Abstract
In this paper, the modelling and thermal performance results of the solar air collector have been presented. This system is conceived to heat the ambient air in order to be used in many domains like drying processes and heating buildings. The pseudo-bond graph methodology was used in modelling this system. Such methodology was very suitable for this thermodynamic process since it allows good management of the non-linearity present in the system.

The simulation of the global model with bond graph software [20-sim] allows us to analyze the temperature variations and the efficiency of the solar collector and compared with experiment results.

Keywords: Solar air collector; Temperature; Collector efficiency; Pseudo bond graph modelling.

1. Introduction
It exists not only several manners to supply of the solar energy, but equally different methods to captivate the solar energy proceeding from an incident radiation.

The solar collector absorbs the solar radiation and transforms him in heat transmitted to a fluid (air) who will be used for else applications (Gao W, Lin W, Liu T and Xia C 2007; Grag H.P and Kumar R 2000; Duffie J.A and Beckman W.A 1991; Dickson W.C and Chemisino P.N 1980).

In this paper we analysed thermal performances of the flat plate solar collector based on the bond graph approach. This technique was developed as an appropriate tool for modelling all types of the engineering systems. It is based on the topologic representation of different mechanisms regarding exchange, storage and energy dissipation in any thermodynamic system (Karnopp D and Rosenberg R. 1983). For thermal and chemical engineering, the classic bond graph behaviours introduce complex variables likes entropy and chemical potential, which do not present simple conservation laws. Therefore, we used the pseudo-bond graph methodology where the product effort and flow is not a power. Some research works haves been published in this area (Karnopp D, Margolis D and Rosenberg R 1990; Thomas J. and Ould Bouamama B 2000; Cellier F. E 1991).

2. System description
The solar air collector is used as a generator of hot air, It is a flat-plate collector with simple glazing and classical absorber (metallic undulate plate carrying chicaneries). The air passes through the flat-plate collector between the absorber plate and the glass cover as shown in Figure 1.

The solar air collector is sensitive of two external parameters: solar radiation and wind velocity. The recovered heat proceeds essentially from thermal convection exchanges between the air, the absorber plate and the glass cover.

The studied device admits following assumptions:
• The solar air collector is placed in north-southern direction with inclination angle 37° and exposed to the solar radiation throughout the day.
• The air flow is unidirectional and mass flow is considered as constant along the channel of the solar air collector.
• Radiation heat transfer between all the absorber plate and the insulation, the insulation and the soil are assumed negligible.
• The system studied is considered with lumped parameters, that has an evenly distributed temperature.

Different heats transfers took in consideration are well described in Figure 1.

3. BOND GRAPH MODELLING IN PROCESS ENGINEERING
The dynamic behavior of the thermo fluid processes is generally described by the non linear differential equations.

Their formulation and resolution by the classic methods are limited (Hadj H and Vafai K 1999). With the bond graph methodology, these equations are associated to the storage and the dissipation of energy.
Therefore, the bond graph tools permit by its graphic description to explicit the power exchanges in the system, as the energy storage and the thermal dissipation.

The bond graph method was developed as an appropriate tool for modelling all types of the engineering systems. It is based on the topologic representation of different mechanisms regarding exchange, storage and energy dissipation in any thermodynamic system (Karnopp D and Rosenberg R 1993).

With this methodology, the parameter energy is represented through two kinds of general variables: the power variables (effort and flow) and the energy variables (momentum and displacement). In thermo fluid process, thermal and hydraulic energies are coupled (Thoma J and Ould Bouamama B 2000; Ould Bouamama B 2003). Their coupling can be represented by small rings around the bond as shown in Figure 2.

3.1. Word pseudo bond graph
The technological level of modelling can be represented by word pseudo bond graph model. In this step, one splits the entire system into simple subsystem as shown on Figure 3 where at the entry and the release of each subsystem we have already used the liaison variables corresponding, and this will be based on an energizing description of the process.

3.2. Pseudo bond graph model
If the words of the word pseudo-bond graph shown in Figure 3 are replaced by the corresponding elements, we obtain the pseudo-bond graph shown in Figure 4.

This model can be represented by two sub-models, the red corresponds to the thermal subsystem and the blue to the hydraulic subsystem.

The thermal sub-model has temperature \( T \) as one effort variable, and heat flow \( Q \), enthalpy flow \( H \) and solar flow \( I \) as many flow variables. On the other hand the hydraulic sub-model has pressure \( P \) as effort variable, and mass flow \( m \) as flow variable.

The pseudo bond graph model contains seven elements: capacitive elements \( \mathbf{C} \), resistive elements \( \mathbf{R} \), effort sources \( \mathbf{Se} \), flow sources \( \mathbf{Sf} \), modulated or controlled flow source \( \mathbf{MSf} \), 0 – junctions and 1 – junctions.

\( \mathbf{C} \) and \( \mathbf{R} \) elements are passive elements because they convert the supplied energy into stored or dissipated energy. \( \mathbf{Se} \), \( \mathbf{Sf} \) and \( \mathbf{MSf} \) elements are active elements because they supply power to the system and 0- and 1-junctions are junction elements that serve to connect \( \mathbf{C} \),
Figure 3: Word bond graph model of the solar air collector.

Figure 4: Pseudo bond graph model of the solar air collector.
R, Se, Sf and MSf and constitute the junction structure of the Pseudo Bond Graph model.

3.2. Bond graph elements and deducted mathematical equations

3.3.1. Flow sources

Sf1 and Sf2 are used as input to the thermal sub-system. Sf1 represent the solar flow absorbed by the cover and the corresponding flow is:

\[ I_c = \alpha_c G A_c \]  

(1)

where \( \alpha_c \) is the absorptivity of solar radiation by the glass cover, \( G \) is the solar radiation rate incident on the glass cover (W/m²) and \( A_c \) is the cover area (m²).

Sf2 represent the solar flow absorbed by the absorber and the corresponding flow is:

\[ I_{ab} = 0.96 \tau_c \alpha_{ab} G A_{ab} \]  

(2)

where the factor “0.96” represents the averaged transmittance–absorptance product, \( \alpha_{ab} \) is the solar-radiation absorptive of the absorber plate, \( \tau_c \) is the solar-radiation emissivity of the glass cover and \( A_{ab} \) is the absorber plate area (m²).

Sf3 is used as input to the hydraulic subsystem, it represents the air mass flow entering the collector. The corresponding flow is \( \dot{m}_f \) (kg/s).

MSf3 is a modulated or controlled flow source. It represents the air enthalpy flow entering the collector in the thermal subsystem. This way the enthalpy input of the air can be calculated as the specific enthalpy associated to the input temperature multiplied by the input mass flow:

\[ H_f = \dot{m}_f H_f = \dot{m}_f C_{pf} T_f \]  

(3)

Where \( C_{pf} \) (J/kg°C) is the specific heat of the fluid (air) and \( T_f \) the fluid temperature at the inlet of collector.

3.3.2. Effort sources

Se is only the effort source used in this pseudo bond graph model and it represents the ambient temperature \( T_a \).

3.3.3. C-fields

The C- fields describe the storage energy phenomena and determine the effort variable or the flow according to the fixed causality using this formulation:

\[ \begin{align*}
    e & = \varphi_C^{-1}(\int f dt) \\
    f & = \frac{d}{dt} \varphi_C(e)
\end{align*} \]  

(4)

Cc represents the energy flow accumulation on the glass cover. The temperature of the glass cover is given by:

\[ T_c = \frac{1}{C_c} \int \dot{Q}_c dt \]  

(5)

Where \( \dot{Q}_c \) is the thermal heat flow accumulated on the glass cover and \( C_c \) is thermal capacity of glass cover.

\[ C_c = \rho_c V_c C_{pc} \]  

(6)

With \( \rho_c \) (kg/m³) the density, \( V_c \) (m³) the volume and \( C_{pc} \) (J/kg°C) the specific heat of the glass cover.

Cab represents the energy flow accumulation on the absorber plate; the temperature of the absorber plate is given by:

\[ T_{ab} = \frac{1}{C_{ab}} \int \dot{Q}_{ab} dt \]  

(7)

Where \( \dot{Q}_{ab} \) is the thermal heat flow accumulated on absorber and \( C_{ab} \) is the thermal capacity of the absorber plate (J/°C).

\[ C_{ab} = \rho_{ab} V_{ab} C_{pab} \]  

(8)

With \( \rho_{ab} \) (kg/m³) the density, \( V_{ab} \) (m³) the volume and \( C_{pab} \) (J/kg°C) the specific heat of the absorber plate.

3.3.4. R-fields

In our case, R-fields illustrate the heat transfer phenomena in thermal process; the effort or flow variable are determined with taking causalities into account and using this formulation:

\[ \begin{align*}
    e &= \varphi_R(f) \\
    f &= \varphi_R^{-1}(e)
\end{align*} \]  

(9)

Besides the thermal resistances \( R \) are equal to the inverse of heat transfer coefficient \( h \) (R=1/h)

Rr,c-s field models the radiation heat transfer phenomenon between the glass cover and the sky, the radiative heat flow is given by:

\[ \dot{Q}_{r,c-s} = \frac{1}{R_{r,c-s}} (T_c - T_a) A_c = h_{r,c-s} (T_c - T_a) A_c \]  

(10)

\( A_c \) is the glass cover area (m²) and \( h_{r,c-s} \) (W/m²°C) is the radiation heat-transfer coefficient from the glass cover to sky, the referred to the ambient air temperature \( T_a \) may be obtained as follows (Zhai X.Q, Dai Y.J and Wang R.Z 2005):

\[ h_{r,c-s} = \alpha e_c (T_c + T_s) (T_c^2 + T_s^2) (T_c - T_s) (T_c - T_a) \]  

(11)

where \( \sigma = 5.67 \times 10^{-8} \) W/m²K⁴ is the Stefan–Boltzmann constant, \( e_c \) is the emissivity of thermal radiation of the glass cover and the sky temperature \( T_s \)
is estimated by the formulation given by Swinbank (Swinbank WC 1963) as: 

$$T_a = 0.0552 T_d^{1.5}$$  (12)

**Rr,ab-c** field models the radiation heat transfer phenomenon between the absorber plate and the glass cover. The radiative heat flow is given by:

$$Q_{r,ab-c} = \frac{1}{R_{r,ab-c}}(T_{ab} - T_a)A_{ab} = h_{r,ab-c}(T_{ab} - T_a)A_{ab}$$  (13)

$A_{ab}$ is the absorber plate area (m$^2$) and $h_{r,ab-c}$ (W/m$^2$°C) is the radiation heat-transfer coefficients between the glass cover and the absorber plate and is predicted by:

$$h_{r,ab-c} = \frac{\sigma(T_{ab}^2 + T_c^2)(T_{ab} + T_c)}{1 + \frac{1}{\varepsilon_{ab} + \varepsilon_c} - 1}$$  (14)

where $\varepsilon_{ab}$ is the emissivity of thermal radiation of the absorber plate.

**Rd** field models the conduction heat-transfer phenomena across the insulation; the thermal loss is given by:

$$Q_d = \frac{1}{R_d}(T_{ab} - T_a)A_{ab} = h_d(T_{ab} - T_a)A_{ab}$$  (15)

$h_d$ (W/m$^2$°C) is the conductive heat-transfer coefficient across the insulation and estimated by:

$$h_d = k_i/d_i$$  (16)

Where $k_i$ (W/m°C) is the thermal conductivity of the insulation and $d_i$ (m) is the average mean thickness of the insulation.

**Re,c-a** field models the convection heat-transfer phenomena between the glass cover and the wind, the convective heat flow is given by:

$$Q_{c,c-a} = \frac{1}{R_{c,c-a}}(T_c - T_a)A_c = h_{c,c-a}(T_c - T_a)A_c$$  (17)

$h_{c,c-a}$ (W/m$^2$°C) is the convective heat-transfer coefficient from the glass cover due to the wind and is recommended by McAdams (McAdams W.H 1954) as:

$$h_{c,c-a} = 5.7 + 3.8V_a$$  (18)

Where $V_a$ (m/s) is the wind velocity of the ambient air.

**Re,c-f** field models the convection heat-transfer phenomena between the cover and the fluid, the convective heat flow is given by:

$$Q_{c,c-f} = \frac{1}{R_{c,c-f}}(T_c - T_f)A_c = h_{c,c-f}(T_c - T_f)A_c$$  (19)

$h_{c,c-f}$ (W/m$^2$°C) is the convective heat-transfer coefficients for the fluid moving on the glass cover and on the absorbing plate, are calculated by:

$$h_{c,c-f} = \frac{Nuk_f}{D_h}$$  (20)

Where $k_f$ is the thermal conductivity of air, $D_h = 2WH_f/(W + H_g)$ (in m) is the hydraulic diameter of the air flow channel formed by the glass cover and the absorbing plate, $W$ (m) is the collector width, $H_g$ (m) is the mean gap-thickness between the glass cover and the absorbing plate as sketched in Figure 1, and $Nu$ is the Nusselt number for the convection in the air flow channel.

In free convection, heat transfer takes place through the fluid motion induced by temperature gradients. In such cases, $Nu$ may be expressed as a function of the Grashof and Prandtl numbers $Gr$ and Pr (Monteith J. L 1973):

$$Nu = C(Gr Pr)^n$$  (21)

Where $Gr = \frac{g \beta \Delta T L^3}{\nu^2}$, Pr=0.7  (22)

In which $g$ the gravitational acceleration (m$^2$/s), $\beta$ the thermal expansion coefficient (1/°C), $\Delta T$ the temperature difference (°C), $L$ length of the air channel (m) and $\nu$ the kinematic viscosity of fluid (m$^2$/s).

$C$ and $n$ are constants that depend on the geometry and flow type and have the following value

$C=0.54$ and $n=1/4$, for laminar flow

$C=0.14$ and $n=1/3$, for turbulent flow

**Re,ab-f** field models the convection heat-transfer phenomena between the absorber plate and the fluid, the convective heat flow is given by:

$$Q_{c,ab-f} = \frac{1}{R_{c,ab-f}}(T_{ab} - T_f)A_c = h_{c,ab-f}(T_{ab} - T_f)A_c$$  (23)

$h_{c,ab-f}$ (W/m$^2$°C) is the convective heat-transfer coefficients for the fluid moving on the glass cover and on the absorbing plate.

Where $h_{c,ab-f} = h_{c,c-f} = \frac{Nuk_f}{D_h}$  (24)

**Rech** field is a multi-port element as shown in Figure 4. It is used to model the heat exchange phenomena between the absorber plate, the glass cover and the fluid.

In the heat exchange process in pipes, we find a continuity of the mass flow, but a variation of temperature and thermal enthalpy. The multi-port element has the following constitutive equations:

$$\dot{Q}_u = \dot{H}_f - \dot{H}_r = \bar{m}C_{pf}(T_{fo} - T_f)$$  (25)

Where $\dot{Q}_u$ (W/m$^2$) is the useful energy gain which heats the air in the channel from the inlet temperature.
to the outlet temperature $T_{f_0}$, resulting in the mean air-temperature

$$T_f = \frac{T_{f_0} + T_{f_1}}{2}$$  \hspace{1cm} (26)

So $T_{f_0} = 2T_f - T_{f_1}$  \hspace{1cm} (27)

Replacing (25) in (24) we obtain:

$$T_f = \frac{\dot{Q}_u}{2m_fC_pf} + T_{f_1}$$  \hspace{1cm} (28)

The efficiency of solar heat gain of the heater is

$$\eta = \frac{\dot{Q}_u}{AG} = \frac{2C_pf m_f (T_f - T_{f_1})}{AG}$$  \hspace{1cm} (29)

3.3.5 1-junctions correspond to the equality of flows

$$\sum_i e_i = 0$$

3.3.6 0-junctions correspond to the equality of effort

$$\sum_i f_i = 0$$

and represent energy flow balances

- The energy balance equation for the cover is written as:

$$\dot{Q}_c = I_c + \dot{Q}_{r,ab-c} - \dot{Q}_{c,f} - \dot{Q}_{c,a} - \dot{Q}_{r,c-s}$$  \hspace{1cm} (30)

Where $\dot{Q}_c$ is the heat flow accumulated on the cover, $I_c$ solar flow absorbed by the cover, $\dot{Q}_{r,ab-c}$ the radiation heat flow between the absorber and the cover, $\dot{Q}_{c,f}$ the convection heat flow between the cover and the fluid, $\dot{Q}_{c,a}$ the convection heat flow between the cover and the environment and $\dot{Q}_{r,c-s}$ is the radiation heat flow between the cover and the sky.

- The energy balance equation for the absorber plate is written as:

$$\dot{Q}_{ab} = I_{ab} - \dot{Q}_{r,ab-c} - \dot{Q}_{c,ab-f} - \dot{Q}_d$$  \hspace{1cm} (31)

Where $\dot{Q}_{ab}$ is the heat flow accumulated on the absorber plate, $I_{ab}$ the solar flow absorbed by the absorber, $\dot{Q}_{r,ab-c}$ the radiation heat flow between the absorber and the cover, $\dot{Q}_{c,ab-f}$ the convection heat flow between the absorber and the fluid and $\dot{Q}_d$ is the conduction heat flow between the absorber and the environment through the insulation.

- The energy balance equation for the air inside the collector is written as:

$$\dot{Q}_u = \dot{Q}_{c,f} + \dot{Q}_{c,ab-f}$$  \hspace{1cm} (32)

$\dot{Q}_u$ is the useful energy gain, $\dot{Q}_{c,f}$ the convection heat flow between the cover and the fluid and $\dot{Q}_{c,ab-f}$ is the convection heat flow between the absorber and the fluid.

4. Results and discussion

For the numerical appreciation of the developed model for the solar air collector thermal performance, the calculations have been made by using the system parameters (Table 1) and climatic data for one day of the month of May 2011 (05-05-2011).

The software 20-sim was used for all simulations. It is dedicated to the Bond Graph simulation. Its use is rather simple and direct.

In a series of experiments conducted, data were recorded for different operating variables to determine the performance of the solar air collector. This section presents the experimental results of solar air collector and, also, a comparison with the predicted values.

Some scatter values can be seen in the experimental data. Fluctuations in the meteorological variables have contributed to the scatter in the results.

Table 1: Values of parameters used in the analysis.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Values</th>
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<tr>
<td>$A_{ab}$</td>
<td>2.2 (m²)</td>
</tr>
<tr>
<td>$A_c$</td>
<td>2 (m²)</td>
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<tr>
<td>$C_{p_ab}$</td>
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<tr>
<td>$C_{p_c}$</td>
<td>840 (J/Kg°C)</td>
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<tr>
<td>$C_{p_f}$</td>
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<tr>
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</tr>
<tr>
<td>$d_i$</td>
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<td>$k_i$</td>
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<td>$k_f$</td>
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<tr>
<td>$\rho_f$</td>
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<td>$\upsilon$</td>
<td>1.88×10⁻⁵ (m²/s)</td>
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<tr>
<td>$\tau_c$</td>
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</table>

Figure 5 shows the variation of solar radiation and ambient temperature for the day 05-05-2011.

Figure 6 shows the variation of the collector inlet and outlet air temperatures; it is clearly that the collector outlet air temperature is very sensitive to the variation of the solar radiation rate.

The influence of the solar radiation rate on the collector air outlet temperature and the efficiency is shown in Figures 7 and 9. It can be seen that for a steady sunny day, the air collector outlet temperature and the efficiency increases linearly with the solar radiation rate.

Figure 8 shows that the great efficiency is attained between 15 and 16 h, when the solar radiation rate is greatest to midday. The solar air collector accumulates
Figure 5: Variation of solar radiation and ambient temperature versus time of the day. (The weather conditions on May 5, 2011).

Figure 6: Variation of collector inlet and outlet air temperature versus time of the day.

Figure 7: Effect of the solar radiation on the collector outlet air temperature.

Figure 8: Variation of predicted and measured collector efficiency versus time of the day.

Figure 9: Effect of the solar radiation on the collector efficiency.

Figure 10: Effect of the collector outlet air temperature on the collector efficiency.
the energy in the morning when the solar radiation rate increases and resituates it the afternoon when the solar radiation rate decreases. The efficiency of the solar air collector depends significantly on the solar radiation and surface geometry of the absorber (Karsli S. 2007). Figure 10 shows the variation of the efficiency with the outlet air temperature, the efficiency increases linearly with the outlet air temperature.

5. Conclusion
This paper presents a performance analysis of air heating flat plate solar collector. The results showed that the efficiency depends on the solar radiation and the conception of the solar air collector. The temperature rises varied almost linearly with the incident radiation. In this paper, we have also demonstrated that the thermal performance of a solar air collector can be studied by the bond graph approach.

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REFERENCES


