# FRACTIONAL ORDER $\text{PI}^{\lambda}\text{D}^{\mu}$ CONTROLLER: APPLIED TO CONTROL A MANIPULATOR ROBOT WRIST

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# ABSTRACT

The work presented in this paper focuses on the use of multicontroller approach to control a robot wrist (STÄUBLI robot RX 90). A description and a nonlinear Mathematical model of process have been presented along with the local parametric models around operating points. Due to the advantage of fractional order PID control compared to conventional PID, a Fractional order PID controller has been developed around each selected operating points for each local parametric models with the Oustaloup recursive approximation method (ORA) method are presented. at the end we present the results obtained in the different simulations with 3D simulation robot model developed in CAO solid Works software and some perspectives for future work.

**Keywords:** Modeling, Local Control, Multi-controller control, Fractional PID controller, Approximation Oustaloup method.

# 1. INTRODUCTION

Invariant linear model for a physical process can only be an approximation. Indeed, a physical process generally has non-linearities (Slotine 1991) that are not taken into account in the modeling process. For some operating points of the physical process a local linear model can be determined. Two ways can be used to derive these linear models the first is based on the priori knowledge of the process and the second using identification. We may then seek to enslave the whole process in operational space using the local information (Balakrishnan 1994), (Chebassier, 1999). The objectives of this work are to develop a control structure in which control law is deduced from a set of controllers that are working together. The controllers parameters are deduced from the local models of the process. The purpose of the multi-controller command (Balakrishnan 1997) is to control the output of any process in space operation using

controls developed by different local controllers. The diagram block of the multi-controllers control approach is represented as follows:



Figure 1: Multi-controller structure approach.

The multi-controller command is used to specify:

- The controller's structures.
- The switching type (Pagès 2000; Duchamp 1998). Different solutions are proposed such as:
  - Fractional order PID controllers (Bensafia 2011).
  - Digital RST controller and Adaptive controllers (Karimi 1998) (Toscano 1997).
  - Frank or fuzzy switching (Pagès 2000; Foulloy 1998).
  - Direct or indirect approach to collaboration control law (Pagès 2000; Foulloy 1998).

In our work we have choose the use of an indirect approach based on local fractional order PID controllers and frank switching for robot wrist control.

# 2. PROCESS MODELING

The geometric series structure model of STÄUBLI Robot Rx-90 is give by the figure 2 (Khalil 2009):



Figure 2: RX-90 Robot model.

This robot has coupling between axis 5 and 6. The actuators are brushless motors and the engine control uses the rotor position to magnetic flux rotate to achieve desired torque value and generally this motor as a DC motor behave (Sabatier 2010). Our process corresponds to a robot wrist (axis 6) can be represented by the following figure:



Figure 3: Process model.

Mathematic dynamic process model is given by the following equations (ZENNIR 2013):

$$\Gamma_{\rm m} - \Gamma_{\rm s} = \left(J_{\rm m} + \frac{J_{\rm s} + M \cdot L^2}{N^2}\right) \cdot \ddot{\theta}_{\rm m} + \left(\gamma_{\rm m} + \frac{\gamma_{\rm s}}{N^2}\right) \cdot \dot{\theta}_{\rm m} \tag{1}$$

With:

$$J_{t} = \left(J_{m} + \frac{J_{s} + M \cdot L^{2}}{N^{2}}\right)$$
(2)

Where:

 $J_m$ : inertia moment applied in the motor shaft. Js: inertia moment applied in the output shaft (output shaft with mass).

$$\gamma_{\rm t} = \gamma_{\rm m} + \frac{\gamma_{\rm s}}{N^2} \tag{3}$$

 $\gamma_m$ : Viscous friction applied to the motor shaft.

 $\gamma_s$ : Viscous friction applied to the output shaft.

The motor torque is given by:

$$\Gamma_{\rm m} = K_{\rm e} \cdot u(t) \tag{4}$$

Where: Ke is the torque constant and u(t) control voltage. Then the nonlinear model is given by:

$$X_1 = \theta_m(t); X_2 = \dot{\theta}_m(t) \quad ; \quad X = \begin{pmatrix} X_1 \\ X_2 \end{pmatrix}$$
(5)

$$\dot{\mathbf{X}} = \begin{bmatrix} 0 & 1\\ \frac{\mathbf{M} \cdot \mathbf{g} \cdot \mathbf{L}}{\mathbf{N} \cdot \mathbf{J}_{t}} & -\frac{\gamma_{t}}{J_{t}} \end{bmatrix} \cdot \begin{bmatrix} \sin\left(\frac{\mathbf{X}_{1}}{\mathbf{N}}\right)\\ \mathbf{X}_{2} \end{bmatrix} + \begin{bmatrix} 0\\ \frac{K_{e}}{J_{t}} \end{bmatrix} \cdot \mathbf{u}(t)$$
(6)

$$Y = \theta_{s}(t) = \begin{bmatrix} -\frac{1}{N} & 0 \end{bmatrix} \cdot X$$
(7)

Which is the state space representation for the nonlinear model? To find the structure of local parametric models, we applied the tangent linearization and hence the linear local model is as follows (zennir 2013):

$$G(p) = \frac{-K_{\rm p}}{{\rm p}^2 + {\rm a}_{{\rm p}^1} \cdot {\rm p} + {\rm a}_{{\rm p}^2}} \tag{8}$$

Where:

$$K_{p} = \frac{K_{e}}{N \cdot J_{t}}, a_{p1} = \frac{\gamma_{t}}{J_{t}} \text{ and } a_{p2} = \frac{M \cdot g \cdot L}{N^{2} \cdot J_{t}} \cdot COS\left(\frac{X_{10}}{N}\right)$$
(9)

After identification of the linear local model near each operating point (zennir 2013). The corresponding continuous linear model is as follows:

- operating points,  $\theta$ s0=0 :  $G(p) = \frac{-111.5}{p^2 + 11.25 \cdot p + 79.14}$ (10)
- operating points,  $\theta s0=\pi/3$  and  $\theta s0=2\pi/3$  respectively:

$$G(p) = \frac{-111.5}{p^2 + 11.25 \cdot p + 39.57}; \quad G(p) = \frac{-111.5}{p^2 + 11.25 \cdot p - 39.57} \quad (11)$$

#### 3. LOCAL CONTROLLERS STRUCTURES

The structure of the local controllers is of type Fractional  $PI^{\lambda}D^{\mu}$ . The applications of the fractional calculus takes the order of integrals and derivatives have been defind in literatures, such as control theory (Yuquan, 2011), (Bagley 1991), (Makroglou 1994), and electro-analytical chemistry (Oldham 1976), (Goto 1975). In control theory, the general conclusion about fractional control system is that it could enlarge the stability region (Podlubny 1994) moreover it gives performance at least as good as its integer counterpart. In the other hand another important advantage is that fractional integrals or derivatives are hereditary functional while the ordinary ones are point functional. Here we should mention that for Fractional order PID controllers, many contributions and studies are presented in the past years

particularly in the tuning rules (Luo 2009), (Xue 2006), approximation and stability conditions (Sabatier, 2010). In generally Fractional-order calculus is an area of mathematics that deals with derivatives and integrals from non-integer orders. In other words, it is a generalization of the traditional calculus that leads to similar concepts and tools, but with a much wider applicability. In recent years, according to the advances in the field of fractional calculus, there had been a great interest to develop a new generation of PID controllers, which is commonly known as the Fractional-order PID (FOPID) or  $PI^{\lambda}D^{\mu}$  controller. The transfer function of FOPID controller, which was initially proposed by Podlubny (Vinagre 1997), is given by:

$$G_{c}(p) = \frac{U(p)}{E(p)} = K_{p} + K_{I} \frac{1}{p^{\lambda}} + K_{D} p^{\mu}, (\lambda, \mu > 0)$$
(12)

Where  $K_p, K_I, K_D \in R$  and  $\lambda, \mu \in R^+$  are the tuning parameters and the controller design problem is to determine the suitable value of these unknown parameters such that a predetermined set of control objectives is met (Vinagre 1997). Note that in (12) the fractional Laplace variable "p" powers are commonly interpreted in the time domain using either the Grunwald-Letnikov, Riemann-Liouville or the Caputo definition (Bettou, 2011). It should be noted that any conventional PID controller is a particular case of the FOPID controller (12) with  $\lambda$ =1 and  $\mu$ =1. Assuming  $\lambda$ =1 and  $\mu$ =0, or  $\lambda$ =0 and  $\mu$ =1 respectively corresponds conventional PI or PD controllers are the special cases of the fractional PI<sup> $\lambda$ </sup>D<sup> $\mu$ </sup> controller given by (12). Functional diagram of local FOPID controller is represented by the following figure:



Figure 4: Structure of FOPID local controller.

Many methods in literature have been proposed to FOPID approximation (Vinagre 1997). In this step of our work we used the Oustaloup recursive approximation method (ORA) (Oustaloup 1995), (Djouambi 2007, Djouambi 2006). With this FOPID we have using the approximation of Oustaloup. This method is based on the approximation of a function of the form:

$$H(p) = p^u \quad , \mu \in \mathbb{R}^+ \tag{13}$$

By a following rational function:

$$\hat{H}(p) = C \prod_{k=-N}^{N} \frac{1 + \frac{p}{w_k}}{1 + \frac{p}{w_k}}$$
(14)

With:

$$\frac{w_{k+1}}{w_k} = \frac{w_{k+1}}{w_k} = \alpha \eta > 1$$
 (15)

$$\frac{\dot{w}_{k+1}}{\dot{w}_{k}} = \eta > 0; \\ \frac{w_{k}}{\dot{w}_{k}} = \alpha > 0; \\ N = \frac{\log(\frac{w_{N}}{w_{0}})}{\log(\alpha\eta)};$$
(16)

 $\dot{w}_0 = \alpha^{-0.5} w_u; \ w_0 = \alpha^{0.5} w_u;$ 

$$\mu = \frac{\log \alpha}{\log(\alpha \eta)} \tag{17}$$

With  $w_u$  being the unit gain frequency and the central frequency of a band of frequencies geometrically distributed around it .That is,  $w_u = \sqrt{w_h w_l}$ , where  $w_h, w_l$  are the high and the low transitional frequencies. The parameters used in the Oustaloup approximation are:

- G(p) : Transfer function of local model of process.
- N=5: Approximation order.
- r<sub>1</sub>=-0.2 & r<sub>2</sub>=0.2: Integration & derivative order respectively.
- $w_1=10^{-2}$ ; low transitional frequency
- $w_h=10^3$ ; high transitional frequency
- $w_u=10$ ; Cutoff frequency

The transfer function of reference model is given by the following function:

$$Y_m(p) = \frac{\gamma^2}{(p+\gamma)^2} \cdot R(p) = G_m(p) \cdot R(p)$$
(18)

With : 
$$G_m(p) = \frac{\gamma^2}{p^2 + \lambda_1 \cdot p + \lambda_0}$$
(19)

With: R (p) is the set of the loop closes.

#### 4. SIMULATION

The synthesis of the controllers is continuous. The simulation is done in continuous time around the following operating points  $\theta_{s0}=0$ rad,  $\theta_{s0}=\pi/3$ rad and  $\theta_{s0}=2\pi/3$ rad chosen with two stable position and unstable. The parameter values of the reference model  $\lambda_0$  and  $\lambda_1$  are:

$$\gamma = 30; \ \lambda_0 = 900; \ \lambda_1 = 60$$
 (20)

The PID controllers parameters determined with  $\lambda=1$  and  $\mu=1$  (Traditional PID) around the operating points are:

TABLE 1: PARAMETERS OF THE LOCAL CONTROLLER

parameters	$K_p$	K <sub>i</sub>	K <sub>d</sub>
Controller ( $\theta_s=0$ )	-2	-5	-0.7
Controller ( $\theta_s = \pi/3$ )	-2	-0.4	-0.4
Controller ( $\theta_s = 2\pi/3$ )	-3	0.1	-0.4

Two simulations have been performed for each operating point in order to verify the role of the integrator, the stability of the closed loop and the proper functioning of the controllers around the operating points. The block diagram of simulation for each operating points is illustrated by the following figure:



Figure 5: Closed loop control around operating point.

# • The simulation around the operating point $\theta_{s0}=0$ rad

With controller parameters around  $\theta_{s0}$ =0rad and reference signal r (t) is equal to:

$$\mathbf{r}(\mathbf{t}) = 0.1 \cdot \sin(5 \cdot \mathbf{t}) \tag{21}$$

The simulation results are illustrated in the followings figures:



Figure 6.a: Control error ec (t).



Figure 6.b: Output of the model and the reference model.



Figure 6.c: Pole Zero map for closed loop.

• Approximation FOPID Transfer function around  $\theta_{s0}=0$  rad

$C1 = \\ -6.5059 (s^2 + 0.004757s + 5.66e-06) (s^2 + 0.02047s + \\ 0.0001048) (s^2 + 0.08798s + 0.001937) (s^2 + 0.3773s + \\ 0.03564) (s^2 + 1.612s + 0.6516) (s^2 + 6.864s + 11.82) \\ (s^2 + 29.09s + 212.6) (s^2 + 122.7s + 3789) (s^2 + 516.3s + \\ 6.709e04) (s^2 + 2168s + 1.183e06) (s^2 + 9080s + \\ 2.072e07) \\ \end{cases}$
(s+5565) (s+4151) (s+1286) (s+959) (s+297) (s+221.5) (s+68.61) (s+51.18) (s+15.85) (s+11.82) (s+3.661) (s+2.731) (s+0.8458) (s+0.631) (s+0.1954) (s+0.1458) (s+0.04514) (s+0.03367) (s+0.01043) (s+0.007779) (s+0.002409) (s+0.001797)

• The simulation around the operating point  $\theta_{s0} = \pi/3$  rad



Figure 7.a: Control error ec (t).



Figure 7.b: Output of the model and the reference model.



Figure 7.c: Pole Zero map for closed loop.

• Approximation FOPID Transfer function around  $\theta_{s0} = \pi/3$  rad



• The simulation around the operating point  $\theta_{s0}=2*\pi/3$  rad



Figure 8.a: Control error ec (t).



Figure 8.b: Output of the model and the reference model.



Figure 8.c: Pole Zero map for closed loop.

• Approximation FOPID Transfer function around  $\theta_{s0}=2^{*}\pi/3$  rad





Figure 9: Frequency response of each local FOPID control.



3D simulation of the robot is constructed in Malab-Simulink with SimMechanics block library. The System (robot) is represented by the following blocks: the body, joints, constraints, and force. The SimMechanics block library provided us the tools to formulate and solve motion equations of complete mechanical system. We used a bridge between solidworks\_matlab with same adaptations (SimMechanics 2 2007), (MATLAB 2010) to operate the robot model that we designed with solidworks. The Simulink modeling then appears (figure 11, figure 12 and figure 13) :



Figure 11: Block diagram of robot Rx-90 model.

To simplify the simulation we have block all robot joints except the terminal element and after we applied a simple control signal. A block diagram of the robot with the actuator and the sensor is illustrated in the following figure:



Figure 12: Control diagram block



Figure 13: CAO (with Solid Works) 3D robot model.

With the obtained results we can observe that:

- All poles and zeros in the left of complex plan (figure 6.c, figure 7.c and figure 8.c).
- Each local system is stable (figure.6.c, figure.7.c and figure.8.c) and error control given by closed loop system around  $\theta_{s0}=0$  is very small compared other local linear model (figure.6.a, figure.7.a, figure.8.a).
- The closed output loop of the local system around  $\theta_{s0}=0$  has the same curve although the reference output model compared other local system (figure.6.b, figure.7.b, figure.8.b).
- The local controller around  $\theta_{s0}=0$  is more robust compared with other local controller figure.9 and more stable figure 10.

- In the figure.11, figure 12 and figure 13 we have developed RX90 Robot CAO Solid works software for simulation with Matla-Simulink.
- We can observed too the order of local controller after approximation its high for realization

## 5. CONCLUSION

In this work, we have presented the modeling of nonlinear process (Wrist of Rx-90 Stäubli Robot). After that the local linear model near each considered operating points has been developed. We have described FOPID controller principal with Oustaloup Recursive Approximation method (ORA).

Based in Simulation results we noted that the application of CRON structure control is very interesting in this case of system but we need more optimal approximation for order minimization and chose of FOPID parameters. The results obtained allow concluding that the local controllers give good results around the operating points. But the results are local. Therefore, we must seek a collaborative approach these local control laws to obtain good results in all operating space.

Finally we will study at the future work another interesting approximation Fractional-order controller method of Charef (singularity function method) (Djouambi 2007) and frank switching with the indirect approach (collaboration between controller), then the same category with other type of controller as example Digital Fractional -order PID controller or RST controller.

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