ABSTRACT

In the industry of flat rolled sheet product, strip crown and shape has been one of the most important factors for quality assurance, productivity improvement, cost effectiveness, and customer satisfaction. The transient work roll profile influences load distribution and imprints an undesirable profile on rolled strip. We designate by a winding process any system applying the cycles of unwinding, transport, treatment and winding to various flat products. This system knows several constraints such as the thermal and mechanical effects that generate dysfunctions due to the influence of the process conditions. For this installations type, the various automatisms functions, often very advanced, are realized in modular systems with distributed architecture. In the present paper, a backstepping control technique is proposed. The control variables are velocities and traction forces along the web winding system. The proposed control law and Lyapunov function guarantee asymptotic stability from all initial values.

Keywords: Web Winding System (WWS), Nonlinear Control, Lyapunov Functions, Backstepping method.

1. INTRODUCTION

In the early days of control theory investigation, most of concepts such as stability, optimality and uncertainty were descriptive rather than constructive. In the recent two decades, a number of new methods have been developed for designing controllers to control nonlinear dynamic systems. These are mainly recursive methods, such as backstepping, forwarding, and various combinations of them. A common concept of the above named basic recursive methods is the design of a globally stable control system, having a cascade structure, for a class of nonlinear dynamic systems. In particular, the backstepping method is based on Lyapunov function theory (La Salle and Lefschetz 1961), but its origin can be found in some theories of linear control, such as the feedback linearisation method or the LQR method.

The beginning of the development of the backstepping method applied to nonlinear control systems design dates back to the end of the 1980s. A list and a discussion of publications issued at that time can be found in the overview by Sontag and Sussmann, (1989), as well as by Kokotovic and Arcak (2001). The backstepping method is based directly on the mathematical model of the examined system, introducing new variables into it in a form depending on the state variables, controlling parameters, and stabilising functions. The controlled system may be in the state equations with a triangular form. The design of the controller pass by several step, in the first step we consider a Lyapounov function for the first error state, then, the virtual control is calculated in the order to guarantee the negativity of the Lyapunov function proposed. For this virtual control, we associate a second error state, between the second state and the virtual control calculated in first step, then we consider the augmented joint Lyapounov function whose the first function and the second error are appear. The second virtual control is calculate with the same reasoning. The exact control will calculate in the last step by using the virtual control laws calculated in the past steps. We can interpret this method by the addition of the integrators after each step (Krstic, Kanellakopulos and Kokotovic 1995).

This paper presents a new concept of web winding system in which control velocities and tensions are derived for nonlinear controllers designed with the Backstepping method. The dynamics of a Web Winding System (WWS) is described by its strongly nonlinear behavior. In all cases of rolling up or unfolding of a web material, the flatness difficulty arises. Considering the complexity of the system due to nonlinearity and the strong coupling between the web velocity and the web tension, it is more convenient to linearize this WWS. However, this model remains very depend on the set point considered and especially on the variation rate of nonlinearities. This situation pushed the researchers to be directed more and more towards the techniques of the nonlinear control based on the Backstepping’s technique. The paper is organized as follows: The web winding model is described in Section 2, with a brief description of the WWS. A detailed description of winding control design methodology is in Section 4. In Section 5 the control system performances are evaluated in simulation of the web winding model. The last section concludes the paper.

2. PLANT MODEL

2.1. WWS description

The web system is very important in a rolling mill, because its parameters determine the strip quality. Among its parameters, we quote:
The entry and exit of the traction forces.
The entry and exit velocities ensured by the winders motors, and the work rolls velocity (Fig.1).
The pressure force or the variations between the work rolls and their parallelism.

The variation of the exit strip flatness evolves because of the thermal dilation of the cylinders (Rabbah and Bensassi, 2006; Rabbah and bensassi 2007a), but also due to the elasticity forces (Schmitz and Herman 1995). To avoid this phenomenon, the traction forces are applied to limit the elasticity of the rolled material.

The thickness control is ensured by programmable automats, which are called AGC (Automatic Gauge Control system), (Ueno and Sorao 2004). Their goal is to maintain the strip thickness uniform in spite of the acting factors to change it. Considering the complexity of the Cold Rolling System (CRM), the modeling and the control of the WWS should be studied to minimize the flatness defaults. With this intention, we start with the development of a mathematical model describing the dynamic behavior of the system.

2.2. Global model
Let us consider the nonlinear model of the wws (Rabbah and Bensassi 2007b; Rabbah and bensassi 2008) defined by the state representation (1) which can be put in the general form of the nonlinear affine control system:

\[ \dot{x} = f(x, u) + g(x)u \]
\[ y = h(x) \]

with:

\[ x = [V_1, T_1, V_2, T_2, V_3]^T = [x_1, x_2, x_3, x_4, x_5]^T, \]
\[ y = [T_1, V_2, T_3]^T \] et \( u = [U_1, U_2, U_3]^T \)

These parameters characterize the system: it is multivariable, strongly coupled, nonlinear and time-varying.

The model (1) is composed of three subsystems: The first one is of the state vector \([V_1, T_1]^T\), controlled by the tension \(U_1\), the second has as a state vector \([V_2]\), is controlled by \(U_2\) and the third subsystem has as a state vector \([T_2, V_3]^T\), is controlled by the tension \(U_3\).

Table 1 contains the operating condition used in this analysis.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Web length between winder and unwinder</td>
<td>1.15 m</td>
</tr>
<tr>
<td>Young’s modulus</td>
<td>0.16 10^9 kg/m²</td>
</tr>
<tr>
<td>Web section</td>
<td>0.19 mm²</td>
</tr>
<tr>
<td>Sliding coefficient</td>
<td>0.8</td>
</tr>
<tr>
<td>Strip or web thickness</td>
<td>1.6 mm</td>
</tr>
<tr>
<td>Strip or web width (largeur)</td>
<td>600 mm</td>
</tr>
<tr>
<td>Diameter or work rolls</td>
<td>0.45 m</td>
</tr>
<tr>
<td>Nominal torque of unwinding/winding motors</td>
<td>700 kN.m</td>
</tr>
<tr>
<td>Nominal velocity of unwinding/ winding motors</td>
<td>120 Rpm</td>
</tr>
<tr>
<td>Rolling speed</td>
<td>1400 mpm</td>
</tr>
</tbody>
</table>

3. THE BACKSTEPING CONTROLLERS
The control problem considered consists in forcing the web velocities and the downstream/upstream web tensions to follow the reference signals given, noted
respectively \( V_1^{\text{ref}}, T_1^{\text{ref}} \) and \( T_3^{\text{ref}} \). This suggests the following errors:

\[
e_1 = T_1^{\text{ref}} - T_1 \quad (2)
\]

\[
e_2 = V_2^{\text{ref}} - V_2 \quad (3)
\]

\[
e_3 = T_3^{\text{ref}} - T_3 \quad (4)
\]

The regulator synthesis will be done in two steps. In the first step, we will put the virtual controls and the stabilising functions associated. In the second step, we determine the control laws able to ensure convergence towards zero of the difference between the virtual orders and the stabilising functions associated.

**Step 1:** The dynamic of the tracking errors \( e_1, e_2 \) and \( e_3 \) are given by:

\[
\dot{e}_1 = T_1^{\text{ref}} - \frac{ES}{L} (V_2 - V_1) + \frac{V_2}{L} T_1 \quad (5)
\]

\[
\dot{e}_2 = V_2^{\text{ref}} + \frac{gs^2}{J_2} (T_2 - T_3) + \frac{1}{\tau_{os2}} V_2 - \frac{r_2}{\tau_{os2} \cdot k_2} U_2 \quad (6)
\]

\[
\dot{e}_3 = T_3^{\text{ref}} - \frac{ES}{L} (V_1 - V_2) + \frac{V_1}{L} T_3 \quad (7)
\]

The tracking error \( e_2 \) tends asymptotically towards zero if; in this case, the virtual control is selected as follows:

\[
\alpha_2 = V_2^{\text{ref}} + \frac{gs^2}{J_2} (T_2 - T_3) + \frac{1}{\tau_{os2}} V_2 \quad (8a)
\]

Thus, the dynamic error \( e_2 \) can be rewritten as follows:

\[
\dot{e}_2 = \alpha_2 - \frac{r_2}{\tau_{os2} \cdot k_2} U_2 = -c_2 e_2 \quad (8b)
\]

The quantities \( \frac{ES}{L} (V_2 - V_1) \) and \( \frac{ES}{L} (V_1 - V_2) \) are posed like virtual control inputs for the system (5) and (7). It follows that the tracking errors \( e_1 \) and \( e_3 \) tend asymptotically towards zero if these virtual controls are selected such as:

\[
\frac{ES}{L} (V_2 - V_1) = \alpha_1 \quad \text{with} \quad \alpha_1 = c_1 e_1 + T_1^{\text{ref}} + \frac{V_2}{L} T_1 \quad (9)
\]

\[
\frac{ES}{L} (V_1 - V_2) = \alpha_3 \quad \text{with} \quad \alpha_3 = c_3 e_3 + T_3^{\text{ref}} + \frac{V_1}{L} T_3 \quad (10)
\]

where \( c_1, c_2 \) and \( c_3 \) are the positive real constants unspecified.

Indeed, by doing this we obtain:

\[
\dot{e}_1 = -c_1 e_1, \quad \dot{e}_2 = -c_2 e_2 \quad \text{and} \quad \dot{e}_3 = -c_3 e_3
\]

The \( \alpha_1, \alpha_2 \) and \( \alpha_3 \) quantities are called stabilising functions. Then, the first Lyapunov candidate function is defined as:

\[
v_1 = \frac{1}{2} (e_1^2 + e_2^2 + e_3^2) \quad (11)
\]

The derivative of the first Lyapunov function takes the form:

\[
\dot{v}_1 = -c_1 e_1^2 - c_2 e_2^2 - c_3 e_3^2 \quad (12)
\]

who shows the influence of the controlling parameters \( c_1, c_2 \) and \( c_3 \) on the convergence of the \( e_1, e_2 \) and \( e_3 \) errors.

For the equations (9) and (10), the found result supposes that \( \frac{ES}{L} (V_2 - V_1) \) and \( \frac{ES}{L} (V_1 - V_2) \) are effective controls. As such is not the case, we cannot impose the equalities (9) and (10). We will only try to tend these controls towards their ideal trajectories which are precisely the stabilising functions \( \alpha_1 \) and \( \alpha_3 \). For this purpose, we introduce the errors:

\[
\alpha_1 = \frac{ES}{L} (V_2 - V_1) \quad (13)
\]

\[
\alpha_3 = \frac{ES}{L} (V_1 - V_2) \quad (14)
\]

The dynamic of the \( e_1 \) and \( e_3 \) errors are expressed in function of \( z_1 \) and \( z_3 \) as follows:

\[
\dot{e}_1 = -c_1 e_1 + z_1 \quad (15)
\]

\[
\dot{e}_3 = -c_3 e_3 + z_3 \quad (16)
\]

The second step in the regulator synthesis consists in forcing all the \( (e_1, e_2, e_3, z_1, z_3) \) errors to converge towards zero with a suitable choice of the effective controls \( U_1, U_2 \) and \( U_3 \).

**Step 2:** The dynamic of the \( e_1 \) error is given by:

\[
\dot{z}_1 = \alpha_1 - \frac{ES}{L} (V_2 - V_1) \quad (17)
\]

\[
= c_1 e_1 + \frac{T_1^{\text{ref}}}{L} V_2 + \frac{1}{L} V_2 \cdot T_1 - \frac{ES}{L} (V_2 - V_1)
\]
Taking into account (13) and (9), we obtain:

\[
\dot{z} = c_0 (-\alpha + z_2) + \frac{1}{\tau_{\text{es}}} T \left( \frac{g_0^2 T_2}{J_2} \frac{g_0^2}{J_2} T_1 + \frac{1}{\tau_{\text{es}}^2} T_2 \right) + \frac{1}{L} E \left( \frac{g_0^2 T_2}{J_2} \frac{g_0^2}{J_2} T_1 + \frac{1}{\tau_{\text{es}}^2} T_2 \right) + \frac{1}{E} \left( \frac{g_0^2 T_2}{J_2} \frac{g_0^2}{J_2} T_1 + \frac{1}{\tau_{\text{es}}^2} T_2 \right)
\]

(18)

To study the stability of the system (8b), (15), (16), (18) and (21), of state vector \((e_1, e_2, e_3, z_1, z_2)\), we consider the Lyapunov candidate function increased:

\[
v_2 = v_1 + \frac{1}{2} (z_1^2 + z_2^2)
\]

(23)

Its derivative with respect to time:

\[
\dot{v}_2 = e_1 e_2 + e_2 e_3 + e_3 v_1 + z_1 z_2
\]

(24)

Taking into account (8b), (15), (16), (18) and (21), we obtain:

\[
i_2 = -e_1 e_2 - e_1 e_3 - d_1 e_3 - d_2 z_2
\]

(25)

where \(d_1\) et \(d_2\) are the positive real constants unspecified.

The preceding equation suggests choosing the \(U_1\), \(U_2\) and \(U_3\) controls such as:

\[
e_1 + d z_2 + \beta \left( \frac{\tau_{\text{es}}}{L} \right) \frac{e_2}{U_1} - \frac{e_1}{U_1}, \frac{e_2}{U_1} \frac{e_2}{U_1} = 0
\]

(26)

we can deduce the three laws control, there forms are:

\[
U_1 = \frac{L}{\tau_{\text{es}} k_2} \frac{e_1 + d z_2 + \beta \left( \frac{\tau_{\text{es}}}{L} \right) \frac{e_2}{U_1} + \frac{e_2}{U_1} \frac{e_2}{U_1}}{\tau_{\text{es}} k_2}
\]

(27)

The derivative \(\dot{v}_2\) becomes:

\[
\dot{v}_2 = -e_1 e_2 - c_1 e_2 - c_1 e_3 - d_1 e_3 - d_2 z_2
\]

(28)
It is a negative definite function of the vector \((e_1, e_2, e_3, z_1, z_2)\). It follows that the system of the state vector \((e_1, e_2, e_3, z_1, z_2)\) has an equilibrium point globally asymptotically stable on the position \((e_1, e_2, e_3, z_1, z_2) = (0, 0, 0, 0, 0)\). That means in particular that the tracking errors (for the winding velocity and the upstream/downstream tensions) tend towards zero whatever the initial conditions.

4. SIMULATION AND RESULTS DISCUSSION
The performances of the regulator worked out in the preceding paragraph will be illustrated now by simulation. We will use the www model quoted in section 2. We took as references \(V_2^{\text{ref}}, T_1^{\text{ref}}\) and \(T_3^{\text{ref}}\), which respectively have as a value 29m/s, 100N and 120N.

The values chosen for the regulator parameters of Backstepping during simulations are:
\(^{(c_0, c_1, c_2, d_1, d_2)} = (60, 32, 78, 110, 224)\).

The simulations results are presented by figures 2, 3 and 4. We note that the continuation objectives are achieved, as well for velocity as for the upstream and downstream tensions. These results show that the continuation errors corresponding to the each parameter are cancelled after 30 seconds for the tensions and 15 seconds for the web velocity.

Figure 2: Upstream tension control by Backstepping.

Figure 3: Web velocity control by Backstepping.

Figure 4: Downstream tension control by Backstepping.

5. CONCLUSION
In this paper, we approached the control problem of the web velocity and the web tensions of a web winding system, using a regulator worked out by the Backstepping technique. The synthesis rested on the standard model, which holds account owing to the fact that all the state variables were supposed to be available. We formally established that the closed loops made up of this regulator and the model from which it is resulting, are overall asymptotically stable. Moreover, the regulator by Backstepping asymptotically ensures a perfect trajectories tracking of the web velocity and tensions references. This result was confirmed by simulation way.

REFERENCES


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