ANTI-WIND-UP SOLUTION FOR A TWO DEGREES OF FREEDOM CONTROLLER

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ABSTRACT

This paper proposes an anti-windup solution for a STR controller with two degrees of freedom (2DoF). The main advantage of this kind of controllers is that the dynamic for tracking references can be different than the one specified for disturbance rejection. As a design condition, it is also proposed to eliminate the steady state error so that, the presence of an integral term in the controller is mandatory. Due to the physical limitations and an unstable factor in the controller, an anti-windup strategy is needed. These logics or non-linear schemes are extensively studied for classical controllers but not for 2DoF strategies.

The proposal has been verified both in simulation and on a physical plant.

Keywords: two-degree control, anti-windup, integral action.

1. INTRODUCTION

Controllers with two degrees of freedom (2DoF) are attractive because they can separate the two control problems: the set point (SP) tracking and the disturbance rejection.

PIDs are extensively used in controllers of two degrees of freedom (PID-2DoF). Tripathi R. and Hanamoto T. (2017) show the improvement between 1-DOF and 2-DOF for the optimization of a dc voltage controller. Sahu R., Panda S., Rout U. and Sahoo D. (2015) compare different strategies and expose a new and better technique. Wang D., Liu T., Sun X., Zhong C. (2016) propose a discrete-time 2-DOF design method for integrating and unstable process with time delay.

Even the anti-windup solutions have been largely studied for PID systems, as can be seen in the review of solutions presented by Espina J., Arias A., Balcells J. and Ortega C. (2009), it is not a closed subject. Particular solutions are always being studied and compared for specific systems; such is the case of Perez T. (2009), who presented an anti-windup design for the dynamic positioning of marine vehicles with control allocation. However, there are no previous thorough studies that provide a satisfactory Anti-Wind-Up solution for an STR 2-DoF system as for PIDs.

Although the PID-2DoF is deeply studied and used, the STR has some advantages since any kind of dynamics can be introduced. Some authors (Matijevic M., Sredojevic R. and Stojanovic V., 2011), argue that the little industrial use of the STR is due to its complexity in the implementation. This paper shows a simple way to solve STR implementation problems, in particular the windup effect.

The main advantage of a STR controller is its structure, which is able to carry out improved performances due to the fact that it stores several states of the set point, the process variable and the manipulated variable. What is more, STR method allows the user to divide the two degrees of freedom into individual and understandable elements and to add terms without being restricted to any amount of degrees.

Nevertheless, a set-back of this structure is that the implementation of traditional anti-windup structures does not work. The anti-windup solution is more complex than the used in the classical controllers, where only one previously manipulated variable value is needed.

Moreover, it is well known that controllers with any unstable term (e.g. integral action) working on a process with limited actuators can cause malfunctioning of the closed loop system or even have stability problems. For instance, if there is an integral term and the calculated control action is unfeasible (due to any actuator limitation), the integral calculation will increase indefinitely (windup effect). When the error changes its sign, the manipulated value variation will be delayed because of the windup. This problem is strictly of a physical nature because the model of the controller does not contemplate any kind of restriction.

The main goal of this paper is to provide an anti-windup solution that works for a 2-DoF controller without adding an extra parameter decision or an extra mathematical resolution.

First, we will provide a characterization of anti-windup and of the model used for investigation. Then we will explain the method used to build the STR control model. Next, we will assess the different anti-windup solutions for STR one degree of freedom controllers. And finally, we will present a solution for STR 2-DoF controller.

2. PROCESS MODEL

The process model adopted is the classical discrete linear model expressed in the following equation:

$$A(z^{-1})y_k = B(z^{-1})u_k$$
(1)

where u(k) is the manipulated variable (MV) and y(k) the process variable (PV).

The degree of $B(z^{-1})$ must be less than the degree of $A(z^{-1})$. The polynomials $A(z^{-1})$ and $B(z^{-1})$ have

no common factor, and the polynomial $A(z^{-1})$ is monic.

3. CONTROL LAW

Next figure shows the STR structure (Astrom and Wittenmark, 1997) where the block "process" represents the plant to be controlled.



Figure1. STR Block Diagram

The resulting linear controller is:

$$R(z^{-1})u_{k} = T(z^{-1})r_{k} - S(z^{-1})y_{k}$$
(2)

The closed loop transfer function becomes

$$\left(A(z^{-1})R(z^{-1}) + B(z^{-1})S(z^{-1})\right)y_k = B(z^{-1})T(z^{-1})sp_k \quad (3)$$

where the characteristic equation

$$\left(A(z^{-1})R(z^{-1}) + B(z^{-1})S(z^{-1})\right) = Alc(z^{-1})$$
(4)

can be solved as a Diophantine equation. There is not a unique solution for the equation and this can be used to solve the 2DoF controller. Defining:

$$R(z^{-1}) = A_{mr}(z^{-1})\overline{R}(z^{-1})\Delta(z^{-1})$$

$$S(z^{-1}) = A_{mr}(z^{-1})\overline{S}(z^{-1})$$

$$T(z^{-1}) = \overline{B}_{mr}(z^{-1})\overline{A}_{lc}(z^{-1})$$
with
$$\Delta(z^{-1}) = 1 - z^{-1}$$

$$\overline{A}_{lc}(z^{-1}) = A(z^{-1})\overline{R}(z^{-1})\Delta(z^{-1}) + B(z^{-1})\overline{S}(z^{-1})$$

$$G_m(z^{-1}) = B(z^{-1})\overline{B}_m(z^{-1})/A_m(z^{-1})$$
(5)

In this way, it can obtain: a) the relation between PV and set point (SP) is $G_m(z^{-1})$, which can be arbitrarily defined; b) due to $\Delta(z^{-1})$, the controller has integral action, and c) the disturbance rejection dynamic is set independently, by the choice of $\overline{A}_{lc}(z^{-1})$.

Without any loss of generality, as an example, a process model of first order will be considered as follows:

$$\begin{cases}
A = 1 + a_1 z^{-1} \\
B = b_1 z^{-1}
\end{cases}$$
(6)

The minimum degree of $\overline{R}(z^{-1})$ and $\overline{S}(z^{-1})$ are:

$$\overline{R}(z^{-1}) = 1 + r_1 z^{-1}$$

$$\overline{S}(z^{-1}) = s_0 + s_1 z^{-1}$$
and
(7)

$$\overline{A}_{lc}(z^{-1}) = 1 + a_{lc1}z^{-1} + a_{lc2}z^{-2} + a_{lc3}z^{-3}$$
(8)

Hence solving the diophantine equation using the Sylvester matrix:

$$\begin{bmatrix} 1 & b_1 & 0 \\ a_1 - 1 & 0 & b_1 \\ -a_1 & 0 & 0 \end{bmatrix} \begin{bmatrix} r_1 \\ s_0 \\ s_1 \end{bmatrix} = \begin{bmatrix} a_{lc1} - a_1 + 1 \\ a_{lc2} + a_1 \\ a_{lc3} \end{bmatrix}$$
(9)
The unknown controller coefficients

 $[r_1 \ s_0 \ s_1]$ always can be found if an only if $A(z^{-1})$ and $B(z^{-1})$ have no common roots.

3.1. Control Law without Anti-Wind-Up

According to (2) and (5), the resulting control law has the following polynomials:

$$\begin{cases} R = 1 + r_1 z^{-1} + r_2 z^{-2} + r_3 z^{-3} + r_4 z^{-4} + r_5 z^{-5} \\ S = s_0 + s_1 z^{-1} + s_2 z^{-2} + s_3 z^{-3} + s_4 z^{-4} \\ T = t_0 + t_1 z^{-1} + t_2 z^{-2} + t_3 z^{-3} \end{cases}$$
(10)

4. ANTI-WINDUP STUDY FOR STR MODEL WITH ONE DEGREE OF FREEDOM

There are two common situations that may lead to the need for an anti-windup solution: a) when a set point greater/less than the physically possible achievable is requested, and the manipulated variable saturates without reaching the set point; b) when a great performance is required, and the system is expected to react very fast so, the control actions are too violent, finding the upper or lower physical limitation.

As is proposed in (Huang, Peng, Wang, 2008), the antiwindup solutions can be divided in two groups: a) conditional integration and b) feedback calculation. In the former, the integration value is frozen and forced to be near the actual plant input (MV). In the later, an internal second feedback, which is related to the integrator term and the saturation element, is introduced.

First, the anti-windup solution for the scheme with one degree of freedom will be analyzed, in order to demonstrate that the difference with PIDs is not only related to the two degrees of freedom condition. In this case the closed loop poles to follow references and to reject disturbances are the same.

4.1. Conditional Integration

As a first case of study, the conditional integration solution will be introduced. The control law, in case of

saturation, will be forced to be near the actual input variable (MV).

Three controllers are initially simulated, namely:

- Controller 1: with no anti-windup logic.
- Controller 2: the actual MV is saved for futures calculations.
- Controller 3: the actual MV is saved for futures calculations, and a "set point setup" is performed.

The "set point setup" holds the actual set point value to a feasible reachable PV. Then, the system will store the saturated value as the new set point. When the stationary value for that situation is finally reached; the error will be zero because the system is in the new desired point.



Figure 2 shows a simulation of the three controllers with all the closed loop poles equal to $p_{Amr} = 0.98$, and compares how the controllers work when set point is greater than the physically possible achievable for process variable and manipulated:

Controller 1: the system gets stuck due to the difference between set point and process variable, which results in an accumulation of error. The control action starts working again after several further error samples of opposite sign, because there is no antiwindup logic being applied.

Controller 2: when the accumulated error is not correctly adjusted, the system may act automatically after the set point has been changed because the calculation of the control variable allows this. However,

it may have an underdamped response, as is the case between times 1500 and 2000 in Figure 2.

Controller 3: It works following the reference model without any delay. As can be seen, the set point error does not influence the performance either (response appears after time 1500 and 2500). Since the system stores previous set point values, these become a key factor in the anti-windup solution.

In order to test Controller 3 against the second typical case where an anti-windup solution is required (that is, when great performance is required), the model poles $p_{Amr} = 0.90$ is used. The simulation result can be seen in Figure 3.



Here, an underdamped behaviour appears. The conclusion is that this technique depends on the required performance. Therefore, the anti-windup logic has not only to limit the MV, but it has also to recalculate its internal value for further states.

4.2. Integrator Feedback

These methods consist of adding a feedback related with the saturation element and the integrator term.

4.2.1. Tracking Feedback

The integral part is separated from the rest of the controller. In Figure 4, it is shown the difference between the calculated manipulated variable (CMV or v in the figure) and the actual manipulated variable (MV or *u* in the figure).

Both signals are compared and the difference affected by a gain, is fed to the integrator input.



Figure 4: Tracking Feedback

If the MV is not saturated, this feedback has no effect; when the actuator is saturated, the feedback attempts to make the error signal equal to zero changing the integral value.

The feedback gain (2, in the figure) has to be adjusted for having the fastest correction.



set point is greater than the physically achievable, $p_{Amr} = 0.98$.

This controller uses several (more than a PID) past samples of PV and MV and SP. Therefore, the tracking feedback scheme presents a delay when the SP is not reachable. This effect can be seen in Figure 5.

Thus, the magnitude of the difference between the set point and the process variable will affect the subsequent performance of the system, especially in the response time.





The CMV, by both methods in the saturation situation is shown in Figure 6 and 8. In the case of Controller 3, the

first points in the saturation present difference between the CMV and the MV. This is due to the fact that the steady state has not been reached and the error is not zero.

However, in the tracking feedback remains in a constant oscillating error that is quickly eliminated after the set point changes, but there is a small delay in the reaction. Figure 7 compares Tracking Feedback, Controller3 and an ideal situation without saturation, for the case of a great performance requirement. Until the manipulated variable is not saturated, both schemes give the same sequence of values and the same plant response. When the MV becomes saturated, the Controller 3 has an undesired underdamped performance while the tracking feedback reaches the set point with overdamping.



Figure 7: Tracking Feedback, Controller3 and Ideal Situation, for great performance case, $p_{Amr} = 0.90$.



Figure 8: Tracking Feedback (green) vs. Controller3 (light blue) for great performance case, $p_{Amr} = 0.90$. CMV.

It is interesting to see the calculated values of the MV in this situation (Figures 8). The conclusion is that the Tracking Feedback scheme is better in this situation, with a drawback: the response depends on the feedback gain that is another parameter to be set.

The aim of this work is to present a method for designing a controller with anti-windup without requiring any extra parameter.

4.2.2. Saturated Integration

Another way to obtain an integral with saturation is shown in Figure 9. The advantage of this scheme is that there is not any unstable block.



Figure 9: Saturated Integration

Figure 10 shows two overlapped SP changes: a) SP changes from 180 to 40 (blue) and b) SP changes from 90 to 40 (green). It can be seen that in case a) there is a greater delay in the MV (right side of figure 10). In this anti-windup scheme does not have into account the SP value, which should be considered to improve the logic. It can be proved that this delay depends on the position of the reference model poles.



Figure 10: The response in Performance Requirement with model poles of $p_{Amr} = 0.90$.

4.2.3. Polynomial Characteristic Observer

Astrom and Wittenmark (1997) propose a different possible scheme that adds a characteristic polynomial observer $W(z^{-1})$ to the system. Figure 12 presents the block diagram.



Figure 11: Polynomial Characteristic Observer

The linear equation of control (Equation 2) can be expressed as:

$$W(z^{-1})v_{k} = T(z^{-1})r_{k} - S(z^{-1})y_{k} + (W(z^{-1}) - R(z^{-1}))u_{k}$$
(11)

This controller is equivalent to Equation 2 while the system is not in a saturated state; when saturated or equal to one of the saturation limits, $W(z^{-1})$ will determine the dynamics of the saturator. However, the authors don't explain how to define $W(z^{-1})$, which is a key factor.

Considering that $R(z^{-1})$ is derived from Equation 5, $W(z^{-1})$ can be defined as any part of $R(z^{-1})$ or a combination of them.

On the other hand, the stability must be guaranteed in the saturation situation or in the linear behavior. To assure the stability in the saturation situation, the polynomial $T(z^{-1})/W(z^{-1})$ must be stable which is easily seen rearranging Figure 11 as can be shown in Figure 12.



Figure 12: Rearrangement of Astrom's and Wittenmark's Polynomial Characteristic Observer

If, by instance, $W(z^{-1}) = 1 + r_1 z^{-1}$, then the system will be unstable in the saturation situation. Figure 13 shows the response in this example.



Figure 13: Polynomial Characteristic Unstable Observer $W(z^{-1}) = 1 + r_1 z^{-1}.$

If $W(z^{-1}) = (1 + r_1 z^{-1}) A_{mr}$, the system is equivalent a pure integration feedback, which is not suitable for STR systems.

If $W(z^{-1}) = A_{mr}$, due to $A_{mr} = A_{lc}$, the dynamic of then $T(z^{-1})/W(z^{-1}) = cte$. Thus, in saturation state, the controller update is made with actual values because they are updated with the manipulated variable that goes into the process (MV) and the process variable (PV). Then, the resulting system has a correct antiwindup solution for all one degree of freedom cases.



Figure 14: Polynomial Characteristic Observer, $W(z^{-1}) = A_{mr}$

Figure 14 shows the block logic for the anti-windup solution for the STR 1-DOF. In the next Figure (15), the evaluated simulation for the example, the anti-windup for the same set of set points when the poles are positioned in 0.9 or 0.98, are shown.



In conclusion, if $A_{mr} = A_{lc}$ and the Control Law method (see section 4) is used to define all the polynomials, then it will always be possible to apply this anti-windup solution to any processes model.

It is interesting to remark that with this solution one degree is added to the internal feedback of the Pure Integrator. This is clearly related to the fact that the STR model has higher degrees than the PIDs and that conventional system is not enough to guarantee antiwindup for any possible situation.

5. ANTI-WIND-UP STUDY FOR STR MODEL WITH TWO DEGREES OF FREEDOM

With the scheme studied above, the anti-windup solution works exclusively when the poles are equal $(A_{mr} = A_{lc})$ because the term T saves previous states. Then, if $A_{mr} \neq \overline{A}_{lc}$, which is the case in a two-degree of freedom systems, there is no anti-windup solution because the dynamics are not being completely canceled with $W(z^{-1})$ (see Figure 16).



 $p_{Amr} = 0.98$ and $p_{Amr} = 0.90$.

5.1. The anti-windup solution for 2DoF Controller Since $T(z^{-1})$ dynamic is not cancelled, this polynomial is introduced in the saturation feedback.



Figure 17: *T* Rearrangement



Figure 18: Final anti-windup Block Diagram

Figure 17 shows the rearrangement of Figure 12 and Figure 18 shows the new strategy introducing \overline{A}_{lc} to the saturated intern loop.

Hence, the Block Diagram is rearranged so that all the terms that use stored data are updated with the actual values of u_k and y_k .

As $T(z^{-1}) = \overline{B}_{mr}(z^{-1})\overline{A}_{lc}(z^{-1})$, then it turns out an open direct loop without dynamic can be used if and

only if \overline{A}_{lc} is introduced in the loop.

Since the internal saturation feedback has the $R(z^{-1})/A_{lc}(z^{-1})$ term inside, this diagram eliminates all previous information in case of saturation, leaving all the values stuck in the actual process-related values without accumulating any kinds of error. What is more, this method offers the possibility to decrees the order of the A_{mr} polynomial

Figure 19 shows the anti-windup solution where A_{mr} has three different degrees.

Note 1: Figure 19 does not attempt to show better response, the regulation of the poles will deal to a better response or not.



Figure 19: Three different degrees of A_{mr} with Anti-Wind-Up solution, poles $p_{Amr} = 0.98$ and $p_{Amr} = 0.90$.

This gives a solution to the problem of anti-windup that is completely independent of the model and that is not subjected to any extra decision. It can be seen that the resulting performance is overdamped for all the times, regardless of the requirement of the system.

6. EXPERIMENTAL APPLICATION

Finally, the STR anti-windup solution for a real plant is presented. It is implemented in a tank, where the process variable is the water level and the manipulated variable is the valve, with limits $u_{\min} = 0$ and $u_{\min} = 0$. The process to be controlled can be described as:

$$\begin{cases} A = 1 - -0.98585z^{-1} \\ B = -0.00528z^{-1} \end{cases}$$
(12)

For modeling the plant, a test by exciting the plant with a pseudo-random binary sequence (PRBS), was performed. The parameters were obtained by least squares identification.

For this model, with $p_{Amr} = 0.98$ and $p_{Amr} = 0.90$, the polynomial controller result as follow:

$$R(z^{-1}) = 1 - 4.6795z^{-1} + 8.7347z^{-2} - 8.127z^{-3} + +3.7677z^{-4} - 0.696z^{-5}$$
$$S(z^{-1}) = -4.795 + 18.7029z^{-1} - 27.3558z^{-2} + +17.7826z^{-3} - 4.3347z^{-4}$$
(13)

$$T(z^{-1}) = -0.0015 + 0.0041z^{-1} - 0.0037z^{-2} + +0.0011z^{-3}$$



Figure 20: Comparing Pure Integration Feedback, Polynomial Characteristic Observer $W = A_{mr}$ and the proposed anti-windup solution. Poles $p_{Amr} = 0.98$ and

$$p_{Amr} = 0.90$$
.

In Figure 20, the response of the physical process is shown for different anti-windup strategies. The setpoint changes in t = 200 seconds and the solution proposed in this paper is the only one with no delay in response.

It should be noted that the manipulated variable has a reaction problem. It is the so-called "stick-slim motion", and it is due to the friction of the actuator that introduces an oscillation in its movement but that is not observed in the process variable.

7. CONCLUSION

This new anti-windup strategy has been successfully analyzed for a STR controller, with either one or two degrees of freedom.

Throughout the analysis of traditional solutions, it was possible to see not only why they are not accurate enough solutions, but also how different terms of the model influence the anti-windup logic, thus gradually leading to the proposed solution.

It was also demonstrated that when the anti-windup is guaranteed to work for Set Point Greater/Less than the Physically Possible Achievable, it does not necessarily guarantee that it also works for great performance requirement (fast reaction), and vice versa. In order to state that the solution works properly, both situations must be verified. The reason for this is that the system is in different physical situations; hence, the process behavior is different and the controller receives other sorts of data.

The solution finally proposed is a simple expression of an internal feedback. In addition to the simulation, it is shown the anti-windup problem and solution for a real process.

This study is relevant to develop different control strategies unlike PIDs. The presented STR method allows the user to effortlessly divide the two degrees of freedom into individual and understandable elements and to add terms without being restricted to any model dimension. More importantly, the saturation is no longer a problem that doesn't allow the use of this kind of algorithms.

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BIOGRAPHY

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