

# PRECISION TRACKING MOTION CONTROL OF AN XY MICROPOSITIONING STAGE DRIVEN BY STEPPER MOTOR

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## ABSTRACT

This paper aims to present a motion coordination control approach for two axis XY stage system driven by PM stepper motors. The proposed coordinated control approach is based on a three tier composite control structure feed-forward, feedback PID and force disturbance observer. The contouring performance of a biaxial system is studied for circular trajectories and has been improved by using the proposed control scheme. To demonstrate the effectiveness of the control system design based on analysis of stage dynamics, typical results of system performance experiments micropositioning motion with high precision tracking and accurate positioning was obtained by simulations developed using Matlab and Simulink software.

Keywords: motion coordination, disturbance observer, XY stage, stepper motor, position control.

## 1. INTRODUCTION

Precision manufacturing has been steadily gathering momentum and attention over the last century in terms of research, development, and application to product innovation. The driving force in this development appears to arise from requirements for much higher performance of motion precision multi-axis machine. Today, ultra-precision multi-axis machine under computer control has a resolution and positioning accuracy in an order better than micrometers. In the new millennium, ultra-precision manufacture is poised to progress further and to enter the nanometer scale regime (nanotechnology) such as Micro-electro-mechanical Systems (MEMS).

In the past decade, dc motors have been widely used in these systems as high-performance drives due to the relative ease in controlling them. This ease of control is due to the fact that the system equations describing a dc motor are linear. However, there are still disadvantages in using such motors for positioning systems. In fact the mechanical commutators and brush assembly make them much more expensive. Besides, they may produce undesired sparks, which are not allowed in some applications. In particular, for high speed repetitive motion, the brushes are subject to

excessive mechanical wear and consequently lead to a decrease in performance. These inherent disadvantages have prompted continual attempts to find better solution instead of dc motor. An attempt was made to use stepper motor. In fact their ability to provide accurate control over speed and position combined with their small size and relatively low cost make stepper motors a popular choice in a range of applications (F. Nollet, T. Floquet and W. Perruquetti, 2008). In particular, permanent magnet stepper motors deliver higher peak torque per unit weight and have a higher torque to inertia ratio than dc motors. Furthermore, they are more reliable and, require less maintenance, however, using the stepper motor in an open loop configuration results in low performance (G.Grellet, G. Clerc, 2000). Due to technological breakthroughs in digital signal processors, continuous time closed-loop control laws for position regulation and to go ahead and consider feedback for these incremental actuators. The Performance here is much better than the open-loop situation.(Zribi and Chiasson, 1991) considered the position control of stepper motors by exact feedback linearization.(Bodson, Chiasson, Novotnak, and Rekowski, 1993), reported on an experimental implementation of a feedback linearizing controller that guarantees position trajectory tracking by using field-weakening techniques and a speed observer. Accordingly, this study proposes a coordinated control approach based on a three tier composite control structure feed-forward, feedback PID and force disturbance observer (T.K. Kiong, L.T.Heng and H.Sunan, 2008, H.Tlijani and M. Benrejeb, 2010). The proposed approach control is applied to a biaxial XY stage motorized by two rotary bipolar PM stepper motors.

This paper is organized as follows. The stepper model and studied system description are given in Section 2. In section 3, feed-forward, feedback PID are given, and a force observer to suppress force disturbances arising from friction due to movement on motors is designed to solve the position tracking problem. The simulation and results for contouring circle trajectory under the control of the proposed scheme is described in Section 4.

## 2. STUDIED SYSTEM DESCRIPTION

Among the various configurations of long travel and high precision multi-axis machine, one of the most popular is known as the moving gantry. In this configuration, two motors which are mounted on two slides move a load simultaneously in tandem. This gantry system consists of four sub-assemblies, the X and Y-axis sub-assemblies, the planar platform, two bipolar stepper motors, and the end effectors. The system is equipped with a high power density due to the dual drives, and it can yield high speed motion with no significant lateral offset when the actuators are appropriately coordinated and synchronized in motion. The main challenges to address in order to harness the full potential of this configuration are mainly in the control system. In addition to precision motion control of the individual motor, efficient synchronization among them is crucially important to minimize the positional offsets which may arise due to different drive and motor characteristics, non-uniform load distribution of the gantry and attached end-effectors. The studied system is given in, figure.1.

The stepper motors, supplied with the gantry system, provide precise movements in response to electrical voltage pulses. These actuators are permanent magnet stepper motors with two-phases labeled as  $\alpha$  and  $\beta$ . The electrical and mechanical parameters of the two stepping motors are given in appendix. The dynamic equations describing the used stepping motors are composed of three non-linear differential equations (1), (2) and (3) (H.Tlijani, K.B.Saad and M.Benrejeb, 2009, H.Tlijani, B.B.Salah and M. Benrejeb, 2005). These equations give a relation between the stator currents, the voltages and the mechanical quantities: torque, speed and angular position.

$$U_{\alpha} = Ri_{\alpha} + L \frac{di_{\alpha}}{dt} - \Omega k \sin(p\theta) \quad (1)$$

$$U_{\beta} = Ri_{\beta} + L \frac{di_{\beta}}{dt} + \Omega k \cos(p\theta) \quad (2)$$

$$C_r = -k \left[ i_{\alpha} \sin(p\theta) - i_{\beta} \cos(p\theta) \right] \quad (3)$$

$$-D \frac{d\theta}{dt} - J \frac{d^2\theta}{dt^2} - C_f \text{sign}(\Omega)$$

$$\text{where } \frac{d\theta}{dt} = \Omega$$

There are various configurations of gantry stages; many of them are intrinsically similar. A typical gantry stage may be considered as a two-degree of freedom (2DF) servo-mechanism, which can be adequately described by the schematics in fig.1.

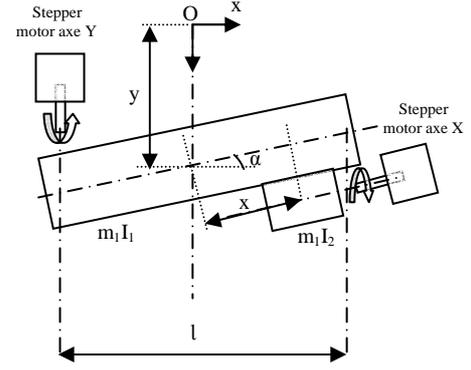


Figure 1: Motion reference using gantry system

A servomotor carries a gantry on which a slider holding the load is mounted. One motor yields a linear displacement  $y$ , while the other yields a linear displacement  $x$ , and also the dynamic loading present due to the translation of the slider along the gantry. The central point of the gantry is thus constrained to move along the dashed line with two degrees of freedom. The displacement of this central point from the origin  $O$  is denoted by  $y$ . The gantry may also rotate about an axis perpendicular to the plane of fig.1 due to the deviation between  $x$  and  $y$ , and this rotational angle is denoted by  $\alpha$ . The slider motion relative to the gantry is represented by  $x$ . It is also assumed that the gantry is symmetric and the distance from its central point to the slider mass center is denoted by  $d$ . With this formulation of the gantry stage, it is imminent to proceed with the dynamic modeling of the gantry stage. This model has been introduced by (T.K.Kiong, L.T.Heng and H.Sunan, 2008, Chuan. Shi, Peqing.Ye, Qiang.Lv, 2006).

Let  $m_1, m_2$  denote the mass of the gantry and slider respectively,  $l$  denotes the length of the gantry arm,  $I_1, I_2$  denote the moment of inertia of the gantry arm and slider respectively, we assume that

$$I_1 = m_1(l/2)^2, I_2 = m_2(l/2 + x)^2$$

$$X = [x \quad \alpha \quad y]^T$$

$$y = y_2 + (y_1 - y_2)/2$$

The positions of  $m_i, i=1,2$  are given by

$$x_1 = 0$$

$$y_1 = y$$

$$x_2 = x \cos \alpha + d \sin \alpha$$

$$y_2 = y + d \cos \alpha - x \sin \alpha$$

Which lead to the corresponding velocities.

$$v_1 = \begin{bmatrix} 0 \\ \dot{y} \end{bmatrix}$$

$$v_2 = \begin{bmatrix} \dot{x} \cos \alpha - x \dot{\alpha} \sin \alpha + d \dot{\alpha} \cos \alpha \\ \dot{y} - d \dot{\alpha} \sin \alpha - \dot{x} \sin \alpha - x \dot{\alpha} \cos \alpha \end{bmatrix}$$

Thus, the total kinetic energy may be computed as

$$K = \frac{1}{2} m_1 v_1^T v_1 + \frac{1}{2} m_2 v_2^T v_2 + \frac{1}{2} (I_1 + I_2) \dot{\alpha}^2$$

$$= \frac{1}{2} (m_1 + m_2) \dot{y}^2 + \frac{1}{2} (I_1 + I_2 + m_2 x^2 + m_2 d^2) \dot{\alpha}^2$$

$$+ \frac{1}{2} m_2 \dot{x}^2 - \dot{y} \dot{\alpha} m_2 (d \sin \alpha + y \cos \alpha) - \dot{x} \dot{y} m_2 \sin \alpha$$

$$+ \dot{\alpha} \dot{x} m_2 d$$

This can be further written as

$$K = \frac{1}{2} \dot{X}^T I \dot{X} \quad (4)$$

Where  $I$  is the inertia matrix given by:

$$I = \begin{bmatrix} m_1 + m_2 & -m_2 d \sin \alpha - m_2 x \cos \alpha & -m_2 \sin \alpha \\ -m_2 d \sin \alpha - m_2 x \cos \alpha & I_1 + I_2 + m_2 x^2 + m_2 d^2 & m_2 d \\ -m_2 \sin \alpha & m_2 d & m_2 \end{bmatrix}$$

The elements of the Coriolis and centrifugal matrix  $A$  can be derived from

$$A_{ij} = \sum_{k=1}^3 a_{ijk} \dot{q}_k$$

Where  $\dot{q}_1, \dot{q}_2$  and  $\dot{q}_3$  represents the derivative of  $x, \alpha$  and  $y$  respectively, and  $c_{ijk}$  are computed as:

$$a_{ijk} = \frac{1}{2} \left( \frac{\partial e_{ij}(q)}{\partial q_k} + \frac{\partial e_{ik}(q)}{\partial q_j} + \frac{\partial e_{jk}(q)}{\partial q_i} \right) \quad (5)$$

Where  $e_{ij}$  represents the element of the inertia matrix  $I$ . Substituting the assumed inertia equation into equation (5), matrix  $A$  can be expressed as

$$A = \begin{bmatrix} 0 & x \dot{\alpha} \cos \alpha - d \dot{\alpha} \cos \alpha - \dot{x} \cos \alpha & -\dot{\alpha} \cos \alpha \\ x \dot{\alpha} \cos \alpha - d \dot{\alpha} \cos \alpha - \dot{x} \cos \alpha & (x \sin \alpha - d \cos \alpha) \dot{y} - \left(\frac{l}{2} + 2x\right) \dot{x} & \left(\frac{l}{2} + 2x\right) \dot{\alpha} - \dot{y} \cos \alpha \\ -\dot{\alpha} \cos \alpha & \left(\frac{l}{2} + 2x\right) \dot{\alpha} - \dot{y} \cos \alpha & 0 \end{bmatrix}$$

Where,  $\cos \alpha = c\alpha, \sin \alpha = s\alpha$ , finally the dynamic model is expressed as

$$I \ddot{X} + A \dot{X} + B F = B U \quad (6)$$

Where

$$B = \begin{bmatrix} 1 & 1 & 0 \\ l \cos \alpha & -l \cos \alpha & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$F = \begin{bmatrix} F_x & F_\alpha & F_y \end{bmatrix}^T, U = \begin{bmatrix} u_x & u_\alpha & u_y \end{bmatrix}^T$$

$F_x, F_\alpha, F_y$  are the frictional forces, and  $u_x, u_\alpha, u_y$  are the generated mechanical forces.

### 3. COORDINATED CONTROL SCHEME

In the proposed coordinated control approach, motors are assigned to horizontal axis  $x$  and vertical axis  $y$ . A supervisory motion program drives the axis through these actuators which share an identical commanded trajectory pre-planned. Each servo loop then has the responsibility of keeping the actual trajectory as closely as possible to the commanded trajectory, since each of motors has its own individual servo loop. Presuming they have tight servo loops, this method provides a tight and smooth link between the motors. Figure.2 provides a block diagram of the proposed coordinated control scheme.

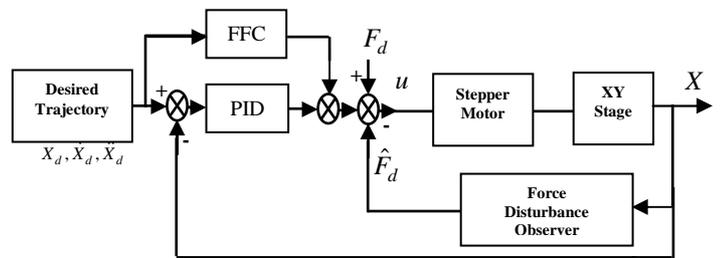


Figure 2: Block diagram of the positioning control system

As shown in the above scheme, a three tier composite control structure is adopted: feed-forward control, feedback control and force disturbance observer. This design possesses several important and useful features. First, it incorporates a feed-forward component to facilitate a high speed response. The feed-forward component addresses model based characteristics relating to the stepper motors. Second, an optimal PID feedback controller is designed and intended to provide optimal command response and stability properties. Third, since the achievable performance of a precision positioning system is unavoidably and very significantly limited by the amount of disturbances present, and the uniformity of their distribution among the motors, a disturbance observer is augmented to the composite controller structure. It provides a fast response to load disturbances and other exogenous signals acting asymmetrically on the two motors. This feature is especially useful since load disturbances are major factors affecting the control performance, especially when the motors jointly carry a dynamical and asymmetrical load such as an additional servo system running across the system (Yo.Tomita, K.Makino and M.Sugimine, 1996). It is used to estimate the actual disturbance, deduced from a disturbance observer, to compensate for the disturbances force.

### 3.1. Feed-forward Control

The servo system at equation (3) can thus be alternatively described by:

$$\begin{aligned}\ddot{\theta} &= -\frac{D}{J}\dot{\theta} - \frac{k}{J}\left[i_{\alpha}\sin(p\theta) - i_{\beta}\cos(p\theta)\right] - \frac{C_r}{J} - \frac{C_f}{J}\text{sign}(\dot{\theta}) \\ \ddot{\theta} &= -\frac{D}{J}\dot{\theta} + \frac{k}{J}\left[-i_{\alpha}\sin(p\theta) + i_{\beta}\cos(p\theta)\right] + \frac{k}{J}g(\theta, \dot{\theta}) \\ g(\theta, \dot{\theta}) &= -\left[\frac{C_r}{k} + \frac{C_f}{k}\text{sign}(\dot{\theta})\right]\end{aligned}$$

$g(\theta, \dot{\theta})$  is assumed to be a smooth non-linear function which may be unknown. With the tracking error  $e$  defined as:  $e = \theta_d - \theta$ , we have :

$$\begin{aligned}\ddot{e} &= -\frac{D}{J}\dot{e} - \frac{k}{J}\left[-i_{\alpha}\sin(p\theta) + i_{\beta}\cos(p\theta)\right] - \frac{k}{J}g(\theta, \dot{\theta}) \\ &\quad - \frac{k}{J}\left[-\frac{J}{k}\ddot{\theta}_d - \frac{D}{k}\dot{\theta}_d\right]\end{aligned}\quad (7)$$

the system state variables are assigned as:

$$x_1 = \int_0^t e(t)dt, \quad x_2 = e, \quad x_3 = \dot{e}$$

denoting:  $X = [x_1 \quad x_2 \quad x_3]^T$ , the equation (7) can then be put into the equivalent state space form:

$$\dot{X} = \Lambda X + \Gamma[-i_{\alpha}\sin(p\theta) + i_{\beta}\cos(p\theta)] + \Gamma g(\theta, \dot{\theta}) + \Gamma\left[-\frac{J}{k}\ddot{\theta}_d - \frac{D}{J}\dot{\theta}_d\right] \quad (8)$$

$$\Lambda = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & -\frac{D+D_0}{J} \end{bmatrix}, \quad \Gamma = \begin{bmatrix} 0 \\ 0 \\ \frac{k_{\phi}}{J} \end{bmatrix}$$

The design of the feed-forward control law is straightforward. From equation (8) the term:

$$\Gamma\left[-\frac{J}{k}\ddot{\theta}_d - \frac{D}{J}\dot{\theta}_d\right]$$

may be neutralised using a feed-forward control term in the control signal. The feed-forward control is thus designed as:

$$u_{FF}(t) = \frac{J}{k}\ddot{\theta}_d + \frac{D}{J}\dot{\theta}_d \quad (9)$$

Clearly, the reference position trajectory must be continuous and twice differentiable; otherwise a pre-compensator to filter the reference signal will be necessary. The only parameters required for the design of the feed-forward control are the parameters of the second-order linear model.

### 3.2. Feedback Control

In the composite control system, PID is used as the feedback control term. While the simplicity in a PID structure is appealing, it is also often proclaimed as the reason for poor control performance whenever it occurs. In this design, advanced optimum control theory is applied to tune PID control gains. The PID feedback controller is designed using the Linear Quadratic Regulator (LQR) technique for optimal and robust performance of the nominal system. The nominal portion of the system is given by:

$$\dot{X} = \Lambda X + \Gamma[-i_{\alpha}\sin(p\theta_m) + i_{\beta}\cos(p\theta_m)]$$

$$u_{PID} = \Psi X = \psi_1 x_1 + \psi_2 x_2 + \psi_3 x_3$$

This is a PID control structure which utilizes a full-state feedback is well known in modern optimal control theory. The PID control is given by

$$u_{PID} = -R^{-1}\Gamma^T \Delta X \quad (10)$$

Where  $\Delta$  is the positive definite solution of the Riccati equation :

$$\Lambda^T \Delta + \Delta \Lambda - \Delta \Gamma R^{-1} \Gamma^T \Delta + Q = 0$$

### 3.3. Design of the Disturbance Observer

As the achievable, performance of a precision positioning is unavoidably and significantly limited by the amount and the uniformity of disturbances, among the motors. A disturbance observer is augmented to the composite control structure to provide a fast response to load disturbances and other exogenous signals acting asymmetrically on the actuators. In figure.3,  $X, u, F$  and

$\hat{F}_d$  denote the position signal, control signal, actual and observed disturbance force signal associated with the axis system.  $H$  and  $H_n$  denote respectively the actual system, and the nominal system.

$$H_n(p) = \frac{a_0}{p^l(p^{m-1} + a_1p^{m-2} + \dots + a_{m-1}p + a_m)}$$

Here, a third order model will be used,  $l = 1, m = 2$  :

$$H_n(p) = \frac{K}{p(Tp + 1)} \quad (10)$$

The disturbance observer incorporates the inverse of the nominal system, and thus a low pass filter  $G$  is required to make the disturbance observer proper and practically realisable. For the choice of a second order model  $H_n$ , a suitable filter is:

$$G(p) = \frac{g_2}{p^2 + g_1p + g_2} \quad (11)$$

the disturbance observer is equivalent to an additional disturbance compensator  $C_{obs}$ , which closes a fast inner loop. It can be shown that:

$$C_{obs} = \frac{G}{1-G} H_n^{-1} \quad (12)$$

For the choice of  $H_n$  and  $G$ , it follows that

$$C_{obs} = \frac{g_2(Tp + 1)}{K(p + g_1)} \quad (13)$$

Therefore,  $C_{obs}$  can be considered as a lead/lag compensator by appropriately designing  $g_1$  and  $g_2$  relatively to  $K$  and  $T$ .

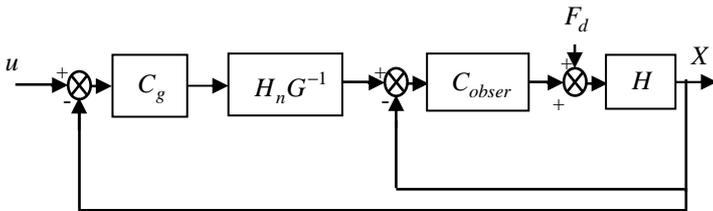


Figure 3: Control system with disturbance observer

The disturbance observer can be designed in many ways. One possible approach is given as follows:

- Identify the nominal model  $K$  and  $T$ , based on which the outer loop controller  $C_f$  can be designed to achieve a desired command response.
- Adjust  $g_1$  and  $g_2$  of the disturbance compensator  $C_{obs}$  to satisfy requirements for robustness and disturbance suppression characteristics. The system sensitivity function and the system transmission function can thus be set independently.
- Carry out simulation and fine tuning till the performance is acceptable.

#### 4. SIMULATION AND RESULTS

Real-time simulation is carried out on biaxial system with one motor and digital encoders each along the x and y direction. The control task in the simulation is to execute planar motion with circular trajectory as straightly and precisely as possible. In the simulation, the two motors have the same dynamical properties. Simulation results are provided to illustrate the effectiveness of the control scheme. The figures (4,5) illustrate respectively the position, and the velocity of axis x and y necessary for the synchronization of biaxial motion. The tracking performance achieved from the use of composite control is given in figure.7, showing that a maximum tracking error of less than 0.01 mm is achieved in the generation of circle profile figure 6.

#### 5. CONCLUSION

This study has presented the coordinated control approach based on three composite controllers, feedback component (PID), a feedforward component (FFC) and a force disturbance observer component. It has shown that is satisfied the desired velocity and acceleration of a stepper motor in the control mode of the incremental motion. It address, also several important challenges to the design of precision motion coordination for two axis XY stage system driven by PM stepper motors. To provide evidence on the effectiveness of the control system design simulation results are obtained by using Matlab and Simulink. It has also shown that the proposed method is more suitable than the conventional method.

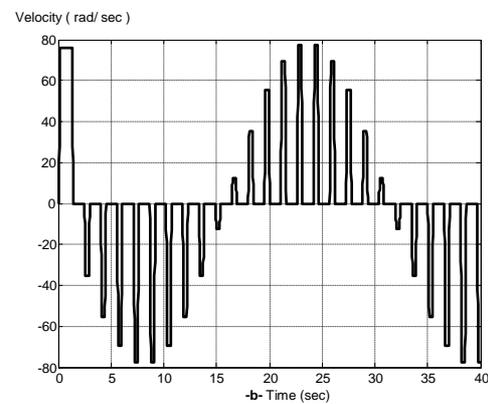
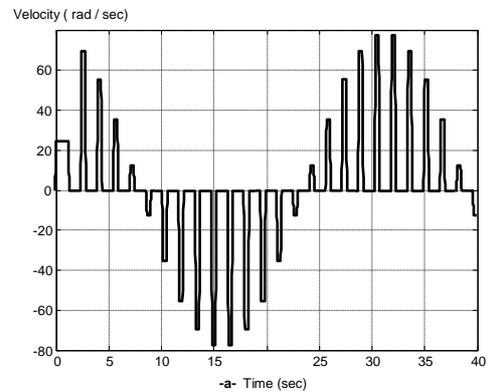


Figure 4: Velocity profile for circular motion: a) axe x – b) axe y

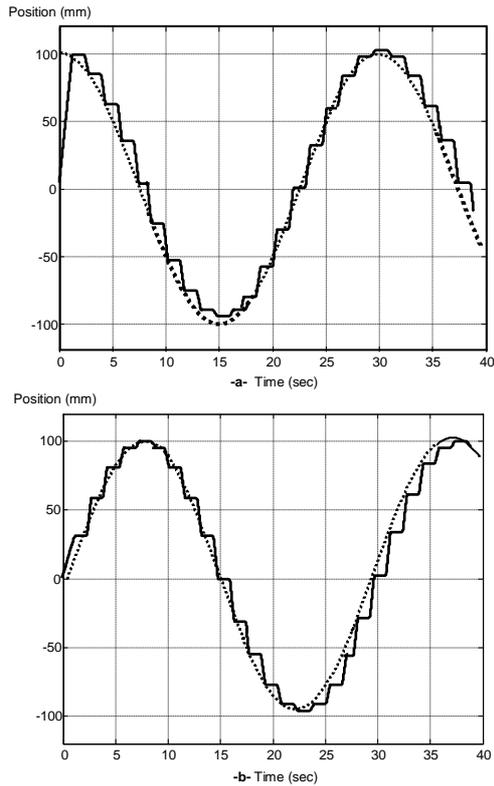


Figure 5: Position profile for circular motion:  
a) axe x - b) axe y, desired position (dotted line)

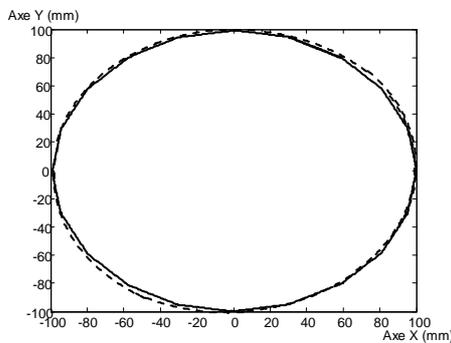


Figure 6: Circular tracking motion: desired circle profile  
(dotted line)

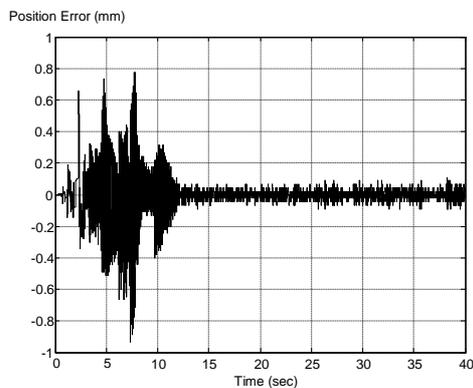


Figure 7: Tracking error position for circular motion

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