NBD may not hold for much of lumpy demand distributions. In the current study, we use a two-stage simulation approach to characterize lumpy demand, as earlier discussed and applied, for instance, by Solis, Longo, Nicoletti, and Yemialyanava (2013). The first stage uses a uniform distribution, with probability z_1 of zero demand, to determine whether or not a demand occurs in the given period. If the first stage leads to a demand actually occurring in the period, the second stage estimates the demand size using an NBD. In the process, $\Pr(X=0)$ in the NBD is applied to adjust the probability z_1 of zero demand in the first stage.

It should be emphasized that this two-stage approach was not intended to "accurately" capture the actual lumpy demand distribution. The objective is to simulate demand distributions that mimic as closely as possible the lumpy demand distributions. We, therefore, address in this study the apparent inadequacy of the NBD, in spite of its "satisfying both theoretical and empirical criteria," for characterizing intermittent demand. As in Solis, Longo, Nicoletti, and Yemialyanava (2013), we illustrate how the two-stage simulation process better characterizes a greater proportion of lumpy demand distributions we investigate.

In this paper, we report on the preliminary results of our empirical investigation of both statistical accuracy and inventory control performance of four intermittent demand forecasting methods which employ exponential smoothing: SES, Croston's, SBA, and SBJ. We evaluate the improvements associated with applying approximate and exact corrections (SBA and SBJ, respectively) which address the positive bias in Croston's method. We first seek to characterize lumpy (i.e., both intermittent *and* erratic) demand by applying the NBD or two-stage approximations. We proceed to evaluate forecast accuracy using a number of error statistics, and then consider inventory control performance.

This paper is organized as follows. In section 2, we discuss the industrial dataset and how data partitioning is conducted for purposes of empirical evaluation, as well our application of the NBD and two-stage approximations to characterize demand. In the next section, we first discuss the statistical measures of forecast accuracy that we use, and proceed to report on our empirical investigation of forecasting performance on the performance blocks of actual data as well as on the simulated demand distributions. In section 4, we report on our empirical investigation of inventory control performance. We present our conclusions in the final section.

2. INDUSTRIAL DATASET AND DEMAND CHARACTERIZATION

2.1. Industrial Dataset and Partitioning

In this paper, we apply the SES, Croston's, SBA, and SBJ methods to stock-keeping units (SKUs) in a regional warehouse of a firm operating in the professional electronics sector. The SKUs represent end items, sub-assemblies, components, and spare parts that are used for building projects, retail sales, or servicing of professional electronic products. The raw data consist of actual withdrawals from stock as reported in the company's enterprise resource planning system over a period of 61 months. The transactional data are aggregated into usage quantities per month, which we treat as a surrogate measure of monthly demand while recognizing that the inventory on hand when a demand occurs may not meet the required quantity. The 61 months of "demand" data are divided into initialization, calibration, and performance measurement blocks (as in Boylan, Syntetos, and Karakostas 2008) with our blocks consisting of 20, 20, and 21 months, respectively.

Syntetos, Boylan, and Croston (2005) proposed a scheme to classify demand patterns into four categories (smooth, erratic, intermittent, and lumpy) for the purpose of establishing 'regions' of superior forecasting performance between Croston's method and SBA. The scheme (hereafter referred to as SBC) is based on the use of two statistics: CV^2 and *ADI*, the squared coefficient of variation of demand and average inter-demand interval, respectively. The four categories are delimited by cutoff values for CV^2 and *ADI* as follows: (i) *smooth*, with *ADI* < 1.32 and CV^2 < 0.49; (ii) *erratic*, with *ADI* < 1.32 and CV^2 > 0.49; (iii) *intermittent*, with *ADI* > 1.32 and CV^2 < 0.49; and (iv) *lumpy*, with *ADI* > 1.32 and CV^2 > 0.49. Recently, Heinecke, Syntetos, and Wang (2013) empirically evaluated the SBC cutoff values for CV^2 and *ADI* in comparison with alternative approaches proposed by Kostenko and Hyndman (2006), and found that SBC results in inferior forecasting performance overall in comparison with the latter alternatives. For purposes of the current study, we will nonetheless continue to apply the relatively simple SBC fixed cutoff values to classify demand patterns. These cutoff values and resulting categories have been cited in various other studies involving intermittent or lumpy demand (e.g., Ghobbar and Friend 2002, 2003; Gutierrez, Solis, and Mukhopadhyay 2008; Altay, Rudisill, and Litteral 2008; Boylan, Syntetos, and Karakostas 2008; Mukhopadhyay, Solis, and Gutierrez 2012).

In this paper, we report findings on a set of ten SKUs which have thus far been evaluated for purposes of the current study. Demand statistics are presented in Table 1. All 10 SKUs exhibit lumpy demand (*ADI* > 1.32 and CV^2 > 0.49) according to the SBC categorization scheme. We have yet to find a SKU with intermittent demand (*ADI* > 1.32 and CV^2 < 0.49) according to the scheme. [Solis, Longo, Nicoletti, and Yemialyanava (2013) earlier investigated a separate set of SKUs (nine with lumpy demand, and six with erratic demand) coming from the firm's central warehouse. That earlier study also evaluated the simple moving average method, but did not consider SBJ. Hence, no comparison was made between the approximate and exact corrections of the bias in Croston's method, which is the focus of the current study.]

Table 1: 10 SKUs with Lumpy Demand

SKU #	**1**	**2**	**3**	**4**	**5**
Mean	2.0492	1.0656	2.4918	1.9672	2.7541
Std Dev	2.8427	1.1954	3.2641	3.0549	4.2531
CV^2	1.9244	1.2585	1.7159	2.4115	2.3848
ADI	1.4186	1.6486	1.3864	1.7429	1.5641
z (% of Zero Demand)	31.15%	40.98%	29.51%	44.26%	37.70%
Mean Nonzero Demand	2.9762	1.8056	3.5349	3.5294	4.4211
Std Dev of Nonzero Demand	2.9999	1.0370	3.3831	3.3596	4.6652
SKU #	**6**	**7**	**8**	**9**	**10**
Mean	6.5410	2.5082	6.9016	3.4426	2.1639
Std Dev	9.1462	3.8454	10.1648	4.3609	2.8412
CV^2	1.9552	2.3506	2.1692	1.6046	1.7240
ADI	1.4878	1.5641	1.6053	1.6053	1.6486
z (% of Zero Demand)	34.43%	37.70%	39.34%	39.34%	40.98%
Mean Nonzero Demand	9.9750	4.0263	11.3784	5.6757	3.6667
Std Dev of Nonzero Demand	9.6728	4.2074	10.9477	4.3208	2.8586

2.2. Demand Characterization Using a Negative Binomial Distribution

As previously stated, Syntetos and Boylan (2006) declared that the NBD "satisfies both theoretical and empirical criteria" to characterize demand. It is a discrete probability distribution with density function

$$f(x;r,p) = \binom{r+x-1}{x} p^r (1-p)^x I_{\{0,1,2,\ldots\}}(x). \tag{1}$$

with two parameters *r* and *p*. The parameter *r* is a positive integer. The parameter *p* is a real number satisfying $0 < p \le 1$, and is a probability of "success" in a Bernoulli trial, while *r* is a target number of successes (e.g., Mood, Graybill, and Boes 1974). The random variable *X* represents the number of failures, in a succession of the Bernoulli trials, preceding the *r*th success. The NBD has mean

$$\mu = E[X] = \frac{r(1-p)}{p} \tag{2}$$

and variance

$$\sigma^2 = V[X] = \frac{r(1-p)}{p^2}. \tag{3}$$

Clearly, the variance of the NBD is greater than its mean. The NBD, when *r* = 1, reduces to a geometric (or Pascal) distribution with density function

$$f(x;p) = p(1-p)^x I_{\{0,1,2,\ldots\}}(x). \tag{4}$$

Solving (2) and (3) simultaneously, we obtain

$$\hat{p} = \frac{\mu}{\sigma^2} \tag{5}$$

and

$$\hat{r} = \frac{\mu^2}{\sigma^2 - \mu} \tag{6}$$

as initial estimates of the parameters of an NBD with mean μ and variance σ^2. We use the mean $\overline{x}$ and the variance s^2 of the 61-month demand time series in place of μ and σ^2, respectively. However, while *r* is supposed to be integer-valued, the expression (6) is real-valued. Thus, in attempting to characterize the actual demand distribution using an NBD approximation, we investigate rounded up and rounded down values of $\hat{r}$ while at the same time adjusting $\hat{p}$ to obtain acceptable NBD parameters *r* and *p*.

We used AnyLogic as our simulation platform. To address mathematical modeling not doable within the AnyLogic standard library, some code was written in Java. For the NBD parameters tested, we performed 100 runs each consisting of 100 months (for a total of 10,000 months) in each experiment.

As a rule of thumb, we operationalize 'reasonably acceptable' approximation in terms of mean, standard deviation, CV^2, and *ADI* of the simulated distribution all being within ±20% of those of the actual demand distribution and with a fairly small difference between simulated and actual proportion z of zero demand.

In each of SKUs 1, 2 and 3, we found the suggested NBD approximation to yield a simulated distribution which fairly closely resembles the actual demand distribution. The simulation results are reported in Table 2. For SKUs 1 and 3, the adjusted r value is 1 (i.e., the NBD reduces to a geometric distribution).

Table 2: Three SKUs with Reasonably Acceptable NBD Approximations

SKU #	1	2	3
Mean	2.0492	1.0656	2.4918
Std Dev	2.8427	1.1954	3.2641
CV^2	1.9244	1.2585	1.7159
ADI	1.4186	1.6486	1.3864
z (% of Zero Demand)	31.15%	40.98%	29.51%
*r^*	0.6962	3.1246	0.7607
*p^*	0.2536	0.7457	0.2339
SIMULATION	**NBD**	**NBD**	**NBD**
r	1	4	1
p	0.3458	0.7897	0.2904
Mean	1.9053	1.0567	2.4137
Std Dev	2.3690	1.1648	2.8935
CV^2	1.5460	1.2151	1.4371
ADI	1.5249	1.6464	1.4100
z (% of Zero Demand)	34.43%	39.27%	29.09%
Simulated vs Actual Mean	93.0%	99.2%	96.9%
Simulated vs Actual Std Dev	83.3%	97.4%	88.6%
Simulated vs Actual CV^2	80.3%	96.6%	83.7%
Simulated vs Actual ADI	107.5%	99.9%	101.7%
Difference in Simulated vs Actual z	3.28%	-1.71%	-0.42%

2.3. Demand Characterization Using a Two-Stage Distribution

For the seven other SKUs (4-10), we were unable to find NBD approximations that are reasonably acceptable (as set forth in the previous sub-section).

We briefly summarize the alternative, two-stage distribution as applied by Solis, Longo, Nicoletti, and Yemialyanava (2013). In each time period, the first stage is based on applying the continuous uniform distribution defined over the real number interval (0,1). Stage 1 can be viewed as a Bernoulli process, which has a fixed probability of “success” or “failure”. It determines whether or not a demand occurs. If the random number generated in stage 1 is less than or equal to z_1 (a “failure”), demand for the period is set equal to zero and the demand generation process moves on to the next period. If the random number generated in stage 1 is greater than z_1 (a “success”), the demand generation process moves to stage 2 in which an NBD is used to simulate the demand size. It follows that there will still be some probability of zero demand in stage 2. At the conclusion of stage 2, the demand generation process moves to the next time period, again starting with stage 1 of the two-stage process.

To estimate the parameters of the NBD in stage 2 of the demand simulation process, the mean $\bar{x}_{nz}$ and variance s_{nz}^2 of the nonzero demands are calculated and used to obtain first approximations of the parameters $\hat{p}_{nz}$ and $\hat{r}_{nz}$ in line with (5) and (6). We round up or down $\hat{r}_{nz}$ to some integer value and adjust $\hat{p}_{nz}$ accordingly. The corresponding negative binomial probability $P_0 = \Pr(X = 0) > 0$ is then used to find z_1 (as applied in the first stage), as follows:

$$z_1 = \frac{z - P_0}{1 - P_0}, \quad (7)$$

provided $z > P_0$. The resulting proportion of zero demand periods arising from the two-stage distribution is then closer to z. We refine the parameter estimate $\hat{p}_{nz}$ while the mean, standard deviation, CV^2, *ADI*, and z of the actual and simulated distributions are compared.

We note that this two-stage approach did not yield acceptable characterizations of demand for SKUs 1 and 2; only the NBD approximations for these two SKUs (as presented in Table 2) were reasonably close to actual demand statistics. In the case of SKU 3, on the other hand, both NBD and two-stage approximations are both fairly close to the actual demand distribution (refer to Table 3), but with the two-stage approximation appearing to yield a somewhat better characterization.

Table 3: SKU 3 – Comparison of NBD and Two-Stage Approximations

SKU #	3	
Mean	2.4918	
Std Dev	3.2641	
CV^2	1.7159	
ADI	1.3864	
z (% of Zero Demand)	29.51%	
*r^*	0.7607	
*p^*	0.2339	
SIMULATION	**NBD**	**Two-Stage**
r	1	1
p	0.2904	0.2602
Mean	2.4137	2.6693
Std Dev	2.8935	3.2648
CV^2	1.4371	1.4960
ADI	1.4100	1.4186
z (% of Zero Demand)	29.09%	29.52%
Simulated vs Actual Mean	96.9%	107.1%
Simulated vs Actual Std Dev	88.6%	100.0%
Simulated vs Actual CV^2	83.7%	87.2%
Simulated vs Actual ADI	101.7%	102.3%
Difference in Simulated vs Actual z	-0.42%	0.01%

We present in Table 4 the simulation results for the demand distribution approximations, using the NBD approximation for SKUs 1 and 2 and the two-stage approach for SKUs 3-10.

We have yet to evaluate a SKU in the current dataset for which neither approximation method leads to an acceptable characterization, albeit with only 10 SKUs evaluated thus far. [We must quickly point out, however, that Solis, Longo, Nicoletti, and Yemialyanava (2013) reported that both the NBD and two-stage approximations fail in the case of SKUs with demand distributions that are lumpier.]

3. EMPIRICAL INVESTIGATION OF FORECASTING PERFORMANCE

3.1. Smoothing Constants and Forecast Accuracy Measures

In the context of intermittent demand, low values of the smoothing constant α have been recommended, and values in the range 0.05-0.20 are considered realistic (Croston 1972; Willemain, Smart, Shockor, and DeSautels 1994; Johnston and Boylan 1996). We test four α values: 0.05, 0.10, 0.15, and 0.20 (as in Syntetos and Boylan 2005, 2006; Gutierrez, Solis, and Mukhopadhyay 2008; Mukhopadhyay, Solis, and Gutierrez 2012).

Table 4: Lumpy Demand Approximations

SKU #	1	2	3	4	5
Mean	2.0492	1.0656	2.4918	1.9672	2.7541
Std Dev	2.8427	1.1954	3.2641	3.0549	4.2531
CV^2	1.9244	1.2585	1.7159	2.4115	2.3848
ADI	1.4186	1.6486	1.3864	1.7429	1.5641
z (% of Zero Demand)	31.15%	40.98%	29.51%	44.26%	37.70%
r^	0.6962	3.1246	0.7607	0.5254	0.4946
p^	0.2536	0.7457	0.2339	0.2108	0.1523
Mean of nonzero demand	2.9762	1.8056	3.5349	3.5294	4.4211
Std Dev of nonzero demand	2.9999	1.0370	3.3831	3.3596	4.6652
r^ nonzero	-	-	1.5796	1.6058	1.1270
p^ nonzero	-	-	0.3089	0.3127	0.2031
SIMULATION	**NBD**	**NBD**	**Two-Stage**	**Two-Stage**	**Two-Stage**
r	1	4	1	1	1
p	0.3458	0.7897	0.2602	0.2726	0.1987
Pr(*X* = 0)	0.3458	0.3889	0.2602	0.2726	0.1987
Final zero proportion in stage 1	-	-	4.72%	23.37%	22.26%
Mean	1.9053	1.0534	2.6693	1.9950	3.1393
Std Dev	2.3690	1.1959	3.2648	2.9137	4.4296
CV^2	1.5460	1.2888	1.4960	2.1331	1.9910
ADI	1.5249	1.7074	1.4186	1.7989	1.6124
z (% of Zero Demand)	34.43%	41.44%	29.52%	44.42%	37.99%
Simulated vs Actual Mean	93.0%	98.9%	107.1%	101.4%	114.0%
Simulated vs Actual Std Dev	83.3%	100.0%	100.0%	95.4%	104.2%
Simulated vs Actual CV^2	80.3%	102.4%	87.2%	88.5%	83.5%
Simulated vs Actual ADI	107.5%	103.6%	102.3%	103.2%	103.1%
D in Simulated vs Actual z	3.28%	0.46%	0.01%	0.16%	0.29%
SKU #	**6**	**7**	**8**	**9**	**10**
Mean	6.5410	2.5082	6.9016	3.4426	2.1639
Std Dev	9.1462	3.8454	10.1648	4.3609	2.8412
CV^2	1.9552	2.3506	2.1692	1.6046	1.7240
ADI	1.4878	1.5641	1.6053	1.6053	1.6486
z (% of Zero Demand)	34.43%	37.70%	39.34%	39.34%	40.98%
r^	0.5548	0.5123	0.4940	0.7609	0.7925
p^	0.0782	0.1696	0.0668	0.1810	0.2681
Mean of nonzero demand	9.9750	4.0263	11.3784	5.6757	3.6667
Std Dev of nonzero demand	9.6728	4.2074	10.9477	4.3208	2.8586
r^ nonzero	1.1904	1.1854	1.1935	2.4791	2.9845
p^ nonzero	0.1066	0.2274	0.0949	0.3040	0.4487
SIMULATION	**Two-Stage**	**Two-Stage**	**Two-Stage**	**Two-Stage**	**Two-Stage**
r	1	1	1	2	2
p	0.0973	0.2073	0.0877	0.2754	0.3868
Pr(*X* = 0)	0.0973	0.2073	0.0877	0.0758	0.1496
Final zero proportion in stage 1	27.36%	21.41%	33.51%	34.37%	30.60%
Mean	6.6577	3.0033	6.8982	3.4690	2.1988
Std Dev	9.1379	4.1380	10.2014	4.3599	2.7861
CV^2	1.8838	1.8984	2.1870	1.5796	1.6056
ADI	1.5108	1.6028	1.6491	1.6548	1.6998
z (% of Zero Demand)	33.82%	37.62%	39.37%	39.58%	41.18%
Simulated vs Actual Mean	101.8%	119.7%	100.0%	100.8%	101.6%
Simulated vs Actual Std Dev	99.9%	107.6%	100.4%	100.0%	98.1%
Simulated vs Actual CV^2	96.4%	80.8%	100.8%	98.4%	93.1%
Simulated vs Actual ADI	101.5%	102.5%	102.7%	103.1%	103.1%
D in Simulated vs Actual z	-0.61%	-0.08%	0.03%	0.24%	0.20%

To compare intermittent demand forecasting methods, Eaves and Kingsman (2004) used three traditional measures of accuracy: *mean absolute deviation* (MAD), *root mean squared error* (RMSE), and *mean absolute percentage error* (MAPE). We apply these three, as well as two additional error statistics.

MAPE is the most widely used scale-free forecast accuracy measure. The traditional MAPE definition fails when demand is intermittent, due to division by zero. We use an alternative specification of MAPE as a ratio estimate (e.g., Gilliland 2002):

$$\text{MAPE} = \left(\sum_{t=1}^{n} |E_t| \Big/ \sum_{t=1}^{n} A_t \right) \times 100 \cdot \qquad (8)$$

We use as a second scale-free error statistic the *mean absolute scaled error* (MASE), which was proposed by Hyndman and Koehler (2006). It uses the in-sample mean absolute error from the naïve forecast as a benchmark. The scaled error for period *t* is

$$q_t = \frac{e_t}{\left(\sum_{i=2}^{n} |Y_i - Y_{i-1}|\right) \Big/ (n-1)} \cdot \qquad (9)$$

MASE is calculated as follows:

$$\text{MASE} = \text{mean}\left(|q_t|\right). \qquad (10)$$

If MASE is less than one, the forecasting method being considered performs better than the in-sample naïve forecasts. In comparing different forecasting methods, a smaller MASE indicates better performance.

A third scale-free error statistic that we used is *percentage best* (PB), which refers to the percentage of time periods in which one particular method outperforms all of the other methods with respect to a specified criterion. We applied smallest absolute error as performance criterion. PB has been used in previous intermittent demand forecasting studies (e.g., Syntetos and Boylan 2005, 2006; Gutierrez, Solis, and Mukhopadhyay 2008; Mukhopadhyay, Solis, and Gutierrez 2012).

3.2. Forecast Accuracy: Performance Block

The exponential smoothing constant α was selected from among the candidate values (0.05, 0.10, 0.15, or 0.20) for each of the SES, Croston's, SBA and SBJ methods, taking into consideration four forecast accuracy measures (RMSE, MAD, MAPE, and MASE). We did not consider PB, as the 20 months in the calibration block are apparently insufficient to apply PB as an appropriate accuracy measure. For all ten SKUs thus far evaluated, minimum values of MAD, MAPE, and MASE were consistently associated with the same α values. We accordingly selected α values (as reported in Table 5) based upon minimum MAD, MAPE, and MASE in the calibration block. (In the case of five SKUs, the minimum RMSE actually yielded α values consistent with the other three error measures.)

We proceeded to calculate the resulting error statistics (MAD, MAPE, and MASE) when applying SES, Croston's, SBA and SBJ methods to actual demand data in the performance block (the final 21 months). These error statistics are summarized in Table 5. SES resulted in the best forecast accuracy for SKU 10. For the other nine SKUs, SBJ and SBA resulted in the best error statistics. As expected, SBJ yielded "slightly better" accuracy measures, though generally only at the fourth or fifth significant digit. The improvement arising from SBJ's 'exact' correction over SBA's approximate correction factor is, therefore, hardly significant.

3.3. Forecast Accuracy: Simulated Demand

In evaluating forecast accuracy over the 10,000 months of simulated demand (100 runs of 100 months each), we found a more pronounced overall superiority of SBJ and SBA over SES and Croston's methods using the three scale-free error statistics (MAPE, MASE, and PB). These error statistics are summarized in Table 6. Once again, SBJ yielded "better" accuracy measures only generally at the fourth or fifth significant digit. The improvement arising from SBJ's 'exact' correction over SBA's approximate correction factor is, therefore, hardly significant.

Table 5: Error Statistics when Applying Forecasting Methods to Actual Demand in the Performance Block

SKU #	1	2	3	4	5
Smoothing Constants Selected in Calibration Block					
SES	0.05	0.05	0.05	0.05	0.05
Croston	0.05	0.05	0.05	0.05	0.05
SBA	0.05	0.05	0.05	0.05	0.05
SBJ	0.05	0.05	0.05	0.05	0.05
MAD					
SES	1.674	0.670	2.223	2.261	1.909
Croston	1.637	0.649	2.179	2.211	2.029
SBA	1.625	0.652	2.158	2.202	1.982
SBJ	1.625	0.652	2.157	2.202	1.980
Best MAD	SBJ/SBA	SBJ/SBA	SBJ	SBJ/SBA	SBJ
MAPE					
SES	95.0%	87.9%	101.5%	101.0%	174.3%
Croston	92.9%	85.2%	99.5%	98.8%	185.2%
SBA	92.2%	85.5%	98.5%	98.4%	180.9%
SBJ	92.2%	85.5%	98.5%	98.4%	180.8%
Best MAPE	SBJ/SBA	SBJ/SBA	SBJ/SBA	SBJ/SBA	SBJ
MASE					
SES	0.676	0.670	0.765	0.669	1.336
Croston	0.661	0.649	0.750	0.654	1.420
SBA	0.656	0.652	0.743	0.651	1.387
SBJ	0.656	0.652	0.743	0.651	1.386
Best MASE	SBJ/SBA	SBJ/SBA	SBJ/SBA	SBJ/SBA	SBJ
SKU #	**6**	**7**	**8**	**9**	**10**
Smoothing Constants Selected in Calibration Block					
SES	0.05	0.05	0.05	0.2	0.05
Croston	0.05	0.05	0.05	0.05	0.2
SBA	0.05	0.05	0.05	0.05	0.2
SBJ	0.05	0.05	0.05	0.05	0.2
MAD					
SES	8.152	2.079	7.538	2.268	2.600
Croston	7.980	2.032	7.164	2.287	2.787
SBA	7.936	1.991	7.064	2.263	2.718
SBJ	7.935	1.990	7.062	2.263	2.710
Best MAD	SBJ	SBJ	SBJ	SBJ/SBA	SES
MAPE					
SES	113.4%	121.3%	135.3%	103.5%	101.1%
Croston	111.0%	118.5%	128.6%	104.4%	108.4%
SBA	110.4%	116.1%	126.8%	103.3%	105.7%
SBJ	110.4%	116.1%	126.8%	103.3%	105.4%
Best MAPE	SBJ/SBA	SBJ/SBA	SBJ/SBA	SBJ/SBA	SES
MASE					
SES	0.751	1.149	1.319	0.744	0.853
Croston	0.735	1.123	1.254	0.751	0.915
SBA	0.731	1.100	1.236	0.743	0.892
SBJ	0.731	1.100	1.236	0.742	0.889
Best MASE	SBJ/SBA	SBJ/SBA	SBJ/SBA	SBJ	SES

4. EMPIRICAL INVESTIGATION OF INVENTORY CONTROL PERFORMANCE

Recent studies that have looked into both forecast accuracy and inventory control performance of intermittent demand forecasting studies have applied a (T,S) periodic review system, where T and S denote the review period and the base stock (or 'order-up-to' level), respectively. These include Eaves and Kingsman (2004), Syntetos and Boylan (2006), Syntetos, Nikolopoulos, Boylan, Fildes, and Goodwin (2009), Syntetos, Babai, Dallery, and Teunter (2009), Syntetos, Nikolopoulos, and Boylan (2010), and Teunter, Syntetos, and Babai (2010).

We simulate the performance of a (T,S) inventory control system over the 10,000 months of simulated demand (100 runs of 100 months each) generated using the NBD or two-stage approximations. We assume full backordering, with inventory reviewed on a monthly basis (T = 1). The reorder lead time for most SKUs is about one month (L = 1).

Table 6: Error Statistics when Applying Forecasting Methods to the Simulated Demand Distributions

SKU #	1	2	3	4	5
Smoothing Constants Selected in Calibration Block					
SES	0.05	0.05	0.05	0.05	0.05
Croston	0.05	0.05	0.05	0.05	0.05
SBA	0.05	0.05	0.05	0.05	0.05
SBJ	0.05	0.05	0.05	0.05	0.05
MAPE					
SES	81.6%	84.5%	90.4%	106.5%	102.7%
Croston	81.8%	83.5%	89.6%	105.0%	102.6%
SBA	81.1%	83.3%	89.0%	104.3%	101.8%
SBJ	81.1%	83.3%	88.9%	104.3%	101.7%
Best MAPE	SBJ/SBA	SBJ/SBA	SBJ	SBJ/SBA	SBJ
MASE					
SES	0.777	0.759	0.761	0.788	0.778
Croston	0.779	0.750	0.755	0.777	0.777
SBA	0.773	0.748	0.750	0.772	0.771
SBJ	0.772	0.748	0.749	0.772	0.771
Best MASE	SBJ	SBJ/SBA	SBJ	SBJ/SBA	SBJ/SBA
PB					
SES	37.5%	34.6%	35.1%	38.5%	39.7%
Croston	20.5%	26.4%	15.4%	13.5%	20.6%
SBA	0.1%	1.2%	0.3%	0.4%	0.3%
SBJ	42.0%	37.8%	49.2%	47.7%	39.5%
Best PB	SBJ	SBJ	SBJ	SBJ	SES
SKU #	**6**	**7**	**8**	**9**	**10**
Smoothing Constants Selected in Calibration Block					
SES	0.05	0.05	0.05	0.2	0.05
Croston	0.05	0.05	0.05	0.05	0.2
SBA	0.05	0.05	0.05	0.05	0.2
SBJ	0.05	0.05	0.05	0.05	0.2
MAPE					
SES	100.9%	100.8%	109.8%	101.4%	98.5%
Croston	99.9%	99.9%	108.4%	97.9%	101.7%
SBA	99.2%	99.2%	107.5%	97.3%	99.1%
SBJ	99.2%	99.2%	107.5%	97.3%	98.8%
Best MAPE	SBJ/SBA	SBJ/SBA	SBJ/SBA	SBJ/SBA	SES
MASE					
SES	0.771	0.778	0.788	0.810	0.783
Croston	0.763	0.771	0.778	0.783	0.808
SBA	0.758	0.766	0.772	0.779	0.788
SBJ	0.758	0.766	0.772	0.779	0.786
Best MASE	SBJ/SBA	SBJ/SBA	SBJ/SBA	SBJ/SBA	SES
PB					
SES	12.4%	13.4%	12.7%	31.4%	29.4%
Croston	11.4%	12.8%	12.2%	18.3%	14.8%
SBA	0.2%	0.3%	0.2%	0.3%	0.7%
SBJ	49.3%	47.4%	47.4%	31.5%	38.4%
Best PB	SBJ	SBJ	SBJ	SBJ	SBJ

The literature suggests a safety stock component to compensate for uncertainty in demand during the 'protection interval' T+L. For each SKU, we calculated s_{tr}, the standard deviation of monthly demand during the 'training sample' (corresponding to the combined initialization and calibration blocks). We seek a safety stock level of $k \cdot s_{tr}$, with 'safety factor' k. This approach is different from that suggested under an assumption that daily demand is identically and independently normally distributed during the protection interval (e.g., Silver, Pyke, and Peterson 1998). The replenishment quantity Q_t at the time of review is

$$Q_t = (T + L) \cdot F_t + k \cdot s_{tr} - I_t + B_t, \tag{11}$$

where F_t is the forecast calculated at the end of month t, and I_t and B_t are, respectively, on-hand inventory and backlog.

4.1. Service Levels

According to Silver, Pyke, and Peterson (1998), the two most commonly specified service level criteria in inventory systems are:

- Probability of no stockout (PNS) per review period, and
- Fill rate (FR), the average percentage of demand to be satisfied from on-hand inventory.

FR is considered to have considerably more appeal for practitioners.

We used two values of the target PNS (90% and 95%) and two values of the target FR (95% and 98%) in simulating inventory control performance. These target PNS and FR values are comparable with 'reasonable' levels tested in inventory systems studies – for instance, 80%, 90%, 95%, or 97.5% for PNS and 95%, 98%, 99% or 99.9% for FR (Solis, Longo, Nicoletti, Caruso, and Fazzari 2014). We performed simulation searches to find the safety factor k that would yield the target PNS or FR.

4.2. Average Inventory on Hand

For a 95% target FR, resulting averages of inventory on hand are reported in Table 7. We proceeded to index the average inventory on hand using Croston's method as base (Croston index = 100). These indices, reported in Table 8, are all very close to 100. In fact, the indices for SBA and SBJ differ by at most 0.1. Moreover, all SBA and SBJ indices are between 98.9 and 100.3, which indicate that average levels of inventory on hand do not differ much from those arising using Croston's method. Indices for a 98% target FR are all even closer to 100.

In the case of a 90% target PNS, average inventory on hand levels are reported in Table 9. In applying an index of 100 to average inventory on hand under Croston's method (see Table 10), we find the resulting indices for SBA and SBJ to be roughly equal for each of the 10 SKUs. These SBA and SBJ indices are all very close to 100 (between 99.8 and 100.4). With a 95% target PNS, the SBA and SBJ indices all fall between 99.9 and 100.7, and are again roughly equal for each SKU.

Table 7: Average Inventory on Hand for a 95% Target Fill Rate

SKU #	1	2	3	4	5
SES	5.3849	2.1224	7.4255	7.6331	11.3890
Croston	5.3636	2.1561	7.4242	7.6112	11.3918
SBA	5.3618	2.1335	7.4268	7.6104	11.4288
SBJ	5.3625	2.1322	7.4273	7.6101	11.4271

SKU #	6	7	8	9	10
SES	22.5350	10.2828	27.3020	9.3131	5.9441
Croston	22.5247	10.2460	27.2997	9.2100	6.0214
SBA	22.5171	10.2382	27.2846	9.2028	5.9952
SBJ	22.5207	10.2404	27.2797	9.2028	5.9916

Table 8: Indices of Average Inventory on Hand for a 95% Target Fill Rate

SKU #	1	2	3	4	5
SES	100.4	98.4	100.0	100.3	100.0
Croston	100.0	100.0	100.0	100.0	100.0
SBA	100.0	99.0	100.0	100.0	100.3
SBJ	100.0	98.9	100.0	100.0	100.3

SKU #	6	7	8	9	10
SES	100.0	100.4	100.0	101.1	98.7
Croston	100.0	100.0	100.0	100.0	100.0
SBA	100.0	99.9	99.9	99.9	99.6
SBJ	100.0	99.9	99.9	99.9	99.5

Table 9: Average Inventory on Hand for a 90% Target Probability of No Stockout

SKU #	1	2	3	4	5
SES	2.9487	1.2462	4.0906	3.5699	5.7023
Croston	2.9623	1.2558	4.0598	3.5567	5.6738
SBA	2.9616	1.2557	4.0614	3.5520	5.6752
SBJ	2.9616	1.2559	4.0614	3.5530	5.6723

SKU #	6	7	8	9	10
SES	12.3393	5.3399	13.7299	6.1841	3.6508
Croston	12.2887	5.3147	13.6642	6.0096	3.6979
SBA	12.3222	5.3185	13.6631	5.9979	3.7144
SBJ	12.3241	5.3185	13.6692	5.9985	3.7122

Table 10: Indices of Average Inventory on Hand for a 90% Target Probability of No Stockout

SKU #	1	2	3	4	5
SES	99.5	99.2	100.8	100.4	100.5
Croston	100.0	100.0	100.0	100.0	100.0
SBA	100.0	100.0	100.0	99.9	100.0
SBJ	100.0	100.0	100.0	99.9	100.0

SKU #	6	7	8	9	10
SES	100.4	100.5	100.5	102.9	98.7
Croston	100.0	100.0	100.0	100.0	100.0
SBA	100.3	100.1	100.0	99.8	100.4
SBJ	100.3	100.1	100.0	99.8	100.4

4.3. Cumulative Backlogs

For reasonable target service levels, the occurrence of backlogs is minimized with the provision of safety stock levels. Therefore, reporting on average backlog per period will lead to averages of well under one unit. We accordingly record the cumulative backlogs over an entire 100-month simulation run.

Table 11 shows the average (across 100 replications) of the cumulative backlogs over 100-month intervals when the target FR is 98%. The absolute differences (with respect to results arising from the use of Croston's method) are all less than 0.1 unit, indicating that there is hardly any difference in performance with respect to 100-month cumulative backlogs for the given target FR. The same observation holds for a target FR of 95%.

Table 11: Mean 100-Month Backlogs for a 98% Target Fill Rate

SKU #	1	2	3	4	5
SES	3.86	2.14	5.41	3.98	6.45
Croston	3.87	2.14	5.43	3.98	6.47
SBA	3.88	2.14	5.44	3.99	6.48
SBJ	3.87	2.14	5.44	3.99	6.48
SKU #	**6**	**7**	**8**	**9**	**10**
SES	13.98	6.26	13.84	7.04	4.51
Croston	14.03	6.24	13.87	7.10	4.51
SBA	13.99	6.25	13.87	7.11	4.52
SBJ	14.01	6.26	13.89	7.09	4.52

Table 12 provides analogous results under a 95% target PNS. The absolute differences in average cumulative backlogs over 100-month intervals of SBA or SBJ with respect to those arising from the use of Croston's method are all less than 0.2 unit. The absolute differences between SES and Croston's method cumulative backlogs are all well under 1 unit. Essentially the same observations apply for a target PNS of 90%

Table 12: Mean 100-Month Backlogs for a 95% Target Probability of No Stockout

SKU #	1	2	3	4	5
SES	14.88	7.50	19.39	19.07	27.92
Croston	14.83	7.68	19.68	19.04	28.03
SBA	14.84	7.68	19.69	19.03	27.88
SBJ	14.84	7.68	19.69	19.03	27.91
SKU #	**6**	**7**	**8**	**9**	**10**
SES	50.97	24.60	59.48	20.00	14.64
Croston	51.43	24.52	58.85	20.56	14.69
SBA	51.35	24.46	58.93	20.44	14.56
SBJ	51.35	24.46	58.96	20.45	14.54

5. CONCLUSION

Croston's method (1972) was developed to forecast intermittent demand, employing separate exponential smoothing estimates of the average demand size and the average interval between demand occurrences. Syntetos and Boylan (2001) reported an error in Croston's mathematical derivation of expected demand, leading to a positive bias. Syntetos and Boylan (2005) then proposed an approximate correction, SBA. Subsequently, Shale, Boylan, and Johnston (2006) derived the expected bias in Croston's method and proposed an 'exact' correction factor, SBJ.

Both the approximate correction (SBA) and the exact correction (SBJ) have been derived analytically. In the current study, we empirically investigate, using an industrial dataset involving SKUs exhibiting lumpy demand, whether or not there are actually significant improvements in terms of statistical forecast accuracy as well as inventory control performance obtained by applying the approximate or exact correction. We evaluate SES (the original exponential smoothing method), Croston's method, SBA, and SBJ by way of modeling and simulation. This paper constitutes a very preliminary report, limited to ten SKUs, all exhibiting lumpy demand, that have thus far been subjected to extensive simulation experiments.

We first attempt to characterize lumpy demand using a suggested NBD approximation (Syntetos and Boylan 2006). Failing to find a reasonably acceptable NBD approximation, we try characterizing the demand distribution using an alternative two-stage simulation approach involving the continuous uniform distribution (stage 1) and the NBD (stage 2). The two-stage alternative allows us to better characterize demand for most of the 10 SKUs. Having characterized demand, we then simulate forecasting performance. Based on the 10 SKUs evaluated, we have, as expected, found overall superior forecast accuracy of the bias corrections (SBA and SBJ) over both Croston's method and SES. However, we have not found significant differences in forecast accuracy between the SBA (approximate) and SBJ (exact) corrections.

Moreover, in terms of inventory control performance, we have observed very minute differences in average inventory on hand and average cumulative backlogs.

We reiterate, nonetheless, that this is a very preliminary report based upon an investigation of 10 SKUs. In particular, Table 1 shows that the mean monthly demands range between 1.07 and 6.54 (with mean nonzero demands ranging between 1.81 and 9.98). We anticipate being able to report, by the time of the conference, on a larger set of SKUs which will also include some with higher mean monthly demands.

REFERENCES

Altay, N., Rudisill, F., Litteral, L.A., 2008. Adapting Wright's modification of Holt's method to forecasting intermittent demand. *International Journal of Production Economics*, 111 (2), 389-408.

Babai, M.Z., Syntetos, A.A., and Teunter, R., 2010. On the empirical performance of (*T*,*s*,*S*) heuristics. *European Journal of Operational Research*, 202 (2), 466-472.

Boylan, J.E., 1997. The centralization of inventory and the modelling of demand. Unpublished Ph.D. Thesis, University of Warwick, UK.

Boylan, J.E. and Syntetos, A.A., 2007. The accuracy of a modified Croston procedure. *International Journal of Production Economics*, 107 (2), 511-517.

Boylan, J.E., Syntetos, A.A., and Karakostas, G.C., 2008. Classification for forecasting and stock

control: a case study. *Journal of the Operational Research Society*, 59 (4), 473-481.
Croston, J.D., 1972. Forecasting and stock control for intermittent demands. *Operational Research Quarterly*, 23 (3), 289-304.
Eaves, A.H.C. and Kingsman, B.G., 2004. Forecasting for the ordering and stock-holding of spare parts. *Journal of the Operational Research Society*, 55 (4), 431-437.
Gilliland, M., 2002. Is forecasting a waste of time? *Supply Chain Management Review*, 6 (4), 16-23.
Ghobbar, A.A. and Friend, C.H., 2002. Sources of intermittent demand for aircraft spare parts within airline operations. *Journal of Air Transport Management*, 8 (4), 221-231.
Ghobbar, A.A. and Friend, C.H., 2003. Evaluation of forecasting methods for intermittent parts demand in the field of aviation: a predictive model. *Computers & Operations Research*, 30 (14), 2097-2114.
Gutierrez, R.S., Solis, A.O., and Mukhopadhyay, S., 2008. Lumpy demand forecasting using neural networks. *International Journal of Production Economics*, 111 (2), 409-420.
Heinecke, G., Syntetos, A.A., and Wang, W., 2013. Forecasting-based SKU classification. *International Journal of Production Economics*, 143 (2), 455-462.
Hyndman, R.J. and Koehler, A.B., 2006. Another look at measures of forecast accuracy. *International Journal of Forecasting*, 22 (4), 679-688.
Johnston, F.R. and Boylan, J.E., 1996. Forecasting for items with intermittent demand. *Journal of the Operational Research Society*, 47 (1), 113-121.
Kostenko, A.V. and Hyndman, R.J., 2006. A note on the categorization of demand patterns. *Journal of the Operational Research Society*, 57 (10), 1256-1257.
Kwan, H.W., 1991. On the demand distributions of slow moving items. Unpublished Ph.D. Thesis, Lancaster University, UK.
Levén, E. and Segerstedt, A., 2004. Inventory control with a modified Croston procedure and Erlang distribution. *International Journal of Production Economics*, 90 (3), 361-367.
Mood, A.M., Graybill, F.A., and Boes, D.C., 1974. *Introduction to the Theory of Statistics*, 3rd ed. New York, USA: Mc-Graw-Hill.
Mukhopadhyay, S., Solis, A.O., and Gutierrez, R.S., 2012. The Accuracy of Non-traditional versus Traditional Methods of Forecasting Lumpy Demand. *Journal of Forecasting*, 31 (8), 721-735.
Schultz CR., 1987. Forecasting and inventory control for sporadic demand under periodic review. *Journal of the Operational Research Society*, 38 (5), 453-458.
Shale, E.A., Boylan, J.E., and Johnston, F.R., 2006. Forecasting for intermittent demand: the estimation of an unbiased estimate. *Journal of the Operational Research Society*, 57 (5), 588-592.
Silver, E.A., Pyke, D.F., and Peterson, R., 1998. *Inventory Management and Production Planning and Scheduling*. New York, USA: John Wiley & Sons.
Solis, A.O., Longo, F., Nicoletti, L., and Yemialyanava, A., 2013. Applying modeling and simulation to evaluate statistical accuracy and inventory control performance of lumpy demand forecasting methods. *Proceedings of the 12th International Conference on Modeling & Applied Simulation*, 212-220, September 25-27, Athens, Greece.
Solis, A.O., Longo, F., Nicoletti, L., Caruso, P., and Fazzari, E., 2014. A modelling and simulation approach to assessment of a negative binomial approximation in a multi-echelon inventory system. *International Journal of Simulation and Process Modelling*, forthcoming.
Syntetos, A.A., 2001. Forecasting of intermittent demand. Unpublished Ph.D. Thesis, Brunel University of Buckinghamshire New University, UK.
Syntetos, A.A. and Boylan, J.E., 2001. On the bias of intermittent demand estimates. *International Journal of Production Economics*, 71 (1-3), 457-466.
Syntetos, A.A. and Boylan, J.E., 2005. The accuracy of intermittent demand estimates. *International Journal of Forecasting*, 21 (2), 303-314.
Syntetos, A.A. and Boylan, J.E., 2006. On the stock control performance of intermittent demand estimators. *International Journal of Production Economics*, 103 (1), 36-47.
Syntetos, A.A. and Boylan, J.E., 2010. On the variance of intermittent demand estimates. *International Journal of Production Economics*, 128 (2), 546-555.
Syntetos, A.A., Boylan, J.E., and Croston, J.D., 2005. On the categorization of demand patterns. *Journal of the Operational Research Society*, 56 (5), 495-503.
Syntetos, A.A., Babai, M.Z., Dallery, Y., and Teunter, R., 2009. Periodic control of intermittent demand items: theory and empirical analysis. *Journal of the Operational Research Society*, 60 (5), 611-618.
Syntetos, A.A., Nikolopoulos, K., and Boylan, J.E., 2010. Judging the judges through accuracy-implication metrics: The case of inventory forecasting. *International Journal of Forecasting*, 26 (1), 134-143.
Syntetos, A.A., Nikolopoulos, K., Boylan, J.E., Fildes, R., and Goodwin, P., 2009. The effects of integrating management judgement into intermittent demand forecasts. *International Journal of Production Economics*, 118 (1), 72-81.
Teunter, R.H., Syntetos, A.A., and Babai, M.Z., 2010. Determining order-up-to levels under periodic review for compound binomial (intermittent) demand. *European Journal of Operational Research*, 203 (3), 619-624.

Willemain T.R., Smart C.N., Shockor J.H., DeSautels P.A., 1994. Forecasting intermittent demand in manufacturing: a comparative evaluation of Croston's method. *International Journal of Forecasting*, 10 (4), 529-538.

AUTHORS BIOGRAPHY

Adriano O. Solis is Associate Professor of Logistics Management and Area Coordinator for Management Science at York University, Canada. After receiving BS, MS and MBA degrees from the University of the Philippines, he joined the Philippine operations of Philips Electronics where he became a Vice-President and Division Manager. He went on to obtain a Ph.D. degree in Management Science from the University of Alabama. He was previously Associate Professor of Operations and Supply Chain Management at the University of Texas at El Paso. He has been a Visiting Professor in the Department of Mechanical, Energy, and Management Engineering, at the University of Calabria, Italy. He has served as Program Chair of the 2013 Summer Computer Simulation Conference of the Society for Modeling & Simulation International.

Francesco Longo obtained his degree in Mechanical Engineering, *summa cum laude*, in 2002 and his Ph.D. in Mechanical Engineering in 2005 from the University of Calabria, Italy. He is currently an Assistant Professor in the Department of Mechanical, Energy, and Management Engineering at the University of Calabria, where he also serves as Director of the Modeling & Simulation Center – Laboratory of Enterprise Solutions (MSC-LES). He is an Associate Editor of *Simulation: Transactions of the Society for Modeling and Simulation International*. He has also been Guest Editor of two issues of *International Journal of Simulation and Process Modelling*. He has been active in the organization and management of a number of international conferences in modeling and simulation, serving as General Chair, Program Chair, Track Chair, and/or member of the International Program Committee.

Somnath Mukhopadhyay is an Associate Professor of Operations and Supply Chain Management in the College of Business Administration at the University of Texas at El Paso. He has published in numerous well-respected journals like *Decision Sciences*, *INFORMS Journal on Computing*, *Journal of Forecasting*, *Communications of the AIS*, *IEEE Transactions on Neural Networks*, *Neural Networks*, *Neural Computation*, *International Journal of Production Economics*, and *Journal of World Business*. He received MS and Ph.D. degrees in Management Science from Arizona State University. He was a visiting research assistant in the parallel distributed processing group of Stanford University. He has over 15 years of industry experience in building and implementing mathematical models.

Letizia Nicoletti is currently a PhD student in Management Engineering at the University of Calabria, Italy. Her research interests include modeling and simulation for inventory management, as well as for training in complex systems, specifically marine ports and container terminals. She also actively supports the organization of international conferences in the modeling and simulation area.

Vittoria Brasacchio is a student in the *Laurea Magistrale* (master's degree) program in Management Engineering at the University of Calabria's Department of Mechanical, Energy, and Management Engineering. She obtained her first degree in Management Engineering in 2012. She was recently an international visiting graduate student scholar at the School of Administrative Studies, York University, Canada.