Disturbance rejection with derivative state feedback

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***Abstract*— The disturbance rejection problem is classical in many practical controlled systems. It is often solved with a state feedback control law for instance, but this approach doesn't always offer a good solution and in the control of mechanical systems using accelerometers as sensors, it is easier to obtain the state-derivative signals than the state signals. This paper proposes a simple new solution to the disturbance rejection problem with a derivative state feedback with a structural analysis of the model properties.**

Keywords: derivative state feedback, disturbance rejection, linear systems

1. Introduction

The disturbance rejection problem by state feedback has received a great deal of attention during the last decades. Solutions of this problem with stability conditions, first proposed in [10], [25] are often defined in terms of the infinite zero structure and in terms of the unstable zero structure through algebraic treatments [24], [13] or geometric approach [1]. The structural invariants play a fundamental role in this problem. They been extensively studied in many papers and books [15]. [3], [17], [11].

Nevertheless, the disturbance rejection problem is not always solvable by a static state feedback control law. Moreover, even if most of the control algorithms developed for state space systems are related to full state feedback or output feedback, in many applications, the sensors directly measure state derivatives rather than states. For instance, acceleration signals can only be modeled as state derivatives (use of accelerometers in many electromechanical systems) with application to control of car wheel suspension systems, and also in aeronautical engineering and civil engineering. In that case, a derivative state feedback control law is proposed [23] and it is also shown in some papers that one advantage over the conventional state feedback is that it results in smaller gains. A geometric theory of derivative state feedback is given in [12] and an application to stabilizability and Disturbance Rejection with Derivative State Feedback (DRDSF) is proposed in [14], but for a particular case.

With the bond graph approach [16], it has been proved that these concepts can be simultaneously used in a unified way, due to the information contained at the same time in the graphical representation and due to the mathematical information contained in the structure of the model. State feedback control law have been used for the disturbance rejection and input-output decoupling problems [2].

The objective of this paper is the development of a new derivative state feedback control law when the classical disturbance rejection problem is not solvable with a static state feedback. A new solution is proposed thanks to the bond graph model with a derivative causality assignment. The properties of the controlled model are studied, with a graphical (structural) approach. The second section gives some properties of a linear state model related to the infinite and finite structures and the condition for disturbance rejection. In the third part, the new derivative state feedback for disturbance rejection is proposed in the SISO case. The properties of the controlled system are studied with a structural approach. Simulations on a the torsion bar system are presented with a brief conclusion.

2. Disturbance rejection with state feedback

Consider a linear system $\Sigma(C,A,B,E)$ described by the classical state space representation written in (1), with $x \in \Re^n$ is the state vector, $u \in \Re^m$ represents the input vector, $y \in \Re^p$ is the vector of output variables to be controlled and $d \in \Re^q$ is the vector of unknown input variables, disturbance variables in this study.

$$\left\{ \begin{array}{l} \dot{x}(t) = Ax(t) + Bu(t) + Ed(t) \\ y(t) = Cx(t) \end{array} \right. \quad (1)$$

In order to study the disturbance rejection problem, the structure of the model $\Sigma(C,A,B,E)$ must be highlighted. The different transfer functions are $T_{yu}(s) = C(sI-A)^{-1}B$ and $T_{yd}(s) = C(sI-A)^{-1}E$.

2.1. Structures of state space models

Infinite structure The infinite structure of an unperturbed multi-variable linear model $\Sigma(C,A,B)$ is characterized by different integer sets: the set of infinite zero orders of the global model $\Sigma(C,A,B)$ and the set of row infinite zero orders of the row sub-systems $\Sigma(c_i,A,B)$, denoted $\{n_i\}$, c_i is the i^{th} row of matrix C. The infinite structure is well defined in case of LTI models [6] with a transfer matrix representation or with a graphical representation (structured approach), [7], or with a bond graph approach [19].

The row infinite zero order (relative degree) for the proper row sub-system $\Sigma(c_i,A,B)$ is the integer n_i, which verifies condition $n_i = \min \left\{ k | c_i A^{(k-1)} B \neq 0 \right\}$. n_i is equal to the number of derivations of the output variable $y_i(t)$ necessary for at least one of the input variables to appear explicitly.

Finite structure The invariant zeros (transmission zeros for controllable/observable models) of model $\Sigma(C,A,B)$ are the zeros of the system matrix defined in equation (2).

$$S(s) = \begin{pmatrix} sI - A & -B \\ C & 0 \end{pmatrix} \quad (2)$$

System $\Sigma(C,A,B)$ is state controllable iff matrix $[sI - A \;\; -B]$ doesn't contain any zero, and observable iff matrix $[sI - A^t \;\; C^t]^t$ doesn't contain any zero. Otherwise, zeros are called input (output) decoupling zeros (respectively) [17].

2.2. Disturbance rejection with state feedback

The disturbance rejection problem for the system described by equation (1) has a solution by a state feedback control law (without measurement of the disturbance variables) $u(t) = Fx(t) + Gv(t)$ iff the infinite structure of matrix $s^{-1}T_{yu}(s)$ is equal to the infinite structure of matrix $[s^{-1}T_{yu}(s) \quad T_{yd}(s)]$. With the measurement of the disturbance variables, the condition is on matrices $T_{yu}(s)$ and $[T_{yu}(s) \quad T_{yd}(s)]$.

At most, the disturbance rejection problem for the system described by equation (1) has a solution with stability iff the zeros of model $\Sigma(C,A,B)$ which are not zeros of model $\Sigma(C,A,B,E)$ are strictly stable [13].

2.3. Bond graph approach

Bond graph with integral causality assignment: BGI

The state space equation (1) can be directly written from the BGI and the infinite structure of the model $\Sigma(C,A,B,E)$ can be highlighted from a graphical approach, as well as the finite structure.

The determination of the row infinite structure of a bond graph model is based on the concept of causal path length. If all the dynamical elements have an integral causality assignment, the causal path length between an input source and an output detector in the bond graph model is equal to the number of dynamical elements met in the path. n_i is equal to the shortest causal path length between the i^{th} output detector associated to the output variable y_i and the set of input sources.

Bond graph with derivative causality assignment: BGD

A different expression of the state space equation is proposed, which is equivalent to draw the bond graph model with a derivative causality assignment. Consider the new state space representation as proposed in equation (3).

$$\begin{cases} x(t) = A^{-1}\dot{x}(t) - A^{-1}Bu(t) - A^{-1}Ed(t) \\ y(t) = CA^{-1}\dot{x}(t) - CA^{-1}Bu(t) - CA^{-1}Ed(t) \end{cases} \quad (3)$$

With the definition of causal paths and causal path length in the BGD, some zeros can be studied, such as input (output) decoupling zeros (non controllable/observable modes) and also the null invariant zeros. As an example, for a bond graph model the state matrix is invertible if it is possible to assign a derivative causality to each dynamical element, and the state model is controllable if it is possible to assign a derivative causality with dualisation of input sources, if necessary [18]. In that case, non controllable poles are equal to 0. For a mono-variable system, if $CA^{-1}B = 0$, thus we can deduce that the causal path length in the BGD between the output detector and the input source is at least equal to 1 and that system $\Sigma(C,A,B)$ contains at least one null invariant zero.

3. Disturbance rejection with derivative state feedback DRDSF: SISO case

In order to simplify the presentation, the mono-variable case is studied, $m = p = q = 1$. The infinite zero order of the system $\Sigma(C,A,B)$ is the integer denoted here r, which verifies condition $r = \min \left\{ k | CA^{(k-1)}B \neq 0 \right\}$, and the row infinite zero order of the system $\Sigma(C,A,E)$ is the integer r_{dist}.

Assumptions

- System $\Sigma(C,A,B)$ is state controllable/observable and the state matrix A is invertible (non-restrictive assumption for bond graph models)
- $CA^{-1}B \neq 0$, model $\Sigma(C,A,B)$ doesn't contain any invariant zero equal to 0
- The disturbance variable is measured or estimated, (UIO approach proposed in [21]), with also the knowledge of the derivative of the state vector
- $r_{dist} < r$, which means that the classical disturbance rejection problem with static state feedback is not solvable
- The invariant zeros of $\Sigma(C,A,B)$ are stable

3.1. DSF: Derivative state feedback

Consider the linear system $\Sigma(C,A,B,E)$ described by the classical state space representation written in (1), and the DSF control law with disturbance defined in (4). $v(t)$ is the new input variable (new control).

$$u(t) = F\dot{x}(t) + Gv(t) + E_m d(t) \quad (4)$$

The controlled system can be written as (5).

$$\begin{cases} (I - BF)\dot{x}(t) = Ax(t) + BGv(t) + (E + BE_m)d(t) \\ y(t) = Cx(t) \end{cases} \quad (5)$$

The properties of this new model are not easy to be highlighted. Moreover, if matrix $(I - BF)$ is not invertible, the state equation in (5) is designed under the names of generalized state space form or descriptor (singular) form and in that case the controlled model contains poles at infinity.

The characteristic equation of the closed loop system (5) is defined as (6).

$$det(sI - sBF - A) = 0 \quad . \quad (6)$$

The degree γ of the characteristic polynomial in equation (6) is the number of system's finite eigenvalues, while $n - \gamma$ is the number of system's eigenvalues at infinity [8]. If matrix $(I - BF)$ is not invertible, the system has thus poles at infinity and properties such as controllability/observability properties must be studied in a new way [4], [5] and [26].

In order to provide new algorithms, the "reciprocal state space" (RSS) is provided in [23] for the vibration control of piezoelectric smart plate. In this approach, if the state matrix is invertible, every state variable can be expressed in terms of state derivative variables and control inputs in order to simplify the design of the state feedback matrix.

From a bond graph approach, it is equivalent to apply a derivative causality assigment, and this approach has been extensively used for the UIO (Unknown Input Observer) design [20], [22] and [21]. This concept is used in the following.

3.2. DRDSF without pole placement

With the control law defined in (4), the equations (3) can now be written as equations (7).

$$\begin{cases} x(t) = (A^{-1} - A^{-1}BF)\dot{x}(t) + (-A^{-1}BE_m - A^{-1}E)d(t) \\ -A^{-1}BGv(t) \\ y(t) = (CA^{-1} - CA^{-1}BF)\dot{x}(t) + \cdots \\ \cdots(-CA^{-1}BE_m - CA^{-1}E)d(t) - CA^{-1}BGv(t) \end{cases} \quad (7)$$

If $(-CA^{-1}BE_m - CA^{-1}E) = 0$ and $(CA^{-1} - CA^{-1}BF) = 0$ then $y(t) = -CA^{-1}BGv(t)$ and the disturbance is rejected. With this simple solution, there is a direct transmission between the new input variable $v(t)$ and the output variable $y(t)$. The matrices F, G and E_m, solution of the disturbance rejection problem are defined in equation (8), with condition $CA^{-1}B \neq 0$.

$$\begin{cases} F = (CA^{-1}B)^{-1}CA^{-1} \\ E_m = -(CA^{-1}B)^{-1}CA^{-1}E \\ G = -(CA^{-1}B)^{-1} \end{cases} \quad (8)$$

The input-output relation is now $y(t) = v(t)$. Remark that if $CA^{-1}E = 0$ (system $\Sigma(C,A,E)$ contains at least one invariant zero equal to 0), it is not necessary to measure (or estimate) the disturbance variable.

3.3. Controlled model without pole placement: properties

With the control law defined in equation (4), the invariant zeros of the controlled system (without disturbance) are the zeros of matrix $S_{CL}(s)$ defined in equation (9).

$$S_{CL}(s) = \begin{pmatrix} sI - A - sBF & -B \\ C & 0 \end{pmatrix} \quad (9)$$

Property 1: The invariant zeros of the model $\Sigma(C,A,B)$ are the same as the one of the controlled model with a DSF (Derivative State Feedback). Proof appendix 1.

Property 2: The controlled system $\Sigma(C,A,B)$ with a DSF control law defined in (4) and (8) is an implicit model [17], of the type $E\dot{x} = Ax + Bu$, with matrix E non invertible.

Proof: consider the new state space model with the DSF control defined in (4). Consider matrix $(I - BF) = I - B(CA^{-1}B)^{-1}CA^{-1}$ with a pre-multiplication by matrix CA^{-1}, it comes $CA^{-1}(I - BF) = (CA^{-1} - CA^{-1}B(CA^{-1}B)^{-1}CA^{-1}) = 0$, thus the rank of matrix $(I - BF)$ is not maximal, which proves the property.

Property 3: The degree γ of the characteristic polynomial $\det(sI - sBF - A)$ of the controlled system $\Sigma(C,A,B)$ with a DSF control law defined in equations (4) and (8) is equal to the number of invariant zeros of $\Sigma(C,A,B)$, i.e. $\gamma = n - r$. The new model contains only $n - r$ finite modes (invariant zeros of $\Sigma(C,A,B)$)

Proof: Consider the determinant of matrix $S_{CL}(s)$. $\det S_{CL}(s) = \det(sI - A - sBF).\det(C(sI - A - sBF)^{-1}B)$. The new transfer function between the new input variable $V(s)$ and the output variable $Y(s)$ is equal to 1 and thus $\det S_{CL}(s)$ = $\det(sI - A - sBF)$.□

The input decoupling zeros (non controllable modes) of system $\Sigma(C,A,B)$ are the zeros of matrix $[sI - A \quad -B]$. For a controllable model, this matrix doesn't contain any zero. With the DSF control law, it can be easily proved that the new matrix $[sI - A - sBF \quad -B]$ is equivalent to the previous one (same product as in equation (13)), and thus this matrix doesn't contain any zero.

The output decoupling zeros (non observable modes) are the zeros of matrix $[sI - A^t \quad C^t]^t$. For an observable model, this matrix doesn't contain any zero, but with the DSF control law, the new model can become non observable. It is a classical property when applying a static state feedback control for the disturbance rejection problem, as well as in the case of the input-output decoupling problem. It is well known that the non observable modes are all or one part of the invariant zeros. Since the new model is an implicit (singular) state model, a must precise analysis should be performed, for the infinite non observable modes, but it is not proposed here. Only finite zeros are studied.

Property 4: The zeros of matrix $[sI - s(BF)^t - A^t \quad C^t]^t$ of the controlled system $\Sigma(C,A,B)$ with a DSF control law defined in equations (4) and (8) are the invariant zeros of the model $\Sigma(C,A,B)$. Proof appendix 2.

3.4. DRDSF with pole placement

In this section, the solution for the DRDSF with pole placement is obtained with matrices E_m and G defined in equation (8) and with the new matrix F defined in equation (10). The set $\{\alpha_1, \alpha_2, \cdots, \alpha_{r_{dist}}\}$ is a set of r_{dist} free parameters used for pole placement.

$$F = (CA^{-1}B)^{-1}[CA^{-1} + \alpha_1 C + \alpha_2 CA + \cdots + \alpha_{r_{dist}} CA^{r_{dist}-1}] \quad (10)$$

Property 5: The differential equation verified by the output variable $y(t)$ with a derivative state feedback control law defined in equations (4) and (8) and matrix F in (10) is written in equation (11). Proof: appendix 3.

$$y + \alpha_1 \dot{y} + \alpha_2 \ddot{y} + \cdots + \alpha_{r_{dist}} y^{(r_{dist})} = v(t) \quad (11)$$

Note that relation $CA^{k-1}B = CA^{k-1}E = 0$ with $k < r_{dist}$ is used in this proof, and thus the maximal number of poles which can be placed is equal to r_{dist}.

Property 6: The degree of the characteristic polynomial $\det(sI - sBF - A)$ of the controlled system $\Sigma(C,A,B)$ with a DSF control law defined in equations (4), (8) (G and E_m) and matrix F in (10) is equal to $(n - r) + r_{dist}$ (number of invariant zeros of $\Sigma(C,A,B)$ + infinite zero order of $\Sigma(C,A,E)$).

Proof: Since Matrices $S(s)$ and $S_{CL}(s)$ are equivalent, $\det S(s) = \det S_{CL}(s)$, which is a polynomial of degree $n - r$, (number of invariant zeros). Moreover, $\det S_{CL}(s) =$

$\det(sI - A - sBF).\det(C(sI - A - sBF)^{-1}B)$, and since the new transfer function $\det(C(sI - A - sBF)^{-1}B)$ is of order r_{dist} with a constant numerator, $\det(sI - A - sBF)$ is a polynomial of degree $(n - r) + r_{dist}$.□

3.5. *Some concluding remarks*

Note that for a SISO system $\Sigma(C,A,B,E)$, when the disturbance rejection problem has a solution with a classical state feedback law (i.e. $r \leq r_{dist}$), only r modes can be assigned and model $\Sigma(C,A,B)$ contains $n - r$ invariant zeros which becomes non observable modes. With this approach, the new model is an implicit model with $(n - r) + r_{dist}$ finite modes, whose r_{dist} can be assigned. The properties of the controlled model must be specified, particularly properties of the infinite poles. The study is not proposed in this paper due to the lack of space.

In many practical application, with a state feedback approach, the state vector must be estimated. In this approach, the derivative of the state vector must be estimated, as well as the disturbance variable. From a bond graph approach, as proposed in [21] for the UIO design, the derivative of the state vector is directly obtained from the observer (bond graph model) and the disturbance can be estimated in the same way. Note that this approach can be easily extended to multi-variable systems.

4. CASE STUDY: TORSION BAR

The experimental mechanical system is presented in Fig. 1. This system is composed of an amplifier, a DC motor, three inertias (one for the motor), two of them are linked with a torsion bar.

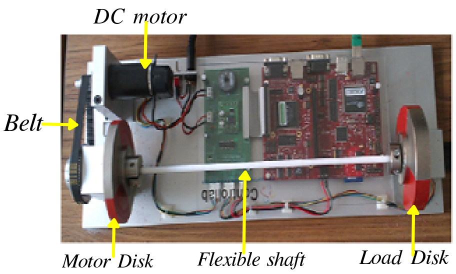

Fig. 1. Experimental torsion bar

The functional schematic model of the torsion bar is drawn in Fig. 2. The system consists of the following components: an amplifier A, a classical DC motor with an electrical part (inductance L_a and resistance R_a) and a mechanical part without friction and with inertia Jm (supposed negligible in the present study), a transmission element that transfers rotation from the motor to the motor disk with transmission ratio k_{belt}, a first rotational disk with an inertial parameter J_1 and a friction coefficient $Disk = R_1$, a flexible shaft modeled as a spring damper element (coefficient C and R_{shaft}), a second rotational disk with an inertial value J_2 and a friction coefficient R_2. The numerical values of system parameters are given in Table I.

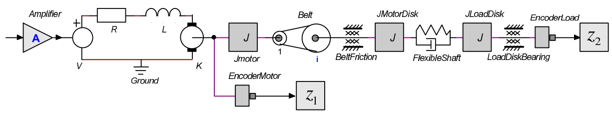

Fig. 2. Schematic model of the torsion bar system

4.1. *BGI model and state space model*

The simplified bond graph model of the system (drawn with 20*Sim*®) is shown in Fig. 3. $MSe : u$ the control input, is an effort source modulated by the control signal $u(t)$. $MSe : d(t)$, the disturbance variable (supposed to be known in this paper), is a torque. There are three output variables associated to output detectors which can be used to estimate the derivatives of the state variables and the disturbance which can be considered as an unknown input [22]. These output variables are considered here as output variables to be controlled. y_3 is a current variable associated to a current output detector (amperemeter), y_1 and y_2 are speed rotational variables associated to the two flow output detectors.

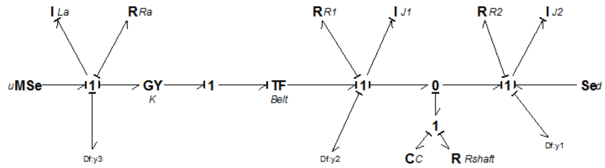

Fig. 3. Simplified BG model of the torsion bar

The state equations (12) are directly obtained from the bond graph model of Fig. 3. The state vector is $x = (x_1, x_2, x_3, x_4)^t$, with energy variables: $x_1 = q_C = q_{C_{shaft}}$ (which represents the displacement), $x_2 = p_{J_2}$, $x_3 = p_{J_1}$ (representing the momentums), and $x_4 = p_{L_a}$. Output variables. The output matrix C can be written as $C = [c_1^t, c_2^t, c_3^t]^t$. Poles of the model are equal to -925.66, $-7.9 \pm j55.4$ and -10.23.

$$\left\{ \begin{array}{l} \dot{x}_1 = -\frac{1}{J_2}x_2 + \frac{1}{J_1}x_3 \\ \dot{x}_2 = \frac{1}{C}x_1 + (-\frac{R_2}{J_2} - \frac{R_{Shaft}}{J_2})x_2 + \frac{R_{Shaft}}{J_1}x_3 + d(t) \\ \dot{x}_3 = -\frac{1}{C}x_1 + \frac{R_{Shaft}}{J_2}x_2 + (-\frac{R_1}{J_1} - \frac{R_{Shaft}}{J_1})x_3 + \frac{k}{L_a.k_{belt}}x_4 \\ \dot{x}_4 = -\frac{k}{J_1.k_{belt}}x_3 - \frac{R_a}{L_a}x_4 + u(t) \\ y_3 = \frac{1}{L_a}x_4 \quad y_2 = \frac{1}{J_1}x_3 \quad y_1 = \frac{1}{J_2}x_2 \end{array} \right. \tag{12}$$

The model has three output variables. The study is proposed for each row model $\Sigma(c_i, A, B, E)$.

4.2. *Structural analysis*

The Bond graph model is controllable/observable and the state matrix in invertible, a derivative causality can be assigned to each dynamical element, Fig. 4.

The infinite zero orders for each output variable y_i, denoted n_i, are equal to the shortest causal path length between the input source $MSe : u(t)$ and the output detector $Df : y_i$. The causal paths are $Df : y_1 \rightarrow I : J_2 \rightarrow R : R_{Shaft} \rightarrow I : J_1 \rightarrow TF : k_{belt} \rightarrow GY : k \rightarrow I : L_a \rightarrow MSe : u$, the second one $Df :$

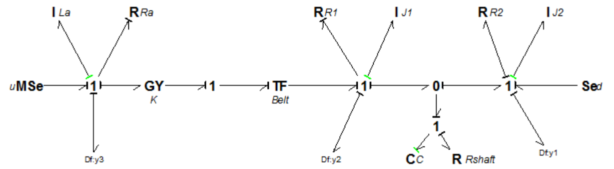

Fig. 4. BG model with derivative causality assignment

$y_2 \to I : J_1 \to TF : k_{belt} \to GY : k \to I : L_a \to MSe : u$ and the last one $Df : y_3 \to I : L_a \to MSe : u$ with respective length $n_1 = 3$, $n_2 = 2$ and $n_3 = 1$. The infinite zero order for row subsystems $\Sigma(c_i, A, E)$ are r_{disti} with $r_{dist1} = 1$, $r_{dist2} = 2$ and $r_{dist3} = 3$ with causal paths $Df : y_1 \to I : J_2 \to Se : d(t)$, the second one $Df : y_2 \to I : J_1 \to R : R_{Shaft} \to I : J_2 \to Se : d(t)$ and $Df : y_3 \to I : L_a \to GY : k \to TF : k_{belt} \to I : J_1 \to R : R_{Shaft} \to I : J_2 \to Se : d(t)$.

The classical disturbance rejection with state feedback is possible for output $y_3(t)$ because $n_3 = 1$ and $r_{dist3} = 3$, with a classical state feedback with measurement (or estimation) of the disturbance for the second output $y_2(t)$ since $n_2 = r_{dist2} = 2$. It is not possible for the first output variable $y_1(t)$ since $n_1 = 3$ and $r_{dist1} = 1$.

A DSF control law is proposed for the disturbance rejection in case of the output variable $y_1(t)$. Some properties of the controlled model are derived from a causal analysis. From the BGD, the causal path lengths between the output detector associated to y_1 and the input source (control and disturbance) are equal to 0, thus $c_1 A^{-1} B \neq 0$ and $c_1 A^{-1} E \neq 0$.

Model $\Sigma(c_1, A, B)$ has one invariant zero since $r_1 = 3$, its value is $z_I = -1/(C.R_{Shaft})$. Some coefficients can be derived from a causal analysis (causal path gains), but here they are directly obtained from formal calculus with Maple. $E_m = -(c_1 A^{-1} B)^{-1} c_1 A^{-1} E = -(R_a.k_{belt})/k$ and $G = (k_{belt}^2.R_1.R_a + k^2 + k_{belt}^2.R_2.R_a)/(k_{belt}.k)$.

Without pole placement, $Y_1(s) = V(s)$ and $\det(sI - A - sBF) = k_{const}.(1 + C.R_{Shaft}s)$ where k_{const} is a constant.

With pole placement, matrix F is defined in equation (10) and $Y_1(s) = \frac{1}{1+\alpha_1 s} V(s)$ (only one pole can be chosen because $n_1 = 1$). In that case, $\det(sI - A - sBF) = k_{const}.(1 + \alpha_1 s)(1 + C.R_{Shaft}s)$ where k_{const} is a constant.

4.3. DRDSF: simulations

Simulations are proposed in case of DRDSF with placement of one pole. In that case, $E_m = -8.42798$, $G = 0.18815$, the invariant zero is $s = -3494.54850$ and $F = [0.188; 8.428 - 134.396\alpha_1; 8.428; 1]$ and $\det(sI - A - sBF) = 2.969.10^7(1 + \alpha_1 s)(1 + 2.861.10^{-4}s)$.

The disturbance variable is a step function with value 0.1Nm in the interval time [1 2] and the new control variable $v(t)$ is also a step function with value $20 rad s^{-1}$ in the interval time [3 4]. The output variable $y(t)$ and the control variable $u(t)$ are studied in the two cases: with and without disturbance rejection, respectively represented in Figures 5 and 6. The simulations prove the accuracy of the proposed methodology, which has been test with other kind of disturbances. The results, as well accurate are not proposed here due to the lack of space.

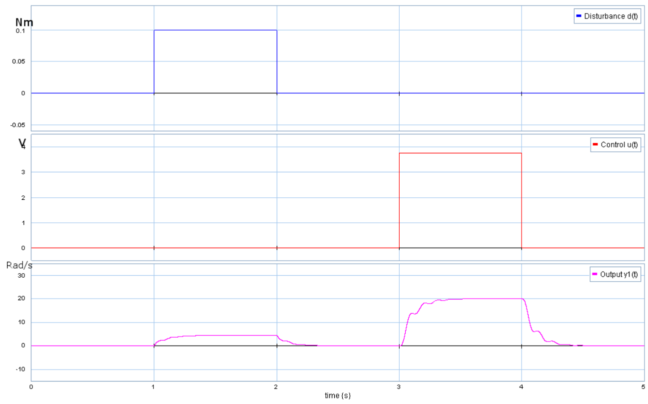

Fig. 5. Output variable $y_1(t)$ and control $u(t)$ without disturbance rejection

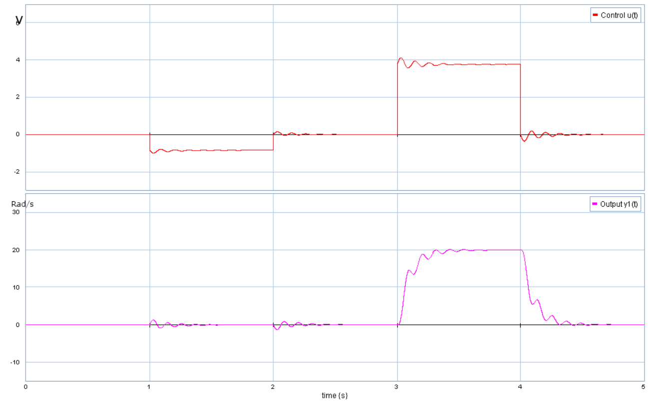

Fig. 6. Output variable $y_1(t)$ and control $u(t)$ with disturbance rejection

5. CONCLUSION

A derivative state feedback control law is proposed for the disturbance rejection problem with analysis of the structural properties of the controlled system. It is proved to be accurate with classical restrictive conditions based on the infinite and finite structure requirements. The application of this new scheme is straight and the approach is similar to the classical disturbance problem with state feedback. As application, simulations are proposed on a torsional bar system. A comparison of the performances of this control will be made with the flat control one on the real system. Theoretical developments will be proposed in a future work for multivariable linear and nonlinear systems.

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Appendix

Appendix 1: Invariant zeros of the model $\Sigma(C,A,B)$

With an elementary right matrix multiplication on matrix $S(s)$ defined in equation (2), matrix $S_{CL}(s)$ is written as:

$$S_{CL}(s) = \begin{pmatrix} sI-A & -B \\ C & 0 \end{pmatrix} \cdot \begin{pmatrix} I & 0 \\ sF & 1 \end{pmatrix} \quad (13)$$

Since in equation (13), the matrix in the right position is unimodular, matrices $S(s)$ and $S_{CL}(s)$ are equivalent, which proves that the invariant zeros are the same for the initial model $\Sigma(C,A,B)$ as for the model with the DSF control law.

TABLE I

NUMERICAL VALUES OF SYSTEM PARAMETERS

Inductance	L_a	$0.0013\,H$
Inertia disk 1	J_1	$9.0662\cdot 10^{-4}\,kg\cdot m^2$
Inertia disk 2	J_2	$0.0014\,kg\cdot m^2$
Spring compliance	C	$0.56\,m/N$
Generator	k	0.1458
Transmission ratio	k_{belt}	1
Resistance	R_a	$1.2288\,\Omega$
Fist disk friction	R_1	$0.005\,Nms/rad$
Second disk friction	R_2	$2.548\cdot 10^{-5}\,Nms/rad$
Damping spring	R_{shaft}	$5.11\cdot 10^{-4}\,N/rad$

Appendix 2: Zeros of matrix $[sI-s(BF)^t-A^t \quad C^t]^t$

Consider the DSF with $F=(CA^{-1}B)^{-1}CA^{-1}$. An elementary left multiplication (with unimodular matrix) is applied on matrix $[sI-s(BF)^t-A^t \quad C^t]^t$, in equation (14).

$$\begin{pmatrix} I & 0 \\ CA^{-1} & 1 \end{pmatrix} \cdot \begin{pmatrix} sI-A-sBF \\ C \end{pmatrix} \quad (14)$$

$$= \begin{pmatrix} I & 0 \\ CA^{-1} & 1 \end{pmatrix} \cdot \begin{pmatrix} sI-A-sB[(CA^{-1}B)^{-1}CA^{-1}] & -B \\ C & 0 \end{pmatrix} \quad (15)$$

$$= \begin{pmatrix} sI-A-sB[(CA^{-1}B)^{-1}CA^{-1}] & -B \\ 0 & -CA^{-1}B \end{pmatrix} \quad (16)$$

Matrix $[sI-s(BF)^t-A^t \quad C^t]^t$ is thus equivalent to matrix defined in equation (16). This elementary calculus proves that with the state (derivative) feedback matrix $F=(CA^{-1}B)^{-1}CA^{-1}$, the finite zeros of matrix $[sI-s(BF)^t-A^t \quad C^t]^t$ are the invariant zeros of $\Sigma(C,A,B)$.

Appendix 3: DRDSF with pole placement

From equation (7), with matrices G and E_m defined in equation (8), the output variable can be written as:

$$y=[CA^{-1}-CA^{-1}BF]\dot{x}(t)+v \quad (17)$$

Thus, with matrix F defined in equation (10), the output variable $y(t)$ is now:

$$y(t)=\{CA^{-1}-CA^{-1}B(CA^{-1}B)^{-1}[CA^{-1}+\alpha_1 C+\alpha_2 CA+\cdots \\ \cdots+\alpha_{r_{dist}}CA^{r_{dist}-1}]\}\dot{x}(t)+v(t) \quad (18)$$

After a first simplification, it comes:

$$y(t)=-\{\alpha_1 C+\alpha_2 CA+\cdots+\alpha_{r_{dist}}CA^{r_{dist}-1}\}\dot{x}(t)+v(t) \quad (19)$$

But $C\dot{x}(t)=\dot{y}(t)$ and $CAx(t)=CA(A^{-1}-A^{-1}BF)\dot{x}(t)+CA(-A^{-1}BE_m-A^{-1}E)d(t)=C(I-BF)\dot{x}(t)-CBE_m d(t)-CEd(t)$. Since $CB=CE=0$ because $r>r_{dist}$, it comes $CAx(t)=C\dot{x}(t)$ and thus $CA\dot{x}(t)=C\ddot{y}(t)$. With a similar calculus, it comes $CA^{(k-1)}\dot{x}(t)=y^{(k)}$ for $k\leq r_{dist}$ and thus relation (11).

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