ABSTRACT
This paper considers a corrugator trim problem for a cardboard boxes manufacturing plant that produces boxes of various sizes for a finished goods inventory that services known customer demand. We present a suboptimal three-step procedure that considers minimizing both trim waste cost and setup time cost where setup cost is measured as the loss of production resulting from stock rolls size changes. The procedure leads to the least-cost method of combining customer orders on the corrugator and the optimal corrugator width to use over an entire shift. The proposed method was motivated by our analysis of the day-to-day scheduling of the corrugator at the UniPaK facility one of the leading manufacturer of cardboard boxes in Lebanon. This paper concludes with an application of the proposed method to derive a lower cost corrugator schedule at UniPak.

Keywords: Corrugator scheduling, heuristic, decision support system

1. INTRODUCTION
The corrugator trim problem is defined in the literature as the problem of determining the least-cost method of combining customer orders on a corrugator where one of the major costs to avoid is waste or excess trim lost from the material used.

The corrugator trim problem is a well studied problem in the literature. There have been many attempts to solve this problem using computer models, heuristics and optimal methods (Haessler and Talbot 1983). Early work was done by Eismann (1957) and Gilmore and Gomory (1961) which proposed linear programming formulations for the general trim problem in the paper industry. It was recognized later that because of the nature of the production process, the corrugator problem could not be completely modeled by means of linear programming which led to the development of sequential heuristic procedures. Viswanthan and Bagchi (1993) developed a best-first tree search algorithm to solve a constrained two-dimensional cutting stock problem where constraints are set on the number of sheets of given dimensions to be cut using only orthogonal guillotine cuts only.

Minimizing trim loss is only one of several major concerns that arise in drawing the optimal corrugator schedule. Other concerns are corrugator width utilization, cutting pattern changes (order changes), avoidance of split orders, and shutdowns costs. The problem becomes far more complex if the corrugator scheduling problem is integrated with other problems of the multi-stage production process, machine failures and unpredictability of customer behavior to ensure “on-time and in-full” deliveries of customer orders (Darley and Sanders 2004).

Krishagina et. al (1998) considered the cutting-stock problem subject to random customer demand and where the objective is to minimize long run expected average costs related to paper waste, shutdowns, backordering, and holding finished good inventory. They used a 2-step procedure with a linear programming model in the first step and a Brownian control in the second step to generate a suboptimal solution to the problem. Simplifying assumptions such as aggregating machines and dedicating machines to the production of a single grade single color papers, and not explicitly modeling shutdown and startup times were used to be able to find a good solution to the problem.

Given the complexity of the corrugator scheduling problem, it would be impossible to make optimal decisions that will achieve all the desired objectives. Thus a hierarchical heuristic approach has been typically adopted to solve the corrugator scheduling problem and this by decomposing it into smaller problems that are solved sequentially. This is why the corrugator trim problem is still for the most part solved manually.

The approach proposed in this paper was motivated by our observations at Unipak, one of the largest cardboard manufacturers in Lebanon and Unipak’s interest in developing a method for scheduling jobs on the corrugator that would minimize both trim waste cost and setup cost resulting form roll width and cutting pattern changes on the corrugator. More specifically, this paper looks at the tradeoff between minimizing trim loss and maximizing roll width utilization in order to minimize waste in material and setup times incurred by changing the roll width and the cutting patterns at every order run. The approach proposed in this paper is a sequential heuristic
programming one where a 3-step sequential procedure is used to draw an optimal daily schedule for UniPak.

2. PRACTICAL CONSIDERATIONS IN SCHEDULING JOBS ON CORRUGATORS

The manufacture of corrugated cardboard boxes involves fabricating a continuous strip of corrugated board then cutting it into sheets of customer-specified dimensions. The corrugator forms a linerboard strip into a fluted shape then sandwiches it between two liners to produce a single-wall board. There are a number of common flute styles. The corrugated strip is next passed over a long set of rollers to allow sufficient time for drying. It is here that the first trim waste is incurred because the board edges are rough and irregular, about 1 cm is removed from each side of the strip. Then the corrugator board is cut into smaller strips corresponding to the specified sheet width, and cut-off to yield the sheet length. Cut-off knives and slitting knives are used to make horizontal cuts and vertical cuts respectively. Most trim waste occurs during the slitting/cut-off stage. The amount of waste is determined by the width of the corrugated strip being produced. This is why linerboard rolls exist in a number of different sizes. Changing from a narrower roll width to a larger roll width normally slows down the machine.

Corrugated boxes are bulky and subject to weather damage, hence the manufacturer would want to have low inventory levels and frequent production runs to ensure on-time delivery. To this end, the production scheduler typically goes through the following steps to schedule customer orders on the corrugator in order to minimize excess trim waste.

- **Selection and classification of Jobs:** The planner divides and arranges the factory tickets received from the customer service department first based on the fluting type of each order and on the liner (paper) type and color. The scheduler then arranges the different sets of factory tickets in ascending order of the different layers GSM (gram per square meter) and not the overall GSM and further group them into sets of orders that have the same GSM requirements within a 5% tolerance (difference). It is a common practice in the corrugated cardboard industry to quality-upgrade orders for practical considerations.

- **Combining Jobs:** The planner combines jobs within a group that can be processed simultaneously on the corrugator in order to minimize trim waste. In combining jobs, the planner has to determine the roll width based on the number of sheets that can be produced for each job per horizontal cut. The roll length is determined later based on the total number of sheets to be produced for each job to satisfy the order quantity. The planner can increment or decrement the order quantity of any job in order to adjust the total length of roll needed to meet the demand of both jobs in a combination that minimizes linear trim waste. This is another common practice in the corrugated cardboard industry where orders specify the quantity with overrun and undertrun tolerances, typically a 10% tolerance. Also in combining jobs, the planner has to accommodate many practical considerations in combining jobs. For example, a maximum of four sheets can be generated per cut on one cutoff and a maximum of six to seven sheets can be generated as a total on both cutoffs. This is important to control serious bottleneck problem down the line at the finishing stage.

- **Sequencing Jobs on the corrugator:** The last step is to schedule the jobs on the corrugator based on the availability of the finishing machines and the due date priority of the orders.

Setup times are incurred when changing rolls and in axle changeovers. Setup times vary with the type of change; from our observation of Unipak operations, an average 12 minutes are needed to change the linerboard roll width and 10 minutes to change the fluting type to the next thicker or thinner fluting type. As for the triplex or triple axle changeovers, 5 minutes are needed on average to fix and set the next order cutting pattern. In this paper an average of 12 minutes setup time is used for roll-width changes including triplex changeovers if any.

It has also been noted that the corrugator speed varies between a maximum of 160 Mpm (Meters/min) and a minimum of 60 Mpm depending on the order length. The smaller the order length is, the slower the corrugator speed. In this paper and for the purpose of assessing the savings in the proposed scheduling approach, we shall assume an average corrugator speed of 100 Mpm. Consequently the money value of the unit setup time could be estimated by multiplying the corrugator speed by the product of the setup time required and the unit cost of paper which is assumed in this paper to be 0.07$ per meter.

3. CORRUGATOR SCHEDULING APPROACH FOR MINIMIZING WASTE AND SETUP TIME

This section presents a three-step suboptimal procedure for drawing the daily corrugator schedule. The procedure returns the least-cost combination of orders to be scheduled on the corrugator over a given shift along with the optimal corrugator roll width to be used to produce all customer orders scheduled for production over that shift at minimal trim loss. Fixing the roll width over the whole shift will reduce setup time incurred by changing roll width for different order runs. The output is a roll width used over an entire production shift and the optimal combination of
customer orders that will be processed during that shift that minimizes trim loss.

The following is a sequential heuristic programming approach for scheduling jobs on a corrugator while minimizing trim loss and maximizing roll width utilization.

1. Solve the Job Selection problem to identify the set $S$ of all possible pairs of jobs $(i, j)$ that can be combined based on paper type, color, fluting and GSM requirements.
2. Fix the roll width $k$ and solve the Job Matching problem to find, for each roll width and for each pair of jobs $(i, j)$ in $S$, the optimal number of horizontal and vertical cuts that minimize total trim waste $W_{ijk}$.
3. For each roll width $k$, solve the Roll Width Optimization problem to find the optimal combination of customer orders that minimizes trim waste for roll width $W_{k}$. Then select the best roll width that minimizes geometric waste and setup time based on equivalent dollar values.

3.1. Step 1: Solve Job Selection Problem

The corrugator under consideration has 2 cutoffs and thus can process 2 different orders at the same time. Jobs are first classified as primary or secondary depending on their due date. Primary jobs are jobs that should be produced now in order to meet their delivery date. Secondary jobs are those that can be run along a primary job in order to minimize trim waste. Orders are paired if they can be processed simultaneously on the corrugator. A pair of orders is formed by combining a primary job with another primary or secondary job provided that the second job has:

- The same fluting type and paper color as the first job
- A gram per square meter (GSM) requirement within 5% of the first job GSM requirement.

Figure 1 shows the algorithm used for solving the Job Selection problem. The Job Selection problem is solved on Excel. The program returns the set of paired orders $S$ that can be processed simultaneously, where $S = \{(i,j) ; i \in POrders; j \in AOrders; P_{ij} = 1\}$. Each pair $(i, j)$ is a possible solution for the Job Matching problem.

3.2. Step 2: Solve Job Matching Problem

Given $S$ the set of paired orders generated in step 1 and $R$ the set of roll widths available, where $R$ for the case of Unipak is $R = \{1.8, 1.9, 2, 2.05, 2.1, 2.10, 2.15, 2.2\}$ in meters, the Job Matching problem consists of finding, for a set roll width $k$, the optimal number of vertical and horizontal cuts for each combination of jobs $(i, j)$ in $S$, that minimizes the geometric trim waste $W_{ijk}$.

**Job_Selection(POrders, AOrders)**

```plaintext
{  
  • Get number ($Np$) of primary orders ($POrders$) and number ($N$) of all orders ($AOrders$). $POrders$ is a subset of $AOrders$. $AOrders$ is ordered by listing $POrders$ first.
  • Get fluting requirements ($F$) of $AOrders$
  • Get paper color ($C$) of $AOrders$
  • Get GSM requirements ($Gsm$) of $AOrders$
  For $i = 1$ to $Np$ do
  |
  |
  For ($j = i+1$ to $N; j++$) |
  |
  if OR {$F_i \neq F_j; C_i \neq C_j$} then $P_{ij} = 0$
  else if $\frac{ABS}{Max\{Gsm_i, Gsm_j\}} \geq 0.05$
  then $P_{ij} = 0$
  Else $P_{ij} = 1$
  
  Figure 1: Job_Selection Algorithm
}
```

There are a number of practical considerations and machine limitations that should be observed in qualifying paired jobs in $S$ for further consideration. Some of these practical considerations and limitations are:

- the total width required for combining the two orders should not exceed the corrugator roll width size
- the two orders combined would need the same run length with a ±10% margin.
- the total number of sheets produced on the two cut-offs should not exceed seven sheets for the case considered in this paper
- the number of sheets produced per cut on each of the upper and lower cut-off should not exceed four sheets also for the case considered in this paper.

Figure 2 shows the algorithm used for solving the Job Matching problem. The algorithm returns for each roll width $k$ in $R$ the set $S_k$ of paired jobs $(i, j)$ from $S$ that can be processed together on the corrugator using a roll width $k$ and the minimum trim waste $W_{ijk}$ resulting from running them together on the corrugator. Note that if the two jobs in a pair cannot be processed
simultaneously on the corrugator for a given roll width because of roll width limitations and thus the number of vertical cuts for either one of the 2 jobs is zero, the pair \((i, j)\) is removed from the set \(S\) and thus from further consideration.

\[
\text{Job\_Matching} (S, R, S_k) \\
\{ \\
\quad \text{Forall } k \in R, \text{ do} \\
\quad \quad \{ \\
\quad \quad \quad \text{Forall } (i, j) \in S, \text{ do} \\
\quad \quad \quad \quad \{ \\
\quad \quad \quad \quad \quad \text{Solve nb\_verticalcuts to find optimal number of} \\
\quad \quad \quad \quad \quad \text{vertical cuts per horizontal cut for order } i (V_{ij}) \text{ and} \\
\quad \quad \quad \quad \quad \text{order } j (V_{ji}) \text{ that minimize linear trim waste} \\
\quad \quad \quad \quad \quad \text{If } V_{ij} \neq 0 \text{ and } V_{ji} \neq 0 \text{ then} \\
\quad \quad \quad \quad \quad \quad \text{Solve nb\_horizontalcuts to find total} \\
\quad \quad \quad \quad \quad \quad \text{length of roll needed to satisfy demand of} \\
\quad \quad \quad \quad \quad \quad \text{primary order } i \text{ while minimizing} \\
\quad \quad \quad \quad \quad \quad \text{geometric trim waste } W_{ijk} \text{ for the} \\
\quad \quad \quad \quad \quad \quad \text{combination } (i, j, k) \\
\quad \quad \quad \quad \quad \quad \text{Append } (i, j, W_{ijk}) \text{ to the set } S_k \\
\quad \quad \quad \} \\
\quad \quad \} \\
\quad \} \\
\{ \\
\quad \text{Forall } i \in POrders, \text{ do} \\
\quad \quad \{ \\
\quad \quad \quad \text{Compute number of vertical cuts per} \\
\quad \quad \quad \quad \text{horizontal cut for order } i \text{ by dividing } k \text{ with the} \\
\quad \quad \quad \quad \text{sheet width of order } i \\
\quad \quad \quad \text{Compute total length of roll needed to satisfy} \\
\quad \quad \quad \quad \text{demand of order } i \text{ by dividing order quantity} \\
\quad \quad \quad \quad \text{of } i \text{ by the number of vertical cuts found} \\
\quad \quad \quad \text{Compute geometric trim waste } W_{iik} \text{ for the} \\
\quad \quad \quad \text{combination } (i, i, k) \\
\quad \quad \quad \text{Append } (i, i, W_{iik}) \text{ to the set } S_k \\
\quad \quad \} \\
\} \\
\}
\]

Figure 2: Job\_Matching Algorithm

3.2.1. Get Optimal Number of Vertical Cuts Per Horizontal Cut (nb\_verticalcuts)

Given a roll width \(k\) and a given combination of jobs \((i, j)\), let \(W_i, L_i, \text{ and } D_i\) be the width, length, and demand in number of sheets for order \(i\). Similarly, let \(W_j, L_j, \text{ and } D_j\) represent the requirements for order \(j\). The following program finds the optimal number of sheets for orders \(i (V_{ij})\) and \(j (V_{ji})\) that can be generated per horizontal cut in order to minimize linear trim waste generated \(LW_{ijk}\).

\[
\text{Min } LW_{ijk} = k - V_{ij}W_i - V_{ji}W_j \\
\text{Subject to} \\
V_{ij}W_i - V_{ji}W_j \leq K - 0.025 \\
D_iL_iV_{ij} \geq 0.9D_iL_iV_{ji} \\
D_jL_jV_{ij} \leq 1.1D_jL_jV_{ji} \\
V_{ij} + V_{ji} \leq 7 \\
V_{ij} \leq 4 \\
V_{ji} \leq 4 \\
V_{ij} \text{ and } V_{ji} \text{ integers}
\]

The constraints in (1) take into account some practical considerations and limitations of the corrugator machine highlighted earlier. In particular, the first and second constraints limit possible job combinations to those that would require the same run length plus or minus a 10% acceptable margin. The above program could be easily adjusted to allow the production of partial orders by relaxing this constraint and updating the list of orders in \(S\) by adding the residual orders with updated remaining demand. This program was solved using LINGO and the optimal number sheets, \(V_{ij}\) and \(V_{ji}\) that minimize the linear trim waste for the combination \((i, j, k)\) is exported to excel for further processing.

3.2.2. Get Optimal Number of Horizontal Cuts (nb\_horizontalcuts)

Given a roll width \(k\) and a job combination \((i, j)\), and given the optimal number of sheets per horizontal cut \(V_{ij}\) and \(V_{ji}\) for orders \(i\) and \(j\) respectively, the following program returns the optimal number of horizontal cuts \(H_{ij}\) and \(H_{ji}\) for orders \(i\) and \(j\) respectively that minimize the geometric trim waste \(W_{ijk}\) for the combination \((i, j, k)\).

\[
\text{Min } W_{ijk} = (k - V_{ij}W_i - V_{ji}W_j)H_{ij}L_i \\
\text{Subject to} \\
0.9D_i \leq V_{ij}H_{ij} \leq 1.1D_i \\
0.9D_jL_i \leq H_{ji}L_jV_{ij} \leq 1.1D_jL_j \\
H_{ij} \text{ and } H_{ji} \text{ integers}
\]

The first constraint in (2) ensures that the total number of sheets produced for each order meets the required demand of order \(i\) within a 10% tolerance range. The second constraint ensures that the total vertical length of the paper roll required is the same for the two orders combined within the 10% acceptable margin. The above model is solved using LINGO. The data is imported from excel and the solution was exported back to the same excel sheet.
3.3. Step 3: Solve Roll Width Optimization Problem

The Roll Width Optimization problem consists of finding, for a set roll width, the optimal combination of orders from \( S_k \) that minimize total trim waste \( W_k \) and where all primary orders are processed either separately or paired with other primary or secondary orders. This process is repeated for all roll widths available and the total trim waste \( W_k \) for each roll width \( k \) is computed.

The roll width with minimum \( W_k \) is selected and used for production during the day/shift under consideration. Note that \( S_k \) is the set of paired jobs that was computed in step 2 where

\[
S_k = \{(i, j, W_{ijk})| i \in POrders, j \in AOrders, j \neq i, V_q \neq 0, V_p \neq 0\}
\]

The roll width optimization problem is modelled and solved as an assignment problem. Let \( O \) be the set of all orders \( i \) and \( j \) such that \( (i, j) \in S_k \). The cost matrix would consist of the waste values \( W_{ijk} \) and \( W_{iik} \) found in step 2. Note that the cost matrix is symmetrical because \( W_{ijk} = W_{iik} \) for a pair of jobs \( i \) and \( j \). The optimal combination of jobs to run on the corrugator for a set roll width \( k \) is found by solving (3).

\[
W_k = \sum_{i \in O} \sum_{j \in O} 0.5Z_{ij}W_{ijk}
\]

Subject to

\[
\begin{align*}
\sum_{i \in O} Z_{ij} &= 1; \forall j \\
\sum_{j \in O} Z_{ij} &= 1; \forall i \\
Z_{ij} &\in \{0,1\}
\end{align*}
\]

(3) is solved for every roll width \( k \) and the roll width that result in the minimum \( W_k \) is set for use during the shift/day under consideration.

4. THE UNIPACK CASE

Motivated by our analysis of the day-to-day scheduling of the corrugator at the UniPaK facility, one of the largest packaging industries in Lebanon, we automated the three-step suboptimal procedure outlined in section 4 to provide UniPaK with a decision support tool that enables them to find a lower cost corrugator schedule. The tool is built in Excel and calls LINGO for solving programs (1), (2) and (3). Excel is used as a platform for preprocessing data files that come from the company and for post-processing results obtained from LINGO.

Customer order data comes in the form of an excel sheet including data such as

- Order name
- Factory Ticket Number
- Order sheet width
- Order sheet length

- Order sheet demand
- Order sheet GSM, fluting, and paper type

A small sample of actual orders that come with due dates in February 2006 are shown in Table 1 for illustration purposes. Customer names and other proprietary information is not shown. Orders are referred to using anonymous customer names.

Table 1: Small Sample of customer Orders for the Month of February

<table>
<thead>
<tr>
<th>Ticket #</th>
<th>Qty</th>
<th>Delivery Date</th>
<th>Customer Name</th>
<th>Sheet Size</th>
<th>Paper Specifications</th>
<th>Flute Type</th>
</tr>
</thead>
<tbody>
<tr>
<td>33453134</td>
<td>334</td>
<td>3/3/2006</td>
<td>AB</td>
<td>348.5 x 120.9</td>
<td>127 W/7 T/75 ISO 127 H/U</td>
<td>CF</td>
</tr>
<tr>
<td>33453123</td>
<td>323</td>
<td>3/3/2006</td>
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<td>348.5 x 120.9</td>
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<td>CF</td>
</tr>
<tr>
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<td>127 W/7 T/75 ISO 127 H/U</td>
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</tr>
<tr>
<td>34653132</td>
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<td>3/3/2006</td>
<td>BB</td>
<td>348.5 x 120.9</td>
<td>127 W/7 T/75 ISO 127 H/U</td>
<td>CF</td>
</tr>
<tr>
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<td>CF</td>
</tr>
<tr>
<td>34653132</td>
<td>323</td>
<td>3/3/2006</td>
<td>DD</td>
<td>348.5 x 120.9</td>
<td>127 W/7 T/75 ISO 127 H/U</td>
<td>CF</td>
</tr>
<tr>
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<td>323</td>
<td>3/3/2006</td>
<td>EE</td>
<td>348.5 x 120.9</td>
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<td>CF</td>
</tr>
<tr>
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<td>3/3/2006</td>
<td>FF</td>
<td>348.5 x 120.9</td>
<td>127 W/7 T/75 ISO 127 H/U</td>
<td>CF</td>
</tr>
<tr>
<td>34653132</td>
<td>323</td>
<td>3/3/2006</td>
<td>GG</td>
<td>348.5 x 120.9</td>
<td>127 W/7 T/75 ISO 127 H/U</td>
<td>CF</td>
</tr>
<tr>
<td>34653132</td>
<td>323</td>
<td>3/3/2006</td>
<td>HH</td>
<td>348.5 x 120.9</td>
<td>127 W/7 T/75 ISO 127 H/U</td>
<td>CF</td>
</tr>
<tr>
<td>34653132</td>
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<td>3/3/2006</td>
<td>II</td>
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<td>MM</td>
<td>348.5 x 120.9</td>
<td>127 W/7 T/75 ISO 127 H/U</td>
<td>CF</td>
</tr>
</tbody>
</table>

Table 2 shows those jobs from Table 1 that have the same fluting type and paper type and color, namely, it shows the list of jobs requiring B fluting and White 1st grade type paper top.

Table 2: Orders sorted by fluting and paper type

<table>
<thead>
<tr>
<th>Ticket #</th>
<th>Qty</th>
<th>Delivery Date</th>
<th>Customer Name</th>
<th>Sheet Size</th>
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<th>Flute Type</th>
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<td>127 W/7 T/75 ISO 127 H/U</td>
<td>CF</td>
</tr>
<tr>
<td>34653123</td>
<td>323</td>
<td>3/3/2006</td>
<td>II</td>
<td>348.5 x 120.9</td>
<td>127 W/7 T/75 ISO 127 H/U</td>
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<td>3/3/2006</td>
<td>JJ</td>
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<td>127 W/7 T/75 ISO 127 H/U</td>
<td>CF</td>
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<tr>
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<td>3/3/2006</td>
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<td>3/3/2006</td>
<td>LL</td>
<td>348.5 x 120.9</td>
<td>127 W/7 T/75 ISO 127 H/U</td>
<td>CF</td>
</tr>
</tbody>
</table>

For the purpose of illustration the Primary orders (POrders) are identified to be AD through MM and the AOrders are the same list of orders. Solving the Job Selection problem (step 1) on the data in Table 1 gave the paired jobs shown in Table 3 and the set

\[ S = \{(AD, EE), (CC, II), (DD, KK), (FF, LL), (GG, HH), (KK, MM)\} \]

Jobs in \( S \) can be paired because they have GSM requirements within acceptable tolerance.
The following shows the computations for a selected roll width of 1.8 m. The same steps are carried for all available roll widths but are not presented in this paper. Tables 4 and 5 show the results of solving the Job Matching problem (step 2) for the selected roll width of 1.8 m. The same computations are carried on all available roll widths and the results saved for the next step (step 3 of the proposed procedure). In particular, Table 4 shows the optimal number of vertical cuts per horizontal cut found by solving nb_verticalcuts for each pair of jobs in S in addition to the optimal number of vertical cuts found if each job is to be processed individually on the corrugator for the same roll width of 1.8 m (shown on the diagonal). For example, for the pair (AD, EE) in S, the number of vertical cuts of order AD is 3 cuts or 3 sheets per horizontal cut and the number of sheets for order EE is 2. The number of cuts or sheets per horizontal cut can be read similarly for the remaining pairs of orders in S. Note that the pair (KK, MM) of S was eliminated from further consideration as a result of applying nb_verticalcuts for a roll width of 1.8 m and this is due to the fact that the run length of both jobs does exceed the 10% accepted tolerance. Hence, the set $S_{1.8}$ formed at the end of this step include the remaining jobs in S in addition to individual jobs.

Table 4: Solution of nb_verticalcuts for a Roll width of 1.8 m

Table 5 shows the results of solving nb_horizontalcuts for a Roll width of 1.8 m. In particular it shows the number of horizontal cuts or sheets that will be generated for each job in a pair to satisfy the demand per job shown earlier in Table 1. Table 6 shows the geometric waste in square meters generated from running jobs in the combinations shown in Table 5 for a roll width of 1.8 m. The geometric waste that would be generated from running jobs individually on a roll width of 1.8 m is shown on the diagonal of Table 6.

Table 5: Solution of nb_horizontalcuts for a Roll width of 1.8 m

Table 6: Waste in square meters generated for a roll width of 1.8 m

Table 7 shows the results obtained by solving the Roll Width Optimization problem (Step 3 of the procedure) for a roll width of 1.8 m. The results indicate the combination of jobs that would result in minimal total geometric waste. Note that the combinations (CC, II) and (FF, LL) were not optimal and thus only (AD, EE), and (DD, KK) were retained from S the remaining jobs are to be run individually if total geometric waste is to be minimized.

Table 7: Solution of Roll Width Optimization problem for a roll width of 1.8 m
schedule because the roll width is fixed over the entire production.

Table 8: Optimal Schedule for a roll width of 1.8 m

<table>
<thead>
<tr>
<th>Combination of Orders</th>
<th>Roll Length (cm)</th>
<th>Production Time (hrs)</th>
<th>Setup Time (hrs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(AD, BE)</td>
<td>(2, 2)</td>
<td>300</td>
<td>10</td>
</tr>
<tr>
<td>B</td>
<td>5</td>
<td>200</td>
<td>15</td>
</tr>
<tr>
<td>C</td>
<td>6</td>
<td>200</td>
<td>20</td>
</tr>
<tr>
<td>(AD, BE)</td>
<td>(2, 2)</td>
<td>300</td>
<td>10</td>
</tr>
<tr>
<td>B</td>
<td>5</td>
<td>200</td>
<td>15</td>
</tr>
<tr>
<td>C</td>
<td>6</td>
<td>200</td>
<td>20</td>
</tr>
<tr>
<td>(AD, BE)</td>
<td>(2, 2)</td>
<td>300</td>
<td>10</td>
</tr>
<tr>
<td>B</td>
<td>5</td>
<td>200</td>
<td>15</td>
</tr>
<tr>
<td>C</td>
<td>6</td>
<td>200</td>
<td>20</td>
</tr>
</tbody>
</table>

Table 9: Summary of waste and other operations information of actual corrugator schedule

<table>
<thead>
<tr>
<th>Combination of Orders</th>
<th>Roll Width (cm)</th>
<th>Waste (m²)</th>
<th>Roll Length (cm)</th>
<th>Production Time (hrs)</th>
<th>Setup Time (hrs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(AD, BE)</td>
<td>(2, 2)</td>
<td>300</td>
<td>10</td>
<td>15</td>
<td>10</td>
</tr>
<tr>
<td>B</td>
<td>5</td>
<td>200</td>
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<td>20</td>
</tr>
<tr>
<td>C</td>
<td>6</td>
<td>200</td>
<td>15</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>(AD, BE)</td>
<td>(2, 2)</td>
<td>300</td>
<td>10</td>
<td>15</td>
<td>10</td>
</tr>
<tr>
<td>B</td>
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</tr>
<tr>
<td>C</td>
<td>6</td>
<td>200</td>
<td>15</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>(AD, BE)</td>
<td>(2, 2)</td>
<td>300</td>
<td>10</td>
<td>15</td>
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</tr>
<tr>
<td>B</td>
<td>5</td>
<td>200</td>
<td>15</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>C</td>
<td>6</td>
<td>200</td>
<td>15</td>
<td>20</td>
<td>20</td>
</tr>
</tbody>
</table>

Table 9 shows the actual schedule for the job selection shown in Table 1. The roll width was changed depending on the combination of jobs to minimize geometric waste or material scrap.

Table 10: Comparison of proposed vs. actual corrugator schedule

<table>
<thead>
<tr>
<th>Actual Schedule</th>
<th>Proposed Schedule</th>
<th>Savings dollar value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$125.57</td>
<td>$125.57</td>
<td>($205.34)</td>
</tr>
<tr>
<td>$420</td>
<td>$420</td>
<td>$0</td>
</tr>
<tr>
<td>$267</td>
<td>$267</td>
<td>$200</td>
</tr>
</tbody>
</table>

5. CONCLUSION

This paper presented a three-step suboptimal procedure that draws a daily corrugator schedule while minimizing both trim waste cost and setup time cost where setup cost is measured as the loss of production resulting from stock rolls size changes. The procedure yields the optimal roll width to use over an entire shift. The proposed method was motivated by our analysis of the day-to-day scheduling of the corrugator at the UniPak facility one of the leading manufacturer of cardboard boxes in Lebanon. An example application of the proposed method to UniPak operations showed that the proposed procedure yields a lower-cost daily schedule of jobs on the corrugator when compared to the cost of the actual schedule for the same day. Indeed and although the actual schedule had a lower total materials waste cost, if we consider the additional setup time associated with changing roll widths and its equivalent dollar value, the proposed procedure gave significant improvements. By fixing the corrugator roll width for a whole shift of eight hours, we were able to achieve significant improvement and savings in terms of the dollar value of paper loss.

ACKNOWLEDGMENTS

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REFERENCES