

# APPLYING MODELING AND SIMULATION TO EVALUATE STATISTICAL ACCURACY AND INVENTORY CONTROL PERFORMANCE OF LUMPY DEMAND FORECASTING METHODS

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## ABSTRACT

A number of time series methods – 13-month simple moving average (SMA13), single exponential smoothing (SES), Croston’s method, and the Syntetos-Boylan approximation (SBA) – are well-referenced methods in the literature on intermittent or lumpy demand forecasting. We apply these four methods to an industrial dataset involving more than 1000 stock-keeping units (SKUs) in the central warehouse of a firm operating in the professional electronics sector. Earlier studies have argued that the negative binomial distribution (NBD) satisfies both theoretical and empirical criteria for modeling intermittent demand. We have found that the NBD often does not provide a good fit. We apply an alternative approach, using a two-stage distribution involving the uniform and negative binomial distributions, in modeling actual demand. We use modeling and simulation to evaluate the four methods in terms of statistical forecast accuracy and, more importantly, inventory system efficiency.

Keywords: lumpy demand forecasting, forecast accuracy, scale-free error statistics, inventory control, modeling and simulation

## 1. INTRODUCTION

When there are time intervals with no demand occurrences for an item of inventory, demand is said to be *intermittent*. Demand is *erratic* when there are large variations in the sizes of actual demand occurrences. Demand that is both intermittent and erratic is referred to as *lumpy* demand.

Syntetos, Boylan, and Croston (2005) proposed to categorize demand patterns into four classes using cutoff values of  $CV^2 = 0.49$  and  $ADI = 1.32$  (where  $CV^2$  and  $ADI$  are, respectively, the squared coefficient of variation of demand and average inter-demand interval) – for the stated purpose of assigning the best forecasting method. The four categories are (i) *smooth*, when  $ADI < 1.32$  and  $CV^2 < 0.49$ ; (ii) *erratic* (but not very intermittent), when  $ADI < 1.32$  and  $CV^2 > 0.49$ ; (iii) *intermittent* (but not very erratic) when  $ADI > 1.32$

and  $CV^2 < 0.49$ ; and (iv) *lumpy*, when  $ADI > 1.32$  and  $CV^2 > 0.49$ . These cutoff values and resulting categories have been cited in various other studies involving intermittent or lumpy demand (e.g., Ghobbar and Friend 2002, 2003; Gutierrez, Solis, and Mukhopadhyay 2008; Boylan, Syntetos, and Karakostas 2008; Mukhopadhyay, Solis, and Gutierrez 2012).

A number of studies (e.g., Syntetos and Boylan 2006; Boylan, Syntetos, and Karakostas 2008; Syntetos, Babai, Dallery, and Teunter 2009) have proposed using a negative binomial distribution (NBD) to model the demand distribution of an item exhibiting intermittent demand. The NBD is a discrete probability distribution which may be specified by the density function:

$$f(x; r, p) = \binom{r+x-1}{x} p^r (1-p)^x I_{\{0,1,2,\dots\}}(x), \quad (1)$$

having two parameters  $p$  and  $r$ , where the real number  $p$  satisfies  $0 < p \leq 1$  and  $r$  is a positive integer. The real number  $p$  is a probability of “success” in a Bernoulli trial, while  $r$  is a target number of successes (e.g., Feller 1957; Mood, Graybill, and Boes 1974). The random variable  $X$  in this case represents, in a succession of the Bernoulli trials, the number of failures preceding the  $r$ th success. The NBD has mean

$$\mu = E[X] = \frac{r(1-p)}{p} \quad (2)$$

and variance

$$\sigma^2 = V[X] = \frac{r(1-p)}{p^2}. \quad (3)$$

Since  $V[X] = E[X]/p$ , it follows that the variance of the NBD is greater than its mean. When  $r = 1$ , the NBD reduces to a geometric (or Pascal) distribution with discrete density function

$$f(x; p) = p(1-p)^x I_{\{0,1,2,\dots\}}(x). \quad (4)$$

Syntetos and Boylan (2006) have argued that the NBD satisfies both theoretical and empirical criteria.

To generate an NBD to approximate the distribution of a random variable with mean  $\mu$  and variance  $\sigma^2$ , we simultaneously solve (2) and (3) to obtain:

$$\hat{p} = \frac{\mu}{\sigma^2} \quad (5)$$

and

$$\hat{r} = \frac{\mu^2}{\sigma^2 - \mu}, \quad (6)$$

as initial estimates of the NBD parameters. These expressions for  $\hat{p}$  and  $\hat{r}$ , however, represent values in the set  $\mathfrak{R}$  of real numbers, while the NBD parameter  $r$  is supposed to be integer-valued. In applying (5) and (6), we generally obtain a non-integer value of  $\hat{r}$ . Thus, in seeking to simulate the actual demand distributions, we have investigated the rounded up and rounded down values of  $\hat{r}$  while adjusting the value of  $\hat{p}$ .

In the intermittent demand forecasting literature, many papers have been published on the relative performance with respect to statistical measures of accuracy of various forecasting methods, most notably simple exponential smoothing (SES), Croston's method (Croston 1972), and an estimator proposed by Syntetos and Boylan (2005). Schultz (1987) suggested that separate smoothing constants,  $\alpha_i$  and  $\alpha_s$ , be used for updating the inter-demand intervals and the nonzero demand sizes, respectively, in place of Croston's single smoothing constant  $\alpha$ . We note, however, that Mukhopadhyay, Solis, and Gutierrez (2012) investigated separate smoothing constants,  $\alpha_i$  and  $\alpha_s$ , in forecasting lumpy demand and did not observe any substantial improvement in forecast accuracy.

Syntetos and Boylan (2001) pointed out a positive bias in Croston's method arising from an error in his mathematical derivation of expected demand. They proposed (Syntetos and Boylan 2005) a correction factor of  $\left(1 - \frac{\alpha_i}{2}\right)$  – where  $\alpha_i$  is the smoothing constant used in updating the inter-demand interval estimate – to be applied to Croston's estimator of mean demand. The revised estimator is now often referred to (e.g., Gutierrez, Solis, and Mukhopadhyay 2008; Boylan, Syntetos, and Karakostas 2008; Babai, Syntetos, and Teunter 2010; Mukhopadhyay, Solis, and Gutierrez 2012) in the intermittent demand forecasting literature as the Syntetos-Boylan approximation (SBA).

We also evaluate the 13-month simple moving average (SMA13) method, which is based upon dividing the 52 weeks in a year into 13 four-week "months". SMA13 has been applied in a number of recent intermittent demand forecasting studies (e.g., Syntetos and Boylan 2005, 2006; Boylan, Syntetos, and

Karakostas 2008) in view of its being built into some commercially available forecasting software.

This paper is organized as follows. In section 2, we discuss the forecasting methods under evaluation, the statistical measures of forecast accuracy that we use, and the nature of the industrial dataset and how data partitioning is performed. In the next section, we propose a two-stage approach to the modeling of demand distribution. We proceed to report on our empirical investigation of forecasting performance, based upon statistical accuracy measures, on the performance block of the actual data and on the simulated demand distribution. The performance of the forecasting methods in terms of inventory systems efficiency is reported in section 4. We present our conclusions in the final section.

## 2. FORECASTING METHODS AND DEMAND DATA

### 2.1. Forecasting Methods and Accuracy Measures

Four methods that are well-referenced in the intermittent demand forecasting literature are evaluated in this paper: SMA13, SES, Croston's, and SBA. For the SES, Croston's, and SBA methods, low values of the exponential smoothing constant  $\alpha$  of up to 0.20 have generally been suggested for lumpy demand (e.g., Croston 1972; Johnston and Boylan 1996). We test four  $\alpha$  values of 0.05, 0.10, 0.15, and 0.20 as used in a number of recent studies (e.g., Syntetos and Boylan 2005; Gutierrez, Solis, and Mukhopadhyay 2008; Mukhopadhyay, Solis, and Gutierrez 2012).

In this paper, we apply three scale-free error statistics. The first is *mean absolute percentage error* (MAPE), which is the most widely used accuracy measure for ratio-scaled data. The traditional MAPE definition, which involves terms of the form  $|E_t|/A_t$  (where  $A_t$  and  $E_t$ , respectively, represent actual demand and forecast error in period  $t$ ), fails when demand is intermittent. We applied an alternative specification (e.g., Gilliland 2002) of MAPE as a ratio estimate, which guarantees a nonzero denominator:

$$\text{MAPE} = \left( \frac{\sum_{t=1}^n |E_t|}{\sum_{t=1}^n A_t} \right) \times 100. \quad (7)$$

This specification of MAPE has been used in evaluating lumpy demand forecasting (e.g., Gutierrez, Solis, and Mukhopadhyay 2008; Mukhopadhyay, Solis, and Gutierrez 2012).

A second scale-free error statistic we use is the *mean absolute scaled error* (MASE). It was fairly recently proposed as a forecast accuracy measurement applicable for intermittent demand, without problems seen in other error statistics (Hyndman and Koehler 2006). Using the naïve method, one-period-ahead forecasts are generated. A scaled error is defined as

$$q_t = \frac{e_t}{\left(\sum_{i=2}^n |Y_i - Y_{t-1}|\right)/(n-1)}, \quad (8)$$

which is independent of the scale of the data. A scaled error is less than one if it arises from a better forecast than the average one-step, naïve forecast computed in-sample. It is greater than one if the forecast is worse. MASE is simply the mean value of  $|q_t|$ .

The third scale-free error statistic we apply in this paper is *percentage best* (PB) – the percentage of time periods in which a particular method performs better than any of the other methods with respect to a specified criterion. PB has been applied in a good number of intermittent demand forecasting studies (e.g., Syntetos and Boylan 2005; Gutierrez, Solis, and Mukhopadhyay 2008; Mukhopadhyay, Solis, and Gutierrez 2012). We used absolute error for assessing performance under the PB approach.

## 2.2. Industrial Dataset and Partitioning

In this paper, we apply the four methods to an industrial dataset involving more than 1000 SKUs in the central warehouse of a firm operating in the professional electronics sector. The raw data consist of actual stock withdrawals reported in the company’s enterprise resource planning system. We first aggregated the transactional data into usage quantities per calendar week, and further aggregated the weekly quantities in terms of 13 four-week “months” in a calendar year. The monthly usage quantities do not constitute actual demand quantities, since the inventory on hand at the time of a stock withdrawal may not meet the required quantity. Since demand is not traditionally tracked as well as actual usage in a transaction-based system, we treat monthly usage quantity as a surrogate measure of monthly demand.

This process yielded 66 months of “demand” data, which we broke down into initialization, calibration, and performance measurement blocks (as in Boylan, Syntetos, and Karakostas 2008) with our blocks consisting of 23, 23, and 20 months, respectively.

For each of the SES, Croston’s and SBA methods, we selected  $\alpha$  based upon the minimum MAPE attained in the calibration block for use as the smoothing constant in the performance block.

Given that the various SKUs represent end items, sub-assemblies, components, and spare parts that are used for building projects, retail sales, or servicing of professional electronic products, it is understandable that we found many of them to actually exhibit erratic or lumpy demand based on the earlier cited categorization scheme (Syntetos, Boylan, and Croston 2005). We have, however, failed to find a SKU with intermittent demand ( $ADI > 1.32$  and  $CV^2 < 0.49$ ) according to this scheme.

In this paper, we report findings on a sample of fifteen SKUs, with demand statistics presented in Table 1. The first six SKUs (1–6) are categorized as having

erratic demand, while the remaining nine SKUs (7–10) exhibit lumpy demand.

Table 1: Sample of 15 SKUs

| SKU #                  | 1       | 2       | 3       | 4       | 5       |
|------------------------|---------|---------|---------|---------|---------|
| Mean                   | 7.1667  | 14.1667 | 11.4091 | 36.2273 | 8.9394  |
| Std Dev                | 8.1930  | 16.3251 | 10.3804 | 32.1050 | 9.5545  |
| $CV^2$                 | 1.3069  | 1.3279  | 0.8278  | 0.7854  | 1.1424  |
| ADI                    | 1.1186  | 1.1000  | 1.1379  | 1.0820  | 1.2692  |
| $z$ (% of Zero Demand) | 10.61%  | 9.09%   | 12.12%  | 7.58%   | 21.21%  |
| Category               | ERRATIC | ERRATIC | ERRATIC | ERRATIC | ERRATIC |

| SKU #                  | 6       | 7      | 8       | 9       | 10     |
|------------------------|---------|--------|---------|---------|--------|
| Mean                   | 3.5152  | 1.0152 | 8.3333  | 29.0152 | 5.3667 |
| Std Dev                | 4.0959  | 1.3977 | 11.8612 | 57.6463 | 7.9723 |
| $CV^2$                 | 1.3577  | 1.8957 | 2.0259  | 3.9472  | 2.2068 |
| ADI                    | 1.2941  | 1.7838 | 1.6500  | 2.2000  | 1.6216 |
| $z$ (% of Zero Demand) | 22.73%  | 43.94% | 39.39%  | 54.55%  | 38.33% |
| Category               | ERRATIC | LUMPY  | LUMPY   | LUMPY   | LUMPY  |

| SKU #                  | 11     | 12     | 13     | 14     | 15     |
|------------------------|--------|--------|--------|--------|--------|
| Mean                   | 3.1818 | 2.3333 | 3.9394 | 5.2879 | 2.2273 |
| Std Dev                | 3.7700 | 2.9053 | 5.3805 | 6.2580 | 2.2790 |
| $CV^2$                 | 1.4039 | 1.5504 | 1.8655 | 1.4006 | 1.0470 |
| ADI                    | 1.6098 | 1.5000 | 1.5000 | 1.3469 | 1.5000 |
| $z$ (% of Zero Demand) | 37.88% | 33.33% | 33.33% | 25.76% | 33.33% |
| Category               | LUMPY  | LUMPY  | LUMPY  | LUMPY  | LUMPY  |

## 2.3. Modeling of Demand Distributions

We sought to simulate demand distributions of the SKUs under study, performing 100 runs each consisting of 100 four-week “months”, for a total of 10,000 months in each experiment. We used AnyLogic as our simulation platform. Some code was written in Java in order to address mathematical modeling which could not be readily undertaken within the standard AnyLogic library (the authors have a remarkable experience in developing simulation model for inventory and supply chain problems investigation, Curcio and Longo, 2009; Bruzzone and Longo, 2010). We used the mean  $\bar{x}$  and the variance  $s^2$  of the 66-month actual demand time series as values of  $\mu$  and  $\sigma^2$ , respectively, in (5) and (6) to generate initial estimates  $\hat{r}$  and  $\hat{p}$  of the NBD parameters. These initial estimates are then adjusted to obtain acceptable NBD parameters  $r$  and  $p$ .

In the case of SKUs 1 and 7 (out of the 15 SKUs that we report on in the current paper), we found the proposed NBD approximation to yield a simulated distribution which fairly closely follows the actual demand distribution. The simulation results are reported in Table 2. In each case, the adjusted  $r$  value is 1 (i.e., the NBD reduces to a geometric distribution) and  $p = \Pr(X = 0)$  closely approximates – in fact, slightly exceeds – the actual proportion  $z$  of zero demand occurrences.

However, we have found that, for most of the other SKUs currently under study, it is not possible to accordingly obtain adjusted values of  $\hat{r}$  and  $\hat{p}$  that would lead to an NBD with mean, standard deviation,  $CV^2$ , and  $ADI$  that are reasonably close to those of the actual demand distribution. To illustrate, in Table 3, we present NBD approximation results for SKUs 2 and 8. In each of these two cases, the NBD reduces to a geometric distribution, with the adjusted  $r$  value being 1. However,  $p = \Pr(X = 0)$  is less than the actual proportion  $z$  of zero demand occurrences, especially in the case of SKU 8 for which demand is lumpy ( $p = 7.87\%$  versus  $z = 39.39\%$ ).

To more directly address the proportion  $z$  of periods with zero demand, we simulate the demand

distribution by way of a two-stage probability distribution (earlier applied by Solis, Nicoletti, Mukhopadhyay, Agosteo, Delfino, and Sartiano 2012).

Table 2: Two SKUs with Good NBD Approximations

| SKU #                                 | 1       | 7      |
|---------------------------------------|---------|--------|
| Mean                                  | 7.1667  | 1.0152 |
| Std Dev                               | 8.1930  | 1.3977 |
| $CV^2$                                | 1.3069  | 1.8957 |
| $ADI$                                 | 1.1186  | 1.7838 |
| $z$ (% of Zero Demand)                | 10.61%  | 43.94% |
| Category                              | ERRATIC | LUMPY  |
| $r^\wedge$                            | 0.8566  | 1.0981 |
| $p^\wedge$                            | 0.1068  | 0.5196 |
| <b>SIMULATION</b>                     |         |        |
| $r$                                   | 1       | 1      |
| $p$                                   | 0.1177  | 0.4800 |
| Mean                                  | 7.4544  | 1.0795 |
| Std Dev                               | 7.8854  | 1.4783 |
| $CV^2$                                | 1.1310  | 1.9056 |
| $ADI$                                 | 1.1319  | 1.9026 |
| $z$ (% of Zero Demand)                | 11.64%  | 47.51% |
| Simulated vs Actual Mean              | 104.0%  | 106.3% |
| Simulated vs Actual Std Dev           | 96.2%   | 105.8% |
| Simulated vs Actual $CV^2$            | 86.5%   | 100.5% |
| Simulated vs Actual $ADI$             | 101.2%  | 106.7% |
| Difference in Simulated vs Actual $z$ | 1.03%   | 3.57%  |

Table 3: Two SKUs with Poor NBD Approximations

| SKU #                                 | 2       | 8       |
|---------------------------------------|---------|---------|
| Mean                                  | 14.1667 | 8.3333  |
| Std Dev                               | 16.3251 | 11.8612 |
| $CV^2$                                | 1.3279  | 2.0259  |
| $ADI$                                 | 1.1000  | 1.6500  |
| $z$ (% of Zero Demand)                | 9.09%   | 39.39%  |
| Category                              | ERRATIC | LUMPY   |
| $r^\wedge$                            | 0.7953  | 0.5247  |
| $p^\wedge$                            | 0.0532  | 0.0592  |
| <b>SIMULATION</b>                     |         |         |
| $r$                                   | 1       | 1       |
| $p$                                   | 0.0591  | 0.0787  |
| Mean                                  | 15.8662 | 11.7009 |
| Std Dev                               | 16.3433 | 11.9177 |
| $CV^2$                                | 1.0716  | 1.0484  |
| $ADI$                                 | 1.0641  | 1.0873  |
| $z$ (% of Zero Demand)                | 5.99%   | 8.00%   |
| Simulated vs Actual Mean              | 82.6%   | 190.4%  |
| Simulated vs Actual Std Dev           | 73.0%   | 137.8%  |
| Simulated vs Actual $CV^2$            | 80.7%   | 51.8%   |
| Simulated vs Actual $ADI$             | 96.7%   | 65.9%   |
| Difference in Simulated vs Actual $z$ | -3.10%  | -31.39% |

The first stage is modeled with a uniform distribution initially based upon  $z_1 = z$ , where  $z$  is the actual proportion of zero demand periods in the 66-month time series. For the second stage, we determine the mean  $\bar{x}_{nz}$  and variance  $s_{nz}^2$  of the nonzero demands and use these to calculate first approximations of the parameters  $\hat{p}_{nz}$  and  $\hat{r}_{nz}$  in line with (5) and (6). The corresponding negative binomial probability  $P_0 = \Pr(X=0) > 0$  is then used to adjust  $z_1$  (as applied in the first stage) downward, as follows:

$$z_1 = \frac{z - P_0}{1 - P_0}, \quad (9)$$

provided  $z > P_0$ . As a result, the proportion of zero demand periods arising from the two-stage distribution

is closer to  $z$ . We refine the parameter estimate  $\hat{p}_{nz}$  as the values of mean, standard deviation,  $CV^2$ , and  $ADI$  of the actual and simulated demand distributions are compared.

We present in Table 4 the simulation results for the demand distributions of the 15 SKUs using the two-stage approach, except for SKUs 1 and 7 (which were modeled using only the NBD approximation). For the other 13 SKUs, we selected  $z_1$  for stage 1 and the parameters  $r$  and  $p$  of the NBD in stage 2 based on what appeared to yield the best combination of values of mean, standard deviation,  $CV^2$ , and  $ADI$  of the simulated distribution in comparison with those of the actual distribution.

Table 4: Simulation of Demand Using the Two-Stage Distribution

| SKU #                               | 1       | 2       | 3       | 4       | 5       |
|-------------------------------------|---------|---------|---------|---------|---------|
| Mean                                | 7.1667  | 14.1667 | 11.4091 | 36.2273 | 8.9394  |
| Std Dev                             | 8.1930  | 16.3251 | 10.3804 | 32.1050 | 9.5545  |
| $CV^2$                              | 1.3069  | 1.3279  | 0.8278  | 0.7854  | 1.1424  |
| $ADI$                               | 1.1186  | 1.1000  | 1.1379  | 1.0820  | 1.2692  |
| $z$ (% of Zero Demand)              | 10.61%  | 9.09%   | 12.12%  | 7.58%   | 21.21%  |
| Category                            | ERRATIC | ERRATIC | ERRATIC | ERRATIC | ERRATIC |
| Mean of nonzero demand              | 8.0169  | 15.5833 | 12.9828 | 39.1967 | 11.3462 |
| Std Dev of nonzero demand           | 8.2639  | 16.4670 | 10.1038 | 31.5958 | 9.4077  |
| $r^\wedge$ nonzero                  | 1.0663  | 0.9502  | 1.8916  | 1.6019  | 1.6684  |
| $p^\wedge$ nonzero                  | 0.1174  | 0.0575  | 0.1272  | 0.0393  | 0.1282  |
| <b>SIMULATION</b>                   |         |         |         |         |         |
| $r$                                 | 1       | 1       | 2       | 2       | 2       |
| $p$                                 | 0.1177  | 0.0589  | 0.1271  | 0.0481  | 0.1491  |
| $\Pr(X=0)$                          | 0.1177  | 0.0589  | 0.0162  | 0.0023  | 0.0222  |
| Final zero proportion in stage 1    | 0.00%   | 3.40%   | 10.68%  | 7.36%   | 19.42%  |
| Mean                                | 7.4544  | 15.3294 | 12.2154 | 37.1695 | 9.0884  |
| Std Dev                             | 7.8854  | 16.3021 | 10.6752 | 29.6808 | 8.9894  |
| $CV^2$                              | 1.1310  | 1.14615 | 0.7738  | 0.6446  | 0.9853  |
| $ADI$                               | 1.1319  | 1.09857 | 1.1392  | 1.0807  | 1.2811  |
| $z$ (% of Zero Demand)              | 11.64%  | 9.02%   | 12.27%  | 7.46%   | 21.97%  |
| Simulated vs Actual Mean            | 104.0%  | 108.2%  | 107.1%  | 102.6%  | 101.7%  |
| Simulated vs Actual Std Dev         | 96.2%   | 99.9%   | 102.8%  | 92.4%   | 94.1%   |
| Simulated vs Actual $CV^2$          | 86.5%   | 86.3%   | 93.5%   | 82.1%   | 86.2%   |
| Simulated vs Actual $ADI$           | 101.2%  | 99.9%   | 100.1%  | 99.9%   | 100.9%  |
| $\Delta$ in Simulated vs Actual $z$ | 1.03%   | -0.07%  | 0.15%   | -0.12%  | 0.76%   |
| <b>SKU #</b>                        |         |         |         |         |         |
| Mean                                | 3.5152  | 1.0152  | 8.3333  | 29.0152 | 5.3667  |
| Std Dev                             | 4.0959  | 1.3977  | 11.8612 | 57.6463 | 7.9723  |
| $CV^2$                              | 1.3577  | 1.8957  | 2.0259  | 3.9472  | 2.2068  |
| $ADI$                               | 1.2941  | 1.7838  | 1.6500  | 2.2000  | 1.6216  |
| $z$ (% of Zero Demand)              | 22.73%  | 43.94%  | 39.39%  | 54.55%  | 38.33%  |
| Category                            | ERRATIC | LUMPY   | LUMPY   | LUMPY   | LUMPY   |
| Mean of nonzero demand              | 4.5490  | 1.8108  | 13.7500 | 63.8333 | 8.7027  |
| Std Dev of nonzero demand           | 4.1246  | 1.4306  | 12.5734 | 71.7573 | 8.6212  |
| $r^\wedge$ nonzero                  | 1.6603  | 13.9098 | 1.3098  | 0.8013  | 1.1541  |
| $p^\wedge$ nonzero                  | 0.2674  | 0.8848  | 0.0870  | 0.0124  | 0.1171  |
| <b>SIMULATION</b>                   |         |         |         |         |         |
| $r$                                 | 1       | 1       | 1       | 1       | 1       |
| $p$                                 | 0.2100  | 0.4800  | 0.0730  | 0.0139  | 0.1150  |
| $\Pr(X=0)$                          | 0.2100  | 0.4800  | 0.0730  | 0.0139  | 0.1150  |
| Final zero proportion in stage 1    | 2.19%   | 0.00%   | 34.62%  | 53.90%  | 30.32%  |
| Mean                                | 3.7000  | 1.0795  | 8.2722  | 31.6675 | 5.4287  |
| Std Dev                             | 4.1823  | 1.4783  | 11.9503 | 56.9295 | 7.7028  |
| $CV^2$                              | 1.2968  | 1.9056  | 2.1254  | 3.3252  | 2.0692  |
| $ADI$                               | 1.2757  | 1.9026  | 1.6384  | 2.2202  | 1.6033  |
| $z$ (% of Zero Demand)              | 21.59%  | 47.51%  | 39.05%  | 54.83%  | 37.57%  |
| Simulated vs Actual Mean            | 105.3%  | 106.3%  | 99.3%   | 109.1%  | 101.2%  |
| Simulated vs Actual Std Dev         | 102.1%  | 105.8%  | 100.8%  | 98.8%   | 96.6%   |
| Simulated vs Actual $CV^2$          | 95.5%   | 100.5%  | 104.9%  | 84.2%   | 93.8%   |
| Simulated vs Actual $ADI$           | 98.6%   | 106.7%  | 99.3%   | 100.9%  | 98.9%   |
| $\Delta$ in Simulated vs Actual $z$ | -1.14%  | 3.57%   | -0.34%  | 0.28%   | -0.76%  |
| <b>SKU #</b>                        |         |         |         |         |         |
| Mean                                | 3.1818  | 2.3333  | 3.9394  | 5.2879  | 2.2273  |
| Std Dev                             | 3.7700  | 2.9053  | 5.3805  | 6.2580  | 2.2790  |
| $CV^2$                              | 1.4039  | 1.5504  | 1.8655  | 1.4006  | 1.0470  |
| $ADI$                               | 1.6098  | 1.5000  | 1.5000  | 1.3469  | 1.5000  |
| $z$ (% of Zero Demand)              | 37.88%  | 33.33%  | 33.33%  | 25.76%  | 33.33%  |
| Category                            | LUMPY   | LUMPY   | LUMPY   | LUMPY   | LUMPY   |
| Mean of nonzero demand              | 5.1220  | 3.5000  | 5.9091  | 7.1224  | 3.3409  |
| Std Dev of nonzero demand           | 3.5930  | 2.9294  | 5.6438  | 6.3002  | 2.0109  |
| $r^\wedge$ nonzero                  | 3.3686  | 2.4108  | 1.3459  | 1.5575  | 15.8781 |
| $p^\wedge$ nonzero                  | 0.3968  | 0.4079  | 0.1855  | 0.1794  | 0.8262  |
| <b>SIMULATION</b>                   |         |         |         |         |         |
| $r$                                 | 3       | 1       | 1       | 1       | 15      |
| $p$                                 | 0.3800  | 0.2764  | 0.1675  | 0.1380  | 0.8130  |
| $\Pr(X=0)$                          | 0.0549  | 0.2764  | 0.1675  | 0.1380  | 0.0448  |
| Final zero proportion in stage 1    | 34.27%  | 7.86%   | 19.92%  | 13.87%  | 30.21%  |
| Mean                                | 3.1966  | 2.3502  | 4.0198  | 5.3850  | 2.3899  |
| Std Dev                             | 3.6903  | 2.9341  | 5.3374  | 6.5807  | 2.3267  |
| $CV^2$                              | 1.3534  | 1.5946  | 1.7937  | 1.5183  | 0.9691  |
| $ADI$                               | 1.6025  | 1.5124  | 1.4889  | 1.3438  | 1.5016  |
| $z$ (% of Zero Demand)              | 37.67%  | 33.75%  | 32.86%  | 25.64%  | 33.56%  |
| Simulated vs Actual Mean            | 100.5%  | 100.7%  | 102.0%  | 101.8%  | 107.3%  |
| Simulated vs Actual Std Dev         | 97.9%   | 101.0%  | 99.2%   | 105.2%  | 102.1%  |
| Simulated vs Actual $CV^2$          | 96.4%   | 102.9%  | 96.2%   | 108.4%  | 92.6%   |
| Simulated vs Actual $ADI$           | 99.5%   | 100.8%  | 99.3%   | 99.8%   | 100.1%  |
| $\Delta$ in Simulated vs Actual $z$ | -0.21%  | 0.42%   | -0.47%  | -0.12%  | 0.23%   |

We must quickly point out, however, that even the two-stage approximation we apply fails in the case of SKUs with demand distributions that are lumpier. In Table 5, we provide three SKUs (16–18) out of a number of SKUs whose demand distributions we were unable to model adequately.

Table 5: Some SKUs for which the Two-Stage Distribution Fails

| SKU #                               | 16       | 17      | 18      |
|-------------------------------------|----------|---------|---------|
| Mean                                | 59.3182  | 3.9848  | 7.7424  |
| Std Dev                             | 117.7808 | 10.2379 | 21.9638 |
| CV <sup>2</sup>                     | 3.9425   | 6.6009  | 8.0475  |
| ADI                                 | 1.5349   | 1.7368  | 1.4667  |
| z (% of Zero Demand)                | 34.85%   | 42.42%  | 31.82%  |
| Category                            | LUMPY    | LUMPY   | LUMPY   |
| Mean of nonzero demand              | 91.0465  | 6.9211  | 11.3556 |
| Std Dev of nonzero demand           | 136.0571 | 12.7775 | 25.8977 |
| r <sup>h</sup> nonzero              | 0.4500   | 0.3064  | 0.1956  |
| p <sup>h</sup> nonzero              | 0.0049   | 0.0424  | 0.0169  |
| <b>SIMULATION</b>                   |          |         |         |
| r                                   | 1        | 1       | 1       |
| p                                   | 0.0080   | 0.0850  | 0.0400  |
| Pr(X = 0)                           | 0.0080   | 0.0850  | 0.0400  |
| Final zero proportion in stage 1    | 34.32%   | 37.08%  | 28.98%  |
| Mean                                | 83.9741  | 6.7120  | 16.9344 |
| Std Dev                             | 117.9103 | 10.2027 | 22.8629 |
| CV <sup>2</sup>                     | 2.0030   | 2.3956  | 1.8701  |
| ADI                                 | 1.5199   | 1.7186  | 1.4643  |
| z (% of Zero Demand)                | 34.20%   | 41.87%  | 32.68%  |
| Simulated vs Actual Mean            | 141.6%   | 168.4%  | 218.7%  |
| Simulated vs Actual Std Dev         | 100.1%   | 99.7%   | 104.1%  |
| Simulated vs Actual CV <sup>2</sup> | 50.8%    | 36.3%   | 23.2%   |
| Simulated vs Actual ADI             | 99.0%    | 99.0%   | 99.8%   |
| Δ in Simulated vs Actual z          | -0.65%   | -0.55%  | 0.86%   |

For such SKUs where even our two-stage approximation of the demand distribution failed, we have understandably been unable to proceed with the evaluation of forecast accuracy and inventory control performance.

### 3. FORECASTING PERFORMANCE

#### 3.1. Forecast Accuracy: Performance Block

The exponential smoothing constant  $\alpha$  selected from among the candidate values (0.05, 0.10, 0.15, or 0.20) for each of the SES, Croston's, and SBA methods, based upon the minimum MAPE in the calibration block, are shown in Table 6. The resulting error statistics when applying SMA13, SES, Croston's, and SBA methods to actual demand data in the performance block (the final 20 months) are likewise reported in the table. There does not appear to be a method that exhibits a superior performance overall across the 15 SKUs.

#### 3.2. Forecast Accuracy: Simulated Demand

When applying the methods to the simulated demand distributions, however, we see in Table 7 that overall the SBA method outperforms SMA13, SES, and Croston's methods – particularly for SKUs with lumpy demand – based on MAPE and MASE. While the forecast accuracy performance with respect to PB appears to be inconclusive, the reported results nevertheless suggest the overall superiority of SBA over the long run.

Table 6: Error Statistics when Applying Various Methods to Actual Demand in the Performance Block

| SKU #   | 1       | 2       | 3       | 4       | 5       |
|---|---------|---------|---------|---------|---------|
| Category  | ERRATIC | ERRATIC | ERRATIC | ERRATIC | ERRATIC |
| Smoothing Constants Selected in Calibration Block |         |         |         |         |         |
| SES   | 0.20    | 0.05    | 0.10    | 0.05    | 0.15    |
| Croston   | 0.20    | 0.05    | 0.05    | 0.05    | 0.05    |
| SBA   | 0.20    | 0.05    | 0.10    | 0.05    | 0.10    |
| MAPE (in %)                                       |         |         |         |         |         |
| SMA13   | 71.92   | 86.83   | 76.34   | 61.59   | 109.87  |
| SES   | 63.47   | 94.56   | 75.16   | 61.42   | 102.88  |
| Croston   | 64.08   | 93.25   | 75.44   | 62.61   | 104.22  |
| SBA   | 63.30   | 92.00   | 75.05   | 63.02   | 104.66  |
| Best MAPE   | SBA     | SMA13   | SBA     | SES     | SES     |
| MASE  |         |         |         |         |         |
| SMA13   | 0.8676  | 0.9343  | 0.7440  | 0.6685  | 1.1332  |
| SES   | 0.7657  | 1.0175  | 0.7325  | 0.6668  | 1.0612  |
| Croston   | 0.7731  | 1.0033  | 0.7352  | 0.6797  | 1.0750  |
| SBA   | 0.7636  | 0.9899  | 0.7314  | 0.6841  | 1.0795  |
| Best MASE   | SBA     | SMA13   | SBA     | SES     | SES     |
| PB (in %)   |         |         |         |         |         |
| SMA13   | 25.00   | 60.00   | 25.00   | 35.00   | 25.00   |
| SES   | 30.00   | 25.00   | 25.00   | 25.00   | 45.00   |
| Croston   | 15.00   | 5.00    | 10.00   | 0.00    | 20.00   |
| SBA   | 30.00   | 10.00   | 40.00   | 40.00   | 10.00   |
| Best PB   | SES/SBA | SMA13   | SBA     | SBA     | SES     |
| <b>SKU #</b>                                      |         |         |         |         |         |
| <b>6</b>  |         |         |         |         |         |
| Category  | ERRATIC | LUMPY   | LUMPY   | LUMPY   | LUMPY   |
| Smoothing Constants Selected in Calibration Block |         |         |         |         |         |
| SES   | 0.05    | 0.05    | 0.10    | 0.20    | 0.20    |
| Croston   | 0.15    | 0.05    | 0.05    | 0.05    | 0.05    |
| SBA   | 0.20    | 0.05    | 0.05    | 0.05    | 0.05    |
| MAPE (in %)                                       |         |         |         |         |         |
| SMA13   | 117.48  | 97.01   | 80.51   | 90.97   | 104.90  |
| SES   | 126.44  | 98.60   | 81.56   | 95.47   | 115.53  |
| Croston   | 112.24  | 90.72   | 78.10   | 93.53   | 96.07   |
| SBA   | 102.76  | 89.98   | 78.24   | 93.60   | 96.03   |
| Best MAPE   | SBA     | SBA     | Croston | SMA13   | SBA     |
| MASE  |         |         |         |         |         |
| SMA13   | 0.7921  | 0.6635  | 0.7520  | 0.7533  | 0.6766  |
| SES   | 0.8524  | 0.6744  | 0.7618  | 0.7906  | 0.7453  |
| Croston   | 0.7567  | 0.6205  | 0.7295  | 0.7746  | 0.6197  |
| SBA   | 0.6928  | 0.6155  | 0.7308  | 0.7751  | 0.6194  |
| Best MASE   | SBA     | SBA     | Croston | SMA13   | SBA     |
| PB (in %)   |         |         |         |         |         |
| SMA13   | 5.00    | 30.00   | 25.00   | 40.00   | 25.00   |
| SES   | 20.00   | 15.00   | 20.00   | 20.00   | 10.00   |
| Croston   | 15.00   | 10.00   | 10.00   | 5.00    | 20.00   |
| SBA   | 60.00   | 45.00   | 45.00   | 35.00   | 45.00   |
| Best PB   | SBA     | SBA     | SBA     | SMA13   | SBA     |
| <b>SKU #</b>                                      |         |         |         |         |         |
| <b>11</b>   |         |         |         |         |         |
| Category  | LUMPY   | LUMPY   | LUMPY   | LUMPY   | LUMPY   |
| Smoothing Constants Selected in Calibration Block |         |         |         |         |         |
| SES   | 0.20    | 0.05    | 0.10    | 0.05    | 0.20    |
| Croston   | 0.20    | 0.05    | 0.10    | 0.05    | 0.05    |
| SBA   | 0.20    | 0.05    | 0.20    | 0.05    | 0.05    |
| MAPE (in %)                                       |         |         |         |         |         |
| SMA13   | 56.58   | 86.50   | 81.57   | 67.36   | 90.21   |
| SES   | 53.46   | 84.31   | 76.97   | 67.53   | 90.51   |
| Croston   | 61.39   | 82.85   | 76.50   | 67.35   | 89.32   |
| SBA   | 63.44   | 82.27   | 78.45   | 67.51   | 89.24   |
| Best MAPE   | SES     | SBA     | Croston | Croston | SBA     |
| MASE  |         |         |         |         |         |
| SMA13   | 0.8909  | 0.7546  | 1.1952  | 0.7359  | 0.6983  |
| SES   | 0.8418  | 0.7356  | 1.1278  | 0.7378  | 0.7006  |
| Croston   | 0.9667  | 0.7228  | 1.1209  | 0.7358  | 0.6914  |
| SBA   | 0.9989  | 0.7178  | 1.1495  | 0.7376  | 0.6908  |
| Best MASE   | SES     | SBA     | Croston | Croston | SBA     |
| PB (in %)   |         |         |         |         |         |
| SMA13   | 25.00   | 20.00   | 15.00   | 35.00   | 30.00   |
| SES   | 40.00   | 25.00   | 35.00   | 20.00   | 35.00   |
| Croston   | 15.00   | 5.00    | 20.00   | 5.00    | 15.00   |
| SBA   | 20.00   | 50.00   | 30.00   | 40.00   | 20.00   |
| Best PB   | SES     | SBA     | SES     | SBA     | SES     |

Table 7: Error Statistics when Applying Various Methods to the Simulated Demand Distributions

| SKU #   | 1       | 2       | 3       | 4       | 5       |
|---|---------|---------|---------|---------|---------|
| Category  | ERRATIC | ERRATIC | ERRATIC | ERRATIC | ERRATIC |
| Smoothing Constants Selected in Calibration Block |         |         |         |         |         |
| SES   | 0.20    | 0.05    | 0.10    | 0.05    | 0.15    |
| Croston   | 0.20    | 0.05    | 0.05    | 0.05    | 0.05    |
| SBA   | 0.20    | 0.05    | 0.10    | 0.05    | 0.10    |
| MAPE (in %)                                       |         |         |         |         |         |
| SMA13   | 80.64   | 81.26   | 69.72   | 64.62   | 79.89   |
| SES   | 81.78   | 79.36   | 68.84   | 62.88   | 80.10   |
| Croston   | 81.46   | 79.07   | 67.56   | 62.38   | 77.82   |
| SBA   | 78.96   | 78.45   | 67.95   | 62.02   | 78.06   |
| Best MAPE   | SBA     | SBA     | Croston | SBA     | Croston |
| MASE  |         |         |         |         |         |
| SMA13   | 0.7637  | 0.7709  | 0.7554  | 0.7578  | 0.7650  |
| SES   | 0.7737  | 0.7529  | 0.7458  | 0.7373  | 0.7667  |
| Croston   | 0.7711  | 0.7501  | 0.7321  | 0.7317  | 0.7451  |
| SBA   | 0.7475  | 0.7442  | 0.7363  | 0.7275  | 0.7474  |
| Best MASE   | SBA     | SBA     | Croston | SBA     | Croston |
| PB (in %)   |         |         |         |         |         |
| SMA13   | 34.53   | 41.24   | 30.05   | 37.56   | 26.29   |
| SES   | 20.75   | 15.43   | 16.32   | 19.96   | 26.02   |
| Croston   | 8.64    | 9.01    | 34.03   | 5.97    | 33.85   |
| SBA   | 36.08   | 34.32   | 19.60   | 36.51   | 13.84   |
| Best PB   | SBA     | SMA13   | Croston | SMA13   | Croston |
| SKU #   | 6       | 7       | 8       | 9       | 10      |
| Category  | ERRATIC | LUMPY   | LUMPY   | LUMPY   | LUMPY   |
| Smoothing Constants Selected in Calibration Block |         |         |         |         |         |
| SES   | 0.05    | 0.05    | 0.10    | 0.20    | 0.20    |
| Croston   | 0.15    | 0.05    | 0.05    | 0.05    | 0.05    |
| SBA   | 0.20    | 0.05    | 0.05    | 0.05    | 0.05    |
| MAPE (in %)                                       |         |         |         |         |         |
| SMA13   | 87.57   | 107.04  | 111.30  | 129.28  | 106.65  |
| SES   | 86.65   | 103.93  | 109.81  | 130.74  | 108.27  |
| Croston   | 87.96   | 102.21  | 105.77  | 121.36  | 102.34  |
| SBA   | 85.61   | 101.45  | 105.08  | 120.55  | 101.68  |
| Best MAPE   | SBA     | Croston | SBA     | SBA     | SBA     |
| MASE  |         |         |         |         |         |
| SMA13   | 0.7801  | 0.8241  | 0.8199  | 0.8495  | 0.8031  |
| SES   | 0.7720  | 0.8000  | 0.8088  | 0.8585  | 0.8148  |
| Croston   | 0.7836  | 0.7869  | 0.7792  | 0.7974  | 0.7709  |
| SBA   | 0.7628  | 0.7810  | 0.7741  | 0.7922  | 0.7660  |
| Best MASE   | SBA     | SBA     | SBA     | SBA     | SBA     |
| PB (in %)   |         |         |         |         |         |
| SMA13   | 23.43   | 36.84   | 31.14   | 22.46   | 22.99   |
| SES   | 30.52   | 13.46   | 19.21   | 32.65   | 46.90   |
| Croston   | 9.65    | 15.71   | 12.73   | 12.20   | 8.43    |
| SBA   | 36.40   | 33.99   | 36.92   | 32.69   | 21.65   |
| Best PB   | SBA     | SMA13   | SBA     | SBA     | SES     |
| SKU #   | 11      | 12      | 13      | 14      | 15      |
| Category  | LUMPY   | LUMPY   | LUMPY   | LUMPY   | LUMPY   |
| Smoothing Constants Selected in Calibration Block |         |         |         |         |         |
| SES   | 0.20    | 0.05    | 0.10    | 0.05    | 0.20    |
| Croston   | 0.20    | 0.05    | 0.10    | 0.05    | 0.05    |
| SBA   | 0.20    | 0.05    | 0.20    | 0.05    | 0.05    |
| MAPE (in %)                                       |         |         |         |         |         |
| SMA13   | 95.95   | 96.13   | 101.66  | 93.39   | 84.25   |
| SES   | 97.25   | 94.61   | 101.00  | 90.98   | 85.34   |
| Croston   | 95.54   | 93.67   | 100.91  | 90.28   | 82.39   |
| SBA   | 93.74   | 92.95   | 98.69   | 89.70   | 82.25   |
| Best MAPE   | SBA     | SBA     | SBA     | SBA     | SBA     |
| MASE  |         |         |         |         |         |
| SMA13   | 0.7939  | 0.7914  | 0.7999  | 0.7805  | 0.7958  |
| SES   | 0.8041  | 0.7788  | 0.7944  | 0.7603  | 0.8056  |
| Croston   | 0.7907  | 0.7714  | 0.7938  | 0.7545  | 0.7786  |
| SBA   | 0.7759  | 0.7654  | 0.7766  | 0.7497  | 0.7774  |
| Best MASE   | SBA     | SBA     | SBA     | SBA     | SBA     |
| PB (in %)   |         |         |         |         |         |
| SMA13   | 24.63   | 39.72   | 27.54   | 39.87   | 23.95   |
| SES   | 29.35   | 15.36   | 12.52   | 14.57   | 32.88   |
| Croston   | 12.97   | 12.13   | 24.18   | 11.43   | 16.65   |
| SBA   | 33.05   | 32.79   | 35.76   | 34.13   | 26.52   |
| Best PB   | SBA     | SMA13   | SBA     | SMA13   | SES     |

#### 4. INVENTORY CONTROL PERFORMANCE

Demand forecasting and inventory control performance have traditionally been considered independently of each other (Tiacci and Saetta 2009). In reality, forecast accuracy may not translate into inventory systems efficiency (Syntetos, Nikolopoulos, and Boylan 2010).

Sani and Kingsman (1997) have recommended a periodic review inventory control system to address intermittent demand. An order-up-to ( $T,S$ ) periodic review system, where  $T$  and  $S$  denote the review period and the base stock (or 'order-up-to' level), respectively, has been applied in recent intermittent demand forecasting studies (e.g., Eaves and Kingsman 2004; Syntetos and Boylan 2006; Syntetos, Nikolopoulos, Boylan, Fildes, and Goodwin, 2009; Syntetos, Babai, Dallery, and Teunter 2009; Syntetos, Nikolopoulos, and Boylan 2010; Teunter, Syntetos, and Babai 2010) that look into both forecast accuracy and inventory control performance.

In this paper, we assume a ( $T,S$ ) system with full backordering, with inventory reviewed on a monthly basis ( $T = 1$ ). The reorder lead time for most SKUs is about one month ( $L = 1$ ). The literature suggests a safety stock component to compensate for uncertainty in demand during the 'protection interval'  $T+L$ . For each SKU, we calculated  $s_{tr}$ , the standard deviation of monthly demand during the initialization and calibration blocks (the first 46 months of actual usage quantities). We apply a 'safety factor'  $k$  to yield a safety stock level of  $k \cdot s_{tr}$ . This approach differs, of course, from that suggested (e.g., Silver, Pyke, and Peterson 1998) under an assumption that daily demand is identically and independently normally distributed during the protection interval. If  $F_t$  is the forecast calculated at the end of month  $t$ , and  $I_t$  and  $B_t$  are, respectively, on-hand inventory and backlog, the replenishment quantity at the time of review is

$$Q_t = (T + L) \cdot F_t + k \cdot s_{tr} - I_t + B_t. \quad (10)$$

##### 4.1. Service Levels

Silver, Pyke, and Peterson (1998) identified the two most commonly specified service level criteria in inventory systems. One is a target average *probability of no stockout* (PNS) per review period. The other is a target *fill rate* (FR), or average percentage of demand to be satisfied from on-hand inventory. FR is noted to have considerably more appeal for practitioners. We simulated inventory control performance with respect to two values of the target FR (95% and 98%) and two values of the target PNS (90% and 95%). We performed simulation searches to find the safety factor  $k$  that would yield the target FR or PNS.

##### 4.2. Average Inventory on Hand

For a target FR of 98%, resulting averages of inventory on hand are reported in Table 8. We proceeded to index

the average inventory on hand, as reported in Table 8, using SBA as base (SBA index = 100). Indices corresponding to a target FR of 98% are reported in Table 9 and depicted graphically in Figure 1. This figure suggests the overall superiority of SBA over SMA13 and SES in terms of resulting average on-hand inventory levels, but does not seem to indicate a similar comparison with respect to Croston's method. In fact, Figure 1 appears to suggest that Croston's method leads to slightly better average on-hand inventory than SBA for erratic demand (SKUs 1–6). When conducting *t* tests, the mean indices for SMA13 and SES exceed 100 at the 1% and 5% levels of significance, respectively. However, testing the mean index for Croston's method does not yield a statistically significant conclusion.

Table 8: Average Inventory on Hand for a 98% Target Fill Rate

| SKU #   | 1       | 2       | 3       | 4       | 5       |
|---------|---------|---------|---------|---------|---------|
| SMA13   | 25.5024 | 53.2450 | 28.7932 | 76.4457 | 24.6869 |
| SES     | 25.8652 | 51.8753 | 28.1311 | 72.4345 | 24.4925 |
| Croston | 25.7067 | 51.7261 | 27.3639 | 72.3506 | 23.7126 |
| SBA     | 25.5968 | 51.7647 | 27.9113 | 72.3894 | 23.9568 |

| SKU #   | 6       | 7      | 8       | 9        | 10      |
|---------|---------|--------|---------|----------|---------|
| SMA13   | 13.9749 | 5.2988 | 44.7181 | 240.5282 | 28.8782 |
| SES     | 13.5985 | 5.0813 | 44.0254 | 242.5799 | 28.8763 |
| Croston | 13.9830 | 5.0115 | 43.1597 | 238.3088 | 28.3105 |
| SBA     | 14.0235 | 5.0069 | 43.2253 | 238.3386 | 28.2883 |

| SKU #   | 11      | 12      | 13      | 14      | 15     |
|---------|---------|---------|---------|---------|--------|
| SMA13   | 10.1452 | 10.0599 | 19.8695 | 22.4635 | 5.3829 |
| SES     | 10.2395 | 9.7714  | 19.6909 | 21.6916 | 5.4407 |
| Croston | 10.2056 | 9.7606  | 19.9021 | 21.6139 | 4.9807 |
| SBA     | 10.0786 | 9.8166  | 19.7692 | 21.5828 | 4.9790 |

Table 9: Indices of Average Inventory on Hand for a 98% Target Fill Rate

| SKU #   | 1     | 2     | 3     | 4     | 5     |
|---------|-------|-------|-------|-------|-------|
| SMA13   | 99.6  | 102.9 | 103.2 | 105.6 | 103.0 |
| SES     | 101.0 | 100.2 | 100.8 | 100.1 | 102.2 |
| Croston | 100.4 | 99.9  | 98.0  | 99.9  | 99.0  |
| SBA     | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 |

| SKU #   | 6     | 7     | 8     | 9     | 10    |
|---------|-------|-------|-------|-------|-------|
| SMA13   | 99.7  | 105.8 | 103.5 | 100.9 | 102.1 |
| SES     | 97.0  | 101.5 | 101.9 | 101.8 | 102.1 |
| Croston | 99.7  | 100.1 | 99.8  | 100.0 | 100.1 |
| SBA     | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 |

| SKU #   | 11    | 12    | 13    | 14    | 15    |
|---------|-------|-------|-------|-------|-------|
| SMA13   | 100.7 | 102.5 | 100.5 | 104.1 | 108.1 |
| SES     | 101.6 | 99.5  | 99.6  | 100.5 | 109.3 |
| Croston | 101.3 | 99.4  | 100.7 | 100.1 | 100.0 |
| SBA     | 100.0 | 100.0 | 100.0 | 100.0 | 100.0 |

Similar results arise for the 95% target FR, as well as for the 90% and 95% PNS target levels – except that the mean index for SES with a target PNS of 95% does not exceed 100 at the 5% significance level. Figure 2 graphically shows indices for the 90% target PNS.

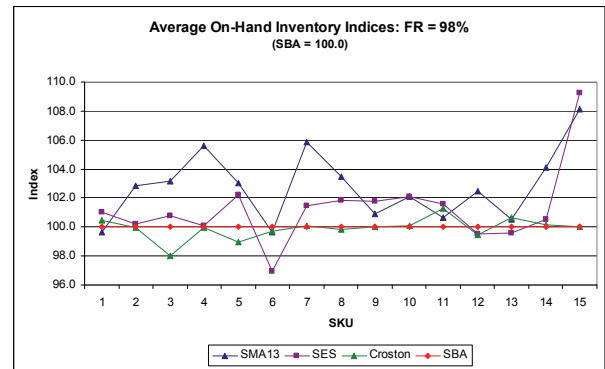


Figure 1: Average On-Hand Inventory Indices for a 98% Target Fill Rate

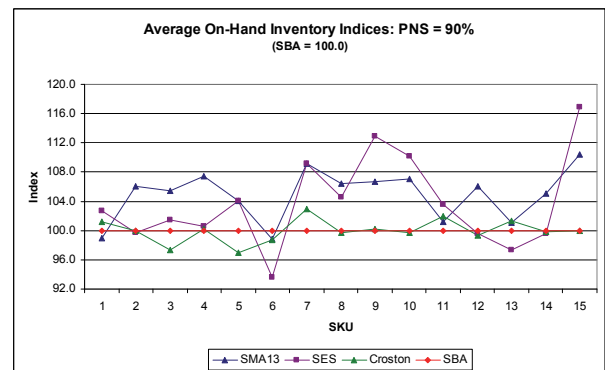


Figure 2: Average On-Hand Inventory Indices for a 90% Target Probability of No Stockout

### 4.3. Cumulative Backlogs

In Table 10, the SBA row shows the average (across 100 replications) of the cumulative backlogs over 100 months when the target FR is 98%. The rows corresponding to SMA13, SES, and Croston's methods indicate respective differences with the SBA value. In the last three rows, a negative figure (in parentheses) represents a lower average, while a positive figure means a higher average. The differences, in absolute terms, are all less than one, indicating that there is hardly any difference in performance with respect to 100-month cumulative backlogs for the given target FR. The same observation holds for a target FR of 95%.

Table 10: Comparison of Mean 100-Month Backlogs (SBA vs. Other Methods) for a 98% Target Fill Rate

| SKU #   | 1      | 2      | 3     | 4      | 5      |
|---------|--------|--------|-------|--------|--------|
| SBA     | 15.16  | 30.89  | 24.28 | 73.73  | 18.47  |
| SMA13   | 0.00   | 0.04   | 0.05  | (0.22) | (0.18) |
| SES     | (0.03) | (0.01) | 0.05  | (0.05) | (0.02) |
| Croston | (0.02) | (0.01) | 0.04  | (0.03) | 0.03   |

| SKU #   | 6      | 7      | 8      | 9      | 10     |
|---------|--------|--------|--------|--------|--------|
| SBA     | 7.49   | 2.27   | 17.28  | 66.53  | 11.19  |
| SMA13   | 0.00   | (0.10) | (0.12) | (0.33) | (0.08) |
| SES     | 0.02   | 0.00   | (0.10) | (0.42) | (0.08) |
| Croston | (0.04) | 0.00   | (0.01) | (0.01) | (0.01) |

| SKU #   | 11     | 12     | 13     | 14     | 15     |
|---------|--------|--------|--------|--------|--------|
| SBA     | 6.44   | 4.73   | 8.04   | 11.07  | 4.66   |
| SMA13   | 0.05   | (0.03) | (0.07) | (0.20) | (0.09) |
| SES     | 0.01   | 0.00   | (0.01) | (0.04) | (0.01) |
| Croston | (0.01) | 0.00   | (0.01) | (0.01) | 0.00   |



Table 11 provides analogous results for a 90% target PNS. With the exception of SKU 9 in which SES and SMA13 respectively exhibit favorable 18-unit and 5-unit average advantages over SBA, the 100-month cumulative backlogs appear to be more or less similar overall across the four methods. We note that SKU 9 is quite lumpy, characterized by relatively high values of  $CV^2$  (3.9472),  $ADI$  (2.2), and  $z$  (54.55%). Similar observations arise under a 95% target PNS.

Table 11: Comparison of Mean 100-Month Backlogs (SBA vs. Other Methods) for a 90% Target PNS

| SKU #   | 1      | 2      | 3      | 4      | 5     |
|---------|--------|--------|--------|--------|-------|
| SBA     | 85.64  | 171.14 | 100.18 | 252.37 | 81.42 |
| SMA13   | (0.17) | 0.83   | (0.66) | 2.56   | 1.45  |
| SES     | (0.40) | 1.20   | (0.71) | (0.85) | 0.90  |
| Croston | 0.31   | 0.50   | 0.23   | (0.51) | 1.26  |

| SKU #   | 6      | 7      | 8      | 9       | 10     |
|---------|--------|--------|--------|---------|--------|
| SBA     | 48.62  | 19.72  | 133.36 | 699.79  | 89.81  |
| SMA13   | 0.45   | (0.13) | 1.28   | (5.26)  | (1.45) |
| SES     | (0.19) | (2.20) | (1.41) | (18.02) | (1.13) |
| Croston | (0.09) | (0.86) | 0.28   | (1.20)  | 0.01   |

| SKU #   | 11     | 12    | 13     | 14    | 15     |
|---------|--------|-------|--------|-------|--------|
| SBA     | 35.67  | 35.78 | 64.28  | 74.35 | 20.03  |
| SMA13   | (0.03) | 0.20  | (0.88) | 1.14  | 0.10   |
| SES     | 0.17   | 0.41  | 0.93   | 1.03  | (1.76) |
| Croston | (0.26) | 0.09  | 0.01   | 0.37  | 0.03   |

## 5. CONCLUSION

Earlier studies have argued that the negative binomial distribution (NBD) satisfies both theoretical and empirical criteria for modeling intermittent demand. We tested the NBD on an industrial dataset involving more than 1000 stock-keeping units (SKUs) in the central warehouse of a firm operating in the professional electronics sector. We have established that the NBD often does not provide a good fit for most of the SKUs tested. We used a two-stage approach (applied preliminarily by Solis, Nicoletti, Mukhopadhyay, Agosteo, Delfino, and Sartiano 2012) involving uniform and negative binomial distributions. In the current paper, we report on 15 SKUs, of which six exhibit erratic demand while the remaining nine have lumpy demand. The simulated demand distributions arising from the two-stage modeling approach more closely approximate the actual demand distributions of the SKUs under consideration. The SMA13, SES, Croston's, and SBA methods are well-referenced in the literature on intermittent or lumpy demand forecasting. We investigated the statistical accuracy of these forecasting methods using three scale-free error statistics. In testing statistical accuracy on the performance block (the final 20 months of the 66-month actual distribution), we found none of the methods under consideration to be consistently superior to the others. However, SBA is found to be the best performing method overall when the four methods are tested over the longer term (100 replications of 100 months, or a total of 10,000 months). We subsequently simulated the inventory control performance of each method, applying the demand estimates on the basis of

the simulated demand distribution for a given SKU. A  $(T,S)$  periodic review inventory control system with full backordering, with a one-month review period and a one-month replenishment leadtime, was assumed. Using either a target fill rate or a target probability of no stockout, we have found SBA to yield the lowest average levels of inventory on hand in almost all cases. At the same time, the expected cumulative backlogs under SBA are comparable to those using the other forecasting methods. Accordingly, it appears that SBA generally leads to better inventory systems efficiency.

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