ABSTRACT

The knowledge of the dynamic behavior of batteries is essential for their correct operation and management, to which aim mathematical models are invaluable tools. This paper presents an improvement of an already existing, commonly used dynamic battery model. The modification allows a better reproduction of the battery output voltage during charge and discharge processes without increasing the model complexity. Three parameter estimation methods are presented for both models. Also results of experimental tests are presented, which were performed in order to provide data for these three estimation methods and for validation purposes.

Keywords: Battery modeling, parameter estimation, battery testing.

1. INTRODUCTION

Nowadays energy storage systems are key elements in electrical systems. They allow for increased integration of renewable energy sources connected to the grid as well as to increase reliability and stability of various systems (Du and Lu 2015) like, for instance, in electric vehicles (Tie and Tan 2013).

Electrochemical batteries are the most widely used (Tie and Tan 2013) and special attention is paid to model them. There are several battery models of diverse complexity and accuracy (Dumbs 1999). A commonly used model was proposed by Tremblay-Dessaint (Tremblay and Dessaint 2009) which allows accurately representing the dynamic behavior of the battery with an easy parameterization method.

This model, implemented in the Matlab-Simulink SimPowerSystems library, takes the charge drain over time and battery current to represent the voltage behavior with an error typically lower than 5%. Its main shortcoming is its performance deterioration when reaching the end of discharge zone and for high current demands.

The objective of this paper is to propose a modification of Tremblay-Dessaint’s battery Model (TDM) which overcomes the above mentioned shortcomings. The new model features a better reproduction of the battery dynamic performance. This modification, here designated as Expanded Battery Model (EBM), does not increase the model complexity nor the parameter estimation process.

An estimation method, which preserves the simplicity of parameter estimation, extended from the method contributed by Tremblay and Dessaint (2009) is presented. In addition, two other new parameterization methods are addressed. The first two are based on the typical constant-current discharge characteristic or “Typical discharge characteristic”, usually provided by manufacturers; in that case, no battery testing is needed. If more accuracy is required and if battery testing is possible, a final estimation method is proposed.

We had focused our work on Li-Ion batteries but the results should be applicable to electrochemical batteries based on other technologies.

The rest of this paper is organized as follows: In section 2, the battery models are described. Section 3 presents the estimation methods. In section 4, the tests performed over the battery in order to estimate the model parameters are addressed and, in section 5, the validation of the EBM as well as the parameters estimations methods are shown. Finally, section 6, presents the conclusions.

2. BATTERY MODELS

An electrochemical battery is an element capable of transforming electrical energy into chemical energy (charging) and vice versa (discharging) through electrochemical reactions. For simulating them, several application-dependent models of varying complexity can be used. The following models use the State of Charge (SoC) and the filtered output current i* as state variables.

2.1. Tremblay-Dessaint’s Model

The Tremblay-Dessaint’s Model (TDM) is a semi-empirical battery model based on Shepherd’s work (Shepherd 1965) and consisting in two voltage equations (discharge and charge) as a function of the charge drain over time i (Ah), the actual current i, and the filtered current i*.

A Li-Ion cell will be utilized along this work. For this type of cell, the battery voltage given by Tremblay and Dessaint (2009) is:
Discharge:
\[ V_{\text{batt-dch}} = E_0 + Ae^{-B_i t} - K \frac{Q}{Q - it} - Ri - Ki' \frac{Q}{Q - it} \]
(1)

Charge:
\[ V_{\text{batt-ch}} = E_0 + Ae^{-B_i t} - K \frac{Q}{Q - it} - Ri - Ki' \frac{Q}{it - 0.1i} \]
(2)

where the variables are the battery voltage \( V \), the charge drain over time form full charge state \( it = \int i \, dt \) (Ah), the output current \( i \) (A), and the filtered current \( i' \) (A); and the parameters are the battery constant voltage \( E_0 \) (V), the polarization constant or polarization resistance \( K \) (V/Ah or \( \Omega \)), exponential zone amplitude \( A \) (V), the exponential zone time constant inverse \( B \) (Ah\(^{-1}\)), the internal resistance \( R \) (\( \Omega \)) and the battery capacity \( Q \) (Ah).

To complete the model the filtered current equation is needed:
\[ T_f \frac{d}{dt} i' + i' = i \]
(3)

where \( T_f \) is the filter time constant.

The State of Charge, \( SoC \), is a widely used variable of battery systems (Zhang and Lee 2011). It is used as an indicator of battery charge left and also to calculate other variables for more complex models such as ageing effect. The \( SoC \) can be calculated as:
\[ SoC = SoC_0 - \frac{\int i dt}{Q} \]
(4)

Where \( SoC_0 \) is the State of Charge initial.

Equations 1 and 2 could be expressed as functions of the \( SoC \), remembering that the \( SoC_0 \) is 1, resulting in the Equation 5.
\[ V_{\text{batt}}(SoC, i, i') = E_0 + Ae^{-B_i (1-SoC)} - KQ \left( \frac{1}{SoC} - 1 \right) - Ri - Ki' \left( \frac{i_{dch}}{SoC} + \frac{i_{ch}}{1.1 - SoC} \right) \]
(5)

where the variables \( i_{dch} \) and \( i_{ch} \) values were introduced to capture both battery voltage equations in only one expression. \( i_{dch} \) is 1 when the battery is discharging and 0 otherwise and \( i_{ch} \) is 1 when the battery is charging and 0 otherwise.

2.2. Expanded Battery Model.
Analyzing the TDM’s voltage Equation 5, two types of terms can be identified. Those depending solely on the \( SoC \) and those depending also on the actual current. The three first terms represent the Open Circuit Voltage (OCV). Special attention is given to the third term modeling the abrupt fall of the voltage for low \( SoC \) which contains the parameter \( K \).

The fourth and fifth terms represent the battery resistance. The last term, polarization-resistance voltage drop, models the abrupt increase of the internal resistance at low \( SoC \) while discharging and at high \( SoC \) while charging. The parameter \( K \) appears also as a multiplying factor.

Even though both effects which involve the \( K \) parameter (OCV voltage drop and polarization resistance) during discharge could likely be due to the active material current density (Shepherd 1965), the fact that both parameters have different units leads to the idea of distinguishing them. This separation, which is proposed in the Expanded Battery Model (EBM), does not increase the parameter estimation complexity and provides better dynamic model performance especially for low \( SoC \).

The voltage state with the EBM is expressed as:
\[ V_{\text{batt}}(SoC, i, i') = E_0 + Ae^{-B_i (1-SoC)} - K_1 Q \left( \frac{1}{SoC} - 1 \right) - Ri - K_2 i' \left( \frac{i_{dch}}{SoC} + \frac{i_{ch}}{1.1 - SoC} \right) \]
(6)

This Expanded Battery Model differentiates \( K_1 \) and \( K_2 \) parameters.

3. PARAMETER ESTIMATION METHODS
One of the main advantages of TDM is its easy parameterization needing only one “Typical discharge characteristic”, normally given by manufacturers. This estimation method will be reviewed and a first modification will be proposed in order to estimate \( K_1 \) and \( K_2 \) without increasing its complexity.

Next, a second method for estimating parameters for both models is presented. In this case, 3 complete constant-current discharge characteristics are used. These characteristics are also normally given by the manufacturer.

Finally, a third method based on the Hybrid Pulse Power Characterization test (HPPC) will be introduced, where all model parameters are estimated. This characterization test excites the frequency spectrum of electrochemical batteries in order to achieve a correct parameter fitting.

Independently of the selected parameterization method the number of parameters to be estimated are 7 for the TDM and 8 for the EBM: \( E_0, K_1, A, B, R, K_2, Q \) and \( T_f \) where the constraint \( K_1 = K_2 = K \) is applied for the TDM.
3.1. Method of the Minimal Equation System (M1)

This method, fully described in Tremblay and Dessaint (2009), can be summarized as follows. From the datasheet provided by manufacturer, the maximum capacity, and the internal resistance, are directly obtained.

A “Typical discharge characteristic” at a constant discharge current (generally \(i_{dch} = 0.2\,C\)) is also provided and three points are extracted. The Full voltage \((0, V_{full})\), the End of exponential zone \((Q_{exp}, V_{exp})\) and the Nominal \((Q_{nom}, V_{nom})\) when the voltage begins to fall abruptly (see Figure 1).

The parameter \(B\) could be approximated to \(3/Q_{exp}\) which is the end of the exponential term.

The steady state of the discharge test (constant current) allows considering \(i = i^* = i_{dch}\) at the two points \((Q_{exp}, V_{exp})\) and \((Q_{nom}, V_{nom})\). And for \((0, V_{full})\) the initial as well as the filtered currents, are zero.

These leads to the following equation system:

\[
\begin{align*}
V_{full} = E_0 + A \\
V_{exp} = E_0 + A\alpha_{(soc_{exp})} - K\beta_{(soc_{exp})} - R_i i_{dch} - K_2 Y_{(soc_{exp}i_{dch})} \\
V_{nom} = E_0 + A\alpha_{(soc_{nom})} - K\beta_{(soc_{nom})} - R_i i_{dch} - K_2 Y_{(soc_{nom}i_{dch})} \\
V_{nomSC} = E_0 + A\alpha_{(soc_{nomSC})} - K\beta_{(soc_{nomSC})} - R_i i_{dchSC} - K_2 Y_{(soc_{nomSC}i_{dchSC})}
\end{align*}
\]  

(7)

where

\[
\begin{align*}
\alpha_{(soc)} &= e^{-B(1-soc)} \\
\beta_{(soc)} &= Q \left( \frac{1}{soc} - 1 \right) \\
Y_{(soc,i^*)} &= \frac{1}{soc} \\
\end{align*}
\]

(8)

Considering \(B\) as known, the previous equation could be easily solved and the parameters \(E_0, A\) and \(K\) can be obtained.

Finally the time constant \(T_f\) is not given by the manufacturer but experimental tests have shown that it can be approximated to 30s (Tremblay and Dessaint 2009).

This completes the 7 parameter estimation method for the TDM.

When relaxing the constraint of equality between the \(K\) parameters, the system of equations became under-determined. This is easily solved by adding a new equation through selecting another point. By choosing a point from a “Typical discharge characteristic” at a higher current (e.g. \(1C\)) and at a low \(soc\) the influence of non-linear resistance is augmented. The point \((Q_{nomSC}, V_{nomSC})\) is selected resulting in Eq. (9),

\[
\begin{align*}
V_{full} = E_0 + A \\
V_{exp} = E_0 + A\alpha_{(soc_{exp})} - K\beta_{(soc_{exp})} - R_i i_{dch} - K_2 Y_{(soc_{exp}i_{dch})} \\
V_{nom} = E_0 + A\alpha_{(soc_{nom})} - K\beta_{(soc_{nom})} - R_i i_{dch} - K_2 Y_{(soc_{nom}i_{dch})} \\
V_{nomSC} = E_0 + A\alpha_{(soc_{nomSC})} - K\beta_{(soc_{nomSC})} - R_i i_{dchSC} - K_2 Y_{(soc_{nomSC}i_{dchSC})}
\end{align*}
\]

This completes the 8 parameter estimation method for the EBM extended from the TDM estimation method.

3.2. Method of Over-determined Equation System (M2)

The previous method is strongly dependent of the selected discharge characteristic curve and the selected points, especially the nominal point. Moreover, only 3 points (4 in the EBM) are used from several complete discharge characteristic curves.

This second estimation method proposes to use the complete discharge characteristic curve to create an over-determined equation system while estimating the parameters \(B, Q\) and \(T_f\) as for the previous method.

The system of equations for the TDM is

\[
\begin{bmatrix}
V_{3} \\
V_{4} \\
V_{5}
\end{bmatrix} =
\begin{bmatrix}
a_{(soc_{3})} & -\beta_{(soc_{3})} & -Y_{(soc_{3}i_{3})} & -l_{i_{3}} & E_0 \\
1 & 1 & 1 & 1 & A \\
1 & 1 & 1 & 1 & K \\
\end{bmatrix}
\]

(10)

The system of equations for the EBM is

\[
\begin{bmatrix}
V_{3} \\
V_{4} \\
V_{5}
\end{bmatrix} =
\begin{bmatrix}
a_{(soc_{3})} & -\beta_{(soc_{3})} & -Y_{(soc_{3}i_{3})} & -l_{i_{3}} & E_0 \\
1 & 1 & 1 & 1 & A \\
1 & 1 & 1 & 1 & K \\
\end{bmatrix}
\]

(11)

For both equation systems, the method of ordinary least squares was used to find an approximate solution. This method states that for a system \(Ax = b\) the least squares formula for solving:

\[
\min_x \|Ax - b\|
\]

is

\[
x = (A^TA)^{-1}A^Tb
\]

3.3. Optimization Method for Estimation of the Complete Parameter Set (M3)

Both previous methods are easily processed from experiments or from data sheets but several assumptions were made in order to simplify the estimation problem. If more precise models were needed and if the necessary tests could be performed, the following method could be used for parametric estimation.
Given voltage measures from data testing, the problem can be formulated as:

$$\min_x \| V_n - V_{\text{batt}(SoC_n,i_n)} \|^2_2$$  \hspace{1cm} (12)$$

where $x$ is the parameters vector ($E_0$, $K$, $A$, $B$, $R$, $Q$, $T_f$ for the TDM, and replace $K$ for $K_1$ and $K_2$ for EBM), $V_n$ is the measured voltage vector and $i_n$ the measured current vector. $SoC_n$ is calculated according to Equation 4, $i_n^*$ according to Equation 3 and $V_{\text{batt}}$ according to Equation 5 or Equation 6 for TDM and EBM respectively.

In order to solve the previous minimization problem, a Trust Region Reflective Method (Coleman and Li 1996) is used. This method needs a suitable starting point to converge. One solution of the previous estimation method is used to initialize the process.

The HPPC test provides a full scope in terms of $SoC$ span and input current.

Based on the constant current discharge characteristics (see Figure 1) for methods M1 and M2 and on the HPPC test (see Figure 2) for the third (M3) method, all parameter results are gathered in Table 1:

Table 1: Battery parameters for the Tremblay Dessaint Model (TDM) and the Extended Battery Model (EBM) using the three estimations methods (M1,M2,M3)

<table>
<thead>
<tr>
<th></th>
<th>TDM</th>
<th>EBM</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>M1</td>
<td>M2</td>
</tr>
<tr>
<td>$E_0$ [V]</td>
<td>3.311</td>
<td>3.297</td>
</tr>
<tr>
<td>$K$ [mVAh$^{-1}$]</td>
<td>0.470</td>
<td>0.370</td>
</tr>
<tr>
<td>$A$ [V]</td>
<td>0.034</td>
<td>0.100</td>
</tr>
<tr>
<td>$B$ [Ah$^{-1}$]</td>
<td>6.010</td>
<td>6.010</td>
</tr>
<tr>
<td>$R$ [mΩ]</td>
<td>5.000</td>
<td>6.511</td>
</tr>
<tr>
<td>$K_2$ [mΩ]</td>
<td>0.470</td>
<td>0.370</td>
</tr>
<tr>
<td>$Q$ [Ah]</td>
<td>36</td>
<td>36</td>
</tr>
<tr>
<td>$T_f$ [s]</td>
<td>30</td>
<td>30</td>
</tr>
</tbody>
</table>

4. BATTERY TESTS

The battery test bench consists in a DC Power Supply and an Electronic DC Load functioning alternately in order to allow two quadrants operation. Data acquisition of voltage, current and temperature was done with a sampling time of 10ms.

A Lithium Iron Phosphate battery was tested (LiFePo4 3.2V-36Ah).

4.1. Typical discharge characteristic

Usually provided by the manufacturer, this test was reproduced in order to achieve parametric estimation of the two first estimation methods.

The “typical discharge characteristic” consists in discharging a fully charged battery at a constant current until minimal voltage is reached. Three different current intensities were selected (0.2C, 0.5C and 1C).

Figure 1: Typical discharge characteristic (1C, 0.5C and 0.2C)

In figure 1 the “Typical discharge characteristic” for the three different discharge currents is shown allowing extraction of the four points used in the first estimation method.

4.2. Hybrid Pulse Power Characterization (HPPC)

The HPPC profile was designed in order to measure the dynamic power capability over a device’s usable charge and voltage range (Hunt 2001, Shim and Striebel 2003). It consists in a series of discharge and charge pulses of constant current at different $SoC$. Pulse duration and intensity depends on test objectives.

In this particular case, because non-linear resistance identification is intended and because battery time constant is approximately 30s, a series of consecutive pulses increasing in intensity were considered with a duration of 45s followed by 45s pause intervals. The phase of pulses is followed by a constant current discharge to change the battery $SoC$ of 10%.

![Figure 1: Typical discharge characteristic (1C, 0.5C and 0.2C) and HPPC profile](image-url)
maximal discharge current while remaining almost the same for the two other discharge currents.

Comparing the model simulations of the second method (M2), see Figure 6, it can be seen that the TDM error is lower than 1% for a SoC > 20% as well as the EBM. Also comparing the NRMSE from Table 2 not a remarkable improvement is seen for this test. As expected, both models improve estimations regarding M1.

For the models estimated with the third method (M3) the errors increase. This was expected because the “typical discharge characteristic” was not used by the third method for estimation.

Table 2: Normalized Root Mean Square of the TDM and EBM models using the 3 parameterization methods.

<table>
<thead>
<tr>
<th>Model</th>
<th>Validation</th>
<th>Estimation</th>
</tr>
</thead>
<tbody>
<tr>
<td>TDM-M1</td>
<td>1,01 0,86 0,40</td>
<td>2,13 1,12 1,33</td>
</tr>
<tr>
<td>EBM-M1</td>
<td>1,97 1,74 1,32</td>
<td>2,13 1,12 1,33</td>
</tr>
<tr>
<td>TDM-M2</td>
<td>0,68 0,79 1,33</td>
<td>1,44 0,97 0,97</td>
</tr>
<tr>
<td>EBM-M2</td>
<td>1,51 1,49 0,97</td>
<td>1,97 1,49 0,97</td>
</tr>
<tr>
<td>TDM-M3</td>
<td>1,03 0,92 0,56</td>
<td>2,37 1,70 1,11</td>
</tr>
<tr>
<td>EBM-M3</td>
<td>2,37 1,70 1,11</td>
<td>1,70 1,47 1,47</td>
</tr>
</tbody>
</table>

5.2. HPPC profile
In Table 2, the reduction of the NRMSE can be seen while using the 2nd estimation method (M2) instead of the M1 while SoC is lower that 20% but, on the contrary, an increment is seen for SoC higher than 20%.

The best simulation result is obtained by the 3rd estimation method (M3) of the EBM. These results are expectable because it was the fitting data for this method.

5.3. Typical Electric Vehicle profile (FTP)
For the M1 estimation method, the maximal error for the TDM is 5% while it is reduced to 4% for the EBM (Figure 11). This improvement is also visible in Table 2 as the reduction of the NRMSE (1.03% to 0.92% for typical SoC and from 2.37% to 1.70% for low SoC).

The performance obtained for the EBM with the method M3 of parameter estimation improves the models estimated using M1 in at least 40% (1.03% to 0.36% for typical SoC and from 2.37% to 1.39% for low SoC).

6. CONCLUSION
In conclusion, this paper shows a battery model improvement from the Tremblay-Dessaint’s Model which does not increase model complexity or parameter estimation difficulty and provides a better prediction of voltage behavior. For SoC higher than 20%, the NRSME indicator is reduced by 10% while for typical SoC is reduced by 28% in the validation (FTP) test.
In addition, two novel estimation methods for both models were developed. One uses the normally given “Typical discharge characteristic”, where no battery tests are needed, and the other uses the “HPPC” profile. Each of these new methods improves the previous one.

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APPENDIX

Figure 4: Battery Test Bench developed in the LAPLACE.

Figure 5: Comparison of typical discharge characteristic of TDM and EM with parametric estimation using method M1 §3.1 and test data.

Figure 6: Comparison of typical discharge characteristic of TDM and EM with parametric estimation using method M2 §3.2 and test data.

Figure 7: Comparison of typical discharge characteristic of TDM and EM with parametric estimation using method M1 §3.3 and test data.

Figure 8: Comparison of HPPC profile of TDM and EM with parametric estimation using method M1 §3.1 and test data.
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