ABSTRACT

In the present paper, the concept of glycemic index of foods is taken as a measure for the dietary control of type 1 diabetes mellitus and it is included as an “objective function” in a linear programming model that will serve to generate a proposal of diet; it takes into consideration the energy requirements and the number of macronutrients (carbohydrates, proteins, and lipids). The linear programming model is based on “equivalent” food groups with standardized measures that, on average, have the same number of macronutrients, thus facilitating the exchange of food within each group. The flexibility of the model allows changes in the matrix of resources and constraints, generating scenarios for various diet proposals for different energy requirements based on a low glycemic index.

Keywords: Dietary control of type 1 diabetes mellitus, Glycemic index, Simulation, Optimization.

1. INTRODUCTION

Diabetes mellitus (DM), or simply diabetes, is a chronic disease that occurs when the pancreas is no longer able to make insulin, or when the body cannot make good use of the insulin it produces. Insulin is a hormone made by the pancreas, that acts like a key to let glucose from the food we eat pass from the blood stream into the cells in the body to produce energy. Carbohydrates (CH) are broken down into glucose in the blood. Insulin helps glucose get into the cells (International Diabetes Federation 2015). For this reason, a control of food intake is very important in people with DM.

Until recently, reducing fasting and pre-meal glucose levels was the basic approach to diabetes control. However, recent studies have shown that there is a strong relationship between high levels of glucose after meals (postprandial glucose) and the risk of diabetic complications (Ceriello and Colagiuri 2007). People with diabetes are at increased risk of developing several serious health problems because of their inability to produce or use insulin effectively, which leads to elevated blood glucose levels (known as hyperglycemia). Eventually, high levels of glucose are associated with damage to the body and multiple organ and tissue insufficiencies (International Diabetes Federation 2015). Table 1 shows fasting and two-hour postprandial blood glucose levels; it helps assess if a person is healthy, if he or she has a higher risk of diabetes (prediabetes) or if he or she has the disease.

Table 1. Optimal Blood Sugar Levels Before and After two hours of Eating

<table>
<thead>
<tr>
<th>Blood sugar classification</th>
<th>Fasting minimum [mg/dl]</th>
<th>Fasting maximum [mg/dl]</th>
<th>2 hours after eating [mg/dl]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal blood sugar</td>
<td>70</td>
<td>99</td>
<td>Less than 140</td>
</tr>
<tr>
<td>Pre-diabetes</td>
<td>100</td>
<td>125</td>
<td>140 to 199</td>
</tr>
<tr>
<td>Established Diabetes</td>
<td>Over 125</td>
<td>Over 125</td>
<td>More than 200</td>
</tr>
</tbody>
</table>

1.1. Glycemic Index

The term glycemic index (GI) was defined as the increase in the area under the blood glucose response curve obtained with 50 grams serving of carbohydrates available in a food, expressed as a percentage of the response, in the same subject, on the intake of 50 grams of anhydride glucose (Mateljan 2001).

Previously, most meal plans designed to improve blood sugar analyzed the total amount of carbohydrates (including sugars and starches) in the foods themselves. The GI goes beyond this approach, looking at the impact of foods on our actual blood sugar. In other words, instead of counting the total amount of CH in foods in their unconsumed state, GI measures the actual impact of these foods on our blood sugar. Therefore, the glycemic index is a systematic way of classifying CH based on their effect on the immediate increase on blood sugar.
levels. Table 2 shows the ranges to classify food based on a low, medium or high glycemic index (Rakel 2008).

Table 2: Reference Ranges of Glycemic Index

<table>
<thead>
<tr>
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</thead>
<tbody>
<tr>
<td>High glycemic index</td>
</tr>
<tr>
<td>Medium glycemic index</td>
</tr>
<tr>
<td>Low glycemic index</td>
</tr>
</tbody>
</table>

1.2. Justification

The glycemic index was conceived and communicated in 1981 by David Jenkins et al., At the University of Toronto Canada, as a weapon for the dietary management of Type 1 Diabetes Mellitus (T1DM). This value was more relevant to the glycemic control of diabetes by reducing the GI in the diet (Fontvieille et al., 1988, 1992; Brand et al., 1991; Frost et al., 1994). Numerous dietary data collected by D. Walter’s group at Harvard University led to the publication of several papers that showed that a diet with a low GI reduced the risk of developing diabetes (Salmerón et al. 1997a, b).

GI is often criticized for its inability to reflect the glycemic effects of food when consumed in a mixture of foods because of added fats and proteins (Hollenbeck et al., 1986; American Diabetes Association 1994, 2002; Franz et al., 1994, 2002). However, the fact that GI is measured only in isolated foods is an important strength of the concept and that is precisely what allows GI to be useful. The point here is that GI is measured in individual foods and it is calculated for a mixture of foods (Wolever 2006).

The GI of the diet is determined by expanding the calculations made for the GI of a meal, that is:

\[
\frac{\sum_{x=1}^{n} IG_n \times g_n}{G} \quad \text{... (A)}
\]

\(x\): is the number of foods in a diet.

\(IG_n\): is the glycemic index of each individual food.

\(g_n\): are the grams of carbohydrates available in each individual food.

\(G\): is the total carbohydrate available in the diet (in grams).

Given equation A (Wolever et al. 1994), to obtain the glycemic index of a diet, it is necessary to make the sum of the product between the \(IG_n\) and the grams corresponding to each individual food \(g_n\) and then divide that result among the total carbohydrates \(G\).

2. MODEL FORMULATION

2.1. Data collection

The data and values were obtained from the food guide for the Mexican population and the Mexican system of equivalent foods (Pérez et al. 2014) and they were compared with the Mexican Official Standard NOM-015-SSA2-2010, for the prevention, treatment, and control of DM (Hernández 2016).

In normative appendix A, we can find the distribution of equivalents and estimate of daily energy requirements for people with diabetes. The number of equivalents varies per the energy requirements and these are calculated per the desirable weight, height, age, sex and physical activity of the individual.

2.2. Development of the model

For this model, the range of values present in the normative appendix A will be a useful basis for the generation of scenarios. So, one will have a minimum energy load of 1200 calories and a maximum load of 2500 calories. With these values, we have all the important considerations to formulate the linear programming model and to do the comparison of results and its validation.

2.3. Model construction

To obtain macronutrient restrictions (in grams) we used appendix A. The respective calculations were performed for each macronutrient, considering the distribution of equivalents.

Table 3: Name of the Model Variables

<table>
<thead>
<tr>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(X1) = Cereals and Tubers without fat</td>
</tr>
<tr>
<td>(X2) = Cereals and Tubers with fat</td>
</tr>
<tr>
<td>(X3) = Vegetables</td>
</tr>
<tr>
<td>(X4) = Fruits</td>
</tr>
<tr>
<td>(X5) = Food of animal origin, very low in fat</td>
</tr>
<tr>
<td>(X6) = Food of animal origin, low in fat</td>
</tr>
</tbody>
</table>

Objective function: Minimize the glycemic index of food in the diet.

\[
\text{Minz} = 56X_1 + 63X_2 + 52X_3 + 49X_4 + 40X_5 + 0X_6 + 30X_7 + 28X_8 + 32X_9 + 0X_{10} + 0X_{11} + 0X_{12} \quad \text{... (1)}
\]
S.T  

\[ 66X_1 + 104X_2 + 22X_3 + 57X_4 + 40X_5 + 55X_6 + 95X_7 + 150X_8 + 112X_9 + 70X_{10} + 45X_{11} + 45X_{12} = (1200 - 2500) \quad \ldots (2) \]

\[ 14X_1 + 15X_2 + 4X_3 + 14X_4 + 0X_5 + 0X_6 + 12X_7 + 12X_8 + 19X_9 + 0X_{10} + 0X_{11} + 0X_{12} < 401 \quad \ldots (3a) \]

\[ 14X_1 + 15X_2 + 4X_3 + 14X_4 + 0X_5 + 0X_6 + 12X_7 + 12X_8 + 19X_9 + 0X_{10} + 0X_{11} + 0X_{12} > 172 \quad \ldots (3b) \]

\[ 1X_1 + 4X_2 + 0.2X_3 + 0.1X_4 + 1X_5 + 3X_6 + 2X_7 + 8X_8 + 1X_9 + 5X_{10} + 5X_{11} + 5X_{12} \leq 163 \quad \ldots (4a) \]

\[ 1X_1 + 4X_2 + 0.2X_3 + 0.1X_4 + 1X_5 + 3X_6 + 2X_7 + 8X_8 + 1X_9 + 5X_{10} + 5X_{11} + 5X_{12} \geq 31 \quad \ldots (4b) \]

\[ 2X_1 + 2X_2 + 1X_3 + 1X_4 + 7X_5 + 7X_6 + 9X_7 + 9X_8 + 7X_9 + 3X_{10} + 0X_{11} + 0X_{12} < 154 \quad \ldots (5a) \]

\[ 2X_1 + 2X_2 + 1X_3 + 1X_4 + 7X_5 + 7X_6 + 9X_7 + 9X_8 + 7X_9 + 3X_{10} + 0X_{11} + 0X_{12} > 73 \quad \ldots (5b) \]

\[ 1X_1 + 1X_2 + 2X_3 + 2X_4 + 6X_9 < 59 \quad \ldots (6a) \]

\[ 1X_1 + 1X_2 + 2X_3 + 2X_4 + 6X_9 > 16 \quad \ldots (6b) \]

\[ X_i \geq 0, \quad \text{para} \ i = 1, 2, 3 \ldots 12 \quad \ldots (7) \]

Where each equation represents:

Objective function: Optimize (minimize) the glycemic index of available foods (1) (Navarro et al. 2016). Equation (2) refers to the calories present in said diet and varies from 1200 to 2500 of 100 in 100. The following equations refer to the restrictions in grams of maximum carbohydrates (3a) and minimum (3b), fats (4a) and (4b), proteins (5a) and (5b), fiber (6a) and (6b), and finally the non-negativity condition applied to each of the variables (7) is included.

Other restrictions included to improve the level of detail of the model are:

\[ X_{10} + X_{11} + X_{12} < 8 \quad \ldots (8) \]

\[ X_1 + X_2 > 5 \quad \ldots (9) \]

\[ X_5 + X_6 < 8 \quad \ldots (10) \]

\[ X_3 > 3 \quad \ldots (11) \]

\[ X_4 > 3 \quad \ldots (12) \]

To increase the level of detail of the model, it was proposed to limit the consumption of lipids (8) and foods of animal origin (10). Also included were minimal amounts of some other foods such as fruits (12), vegetables (11), and cereals (9) that are indispensable for a balanced and healthy diet.

3. VALIDATION OF THE MODEL

To verify and validate the model, the results of the first run (with 1200 calories) are compared to the values in appendix A. The macronutrient values are calculated and compared with the model results.

Given appendix A, an amount of 3 equivalents of animal products, 1 milk equivalent, 1 legume, 5 vegetables, 5 cereals and tubers, and 3 servings of fruits are required for a 1200 calorie diet.

For the calculation of proteins, we have

\[ Proteins = 3(7) + 9 + 8 + 5(2) + 5(2) + 5(3) = 73 \ [g] \]

\[ Carbohydrates = 3(0) + 12 + 20 + 5(4) + 5(15) + 3(15) = 172 \ [g] \]

\[ Lipids = 3(1) + 2 + 1 + 5(0) + 5(0) + 5(5) = 31 \ [g] \]

Table 4: Calculation of the GI of the meal considering the Distribution of Equivalents Recommended for Appendix A

<table>
<thead>
<tr>
<th>Portions</th>
<th>GI of foods</th>
<th>Portion of carbohydrates</th>
<th>IG of the food mixture</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>63</td>
<td>44%</td>
<td>27</td>
</tr>
<tr>
<td>3</td>
<td>49</td>
<td>26%</td>
<td>13</td>
</tr>
<tr>
<td>5</td>
<td>52</td>
<td>12%</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
<td>0%</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>32</td>
<td>12%</td>
<td>4</td>
</tr>
<tr>
<td>1</td>
<td>28</td>
<td>7%</td>
<td>2</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>0%</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>1</td>
</tr>
</tbody>
</table>

And,
The macronutrients are calculated per the model.

**Proteins**

\[\text{Proteins} = 5(2) + 3(1) + 3(1) + 4.43(7) + 0.67(9) + 1.13(9) + 1.38(7) = 73 \text{ [gr]}\]

**Carbohydrates**

\[\text{Carbohydrates} = 5(14) + 3(4) + 3(14) + 4.43(0) + 0.67(12) + 1.13(12) + 1.38(19) = 172 \text{ [gr]}\]

**Lipids**

\[\text{Lipids} = 5(1) + 3(0.1) + 3(0.2) + 4.43(3) + 0.67(2) + 1.13(8) + 1.38(1) = 31 \text{ [gr]}\]

Table 5: Calculation of the GI of the diet resulting from the Model, Considering the Proportion of Carbohydrates Available for each Food

<table>
<thead>
<tr>
<th>Portions</th>
<th>GI of foods</th>
<th>Portion of carbohydrates</th>
<th>IG of the food mixture</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
<td>56</td>
<td>41%</td>
<td>23</td>
</tr>
<tr>
<td>3</td>
<td>49</td>
<td>24%</td>
<td>12</td>
</tr>
<tr>
<td>3</td>
<td>52</td>
<td>7%</td>
<td>4</td>
</tr>
<tr>
<td>4.43</td>
<td>0</td>
<td>0%</td>
<td>0</td>
</tr>
<tr>
<td>0.67</td>
<td>30</td>
<td>5%</td>
<td>1</td>
</tr>
<tr>
<td>1.13</td>
<td>28</td>
<td>8%</td>
<td>2</td>
</tr>
<tr>
<td>1.39</td>
<td>32</td>
<td>15%</td>
<td>5</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td></td>
<td>46.9</td>
</tr>
</tbody>
</table>

As shown in the above calculations, the amounts of macronutrients in the two cases are similar. Both the values calculated by the model results and those calculated based on appendix A are the same, although not all food groups are considered. Therefore, it can be said that the model presents realistic results and that, in addition, it will always comply with the criterion of minimization of the GI. With these results the model is validated.

4. RESULTS

To understand the results of the model, it was necessary to make use of several graphs, which are presented in figures 2, 3, 4 and 5.

Figure 2 shows the behavior of macronutrients in relation to energy requirements. Growth in protein and lipid amounts is observed. However, the amount of carbohydrates remains constant up to 2100 calories. This means that the model's priority is to stabilize the amount of carbohydrates to be consumed. One issue to keep in mind is that in order to optimize blood glucose levels, it is not necessary to decrease carbohydrate consumption, nor to abruptly increase the consumption of fats or proteins, since the values of the macronutrients are below the maximum values set as constraints in the model.

Figure 3 shows the behavior of the macronutrients (in percentages) with respect to the energy requirements. Compared with the previous figure, there is an almost linear decrease in the proportion of carbohydrates; this effect occurs because the consumption of carbohydrates (at least in quantity) remains constant while the amount of proteins and lipids continues to increase.
Figure 4 shows the way in which the model chooses the portions of foods per group (groups with no glycemic index are omitted). In this way, one can see how each food group will affect the glycemic index of the diet (the total food mix). For example, Figure 5 shows a peak (in the 2200 calories) in the glycemic index that is caused by the increase of the variables X8 (dairy) and X2 (cereals and tubers). Afterwards, a decrease is observed in the GI because the consumption of dairy products increases, while the consumption of cereals and tubers remains constant. If the values of the objective function are checked, they will realize that the glycemic index of X2 is greater than that of X8, that is, X2 has a medium glycemic index. This effect occurs because the consumption of foods with low glycemic indexes increases and (per the formula for calculating the GI of the meals) causes the total glycemic index to decrease.

5. CONCLUSIONS

It was possible to optimize the glycemic index of the proposed foods, since in calculating the GI of the proposed diet, the sum of the foods results in a low GI and the validity of the GI is verified when compared with the values of normative appendix A. However, the model would not make much sense without the correct method to calculate the GI of a food mixture, since there are other factors that could influence the glycemic impact of such a mixture, such as: the GI of each individual food, the total amount of carbohydrates available in the diet, and the proportion of carbohydrates per food. Factors that are considered in equation A.

The accuracy of the calculation of the GI of the diet depends on a good selection of data, since it makes use of food groups and the variation could affect the results. However, the values that were taken to obtain the model are based on standardized portions for each food group, which implies that foods have on average the same amount of carbohydrates, proteins and lipids, which facilitates the exchange of food within their respective group, obtaining a proposal of a varied and balanced diet, with a low glycemic index.

The viability of the proposed diets will be subject to the consideration and validation of a professional in nutrition, since in the calculation of the diets more factors intervene than those considered in the elaboration of the model, but it could easily be included as additional restrictions. This observation is made because a nutrition professional might consider that the model does not have a level of detail adequate enough to generate a real diet, because it could consider other factors that may modify macronutrient needs for each person. However, the level of detail that is achieved with the model serves to predict the behavior of macronutrients in the diet and facilitates the collection of certain indicators that could serve for a more qualitative measurement of the carbohydrates consumed in a diet. Finally, it is intended that this model serves as a tool in which these professionals can rely on to generate diets with low GI.

APPENDIX

NORMATIVE APPENDIX A

<table>
<thead>
<tr>
<th>Distribution of equivalents in a meal plan for people with diabetes. By NOM-015SSA22010.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Food group</td>
</tr>
<tr>
<td>Kcal</td>
</tr>
<tr>
<td>Food of animal origin</td>
</tr>
<tr>
<td>Dairy products</td>
</tr>
</tbody>
</table>
### REFERENCES


Pérez Lizaur A. B., Marván Laborde L. Palacios B. Sistema Mexicano de Alimentos Equivalentes. 3era. Ed. Fomento de Nutrición y Salud, A.C. México.


Available from:


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Maximino Navarro is currently studying the industrial engineering career at the Faculty of Engineering of the National Autonomous University of Mexico. He has participated in poster contests organized by the engineering postgraduate around operations research. His first participation as a research student was during the Latin American Congress of Operations Research CLAIO 2016 in which he presented a written work. His areas of interest include the topics of linear programming, simulation, and row theory (within operations research). It has also addressed issues of transport (mainly road congestion) and supply chain.

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