ABSTRACT
This work proposes the use of numerical optimization based on Direct Transcription as a method for the design of the control system for hydroelectric turbines. The control design for this application involves constraints but the usual control techniques do not allow explicitly to incorporate constraints in its formulation or are very sensitive to initial estimates of the optimization problem implying in convergence issues. The Direct Transcription is an alternative optimization-based design where the dynamics are discretized and included as constraints of the optimization problem causing the errors due to the quality of the initial estimate to be diluted over the discretization nodes. The constraints considered are related to the actuator and other operating limits during a change maneuver of operating point.

Keywords: Hydraulic Turbines Governors, Numerical Optimization, Direct Transcription.

1. INTRODUCTION
The majority of existing hydroelectric plants use Francis type turbines, and among these, the most of them use speed governors equipped with electro-hydraulic controllers, usually processing algorithms PID. In recent years, new techniques are under analysis to improve the regulation of the speed and power of the hydroelectric turbines.

Jiang, Ma and Wang (2006) proposed an evolutionary programming method based on a mutating factor for determining the optimized parameters of a PID controller of a speed governor for power hydroelectric turbines. The authors argue that it is possible to optimize the PID parameters efficiently and the system keeps stability characteristics, low variations and quick responses.

Qian, Yi and Liu (2011), using the order reduction feature, applied the Sliding Mode Control (SMC) technique. The authors present a case study emphasizing the robustness of the control problem.

In their paper, Ding and Sinha (2011) combined the SMC and H∞ control techniques. The result are compared with those of traditional PI and LQR controllers, demonstrating that the proposed control technique improves system performance against load disturbances and parametric uncertainties, with significant advantage.

Liu, Li and Huang (2012), in turn, propose a robust nonlinear controller based on a high gain observer. The process of adjusting the controller parameters is simplified and only one control parameter needs to be tuned. The objective function defined reflects the regulatory system characteristics of hydroelectric turbines, i.e., the actuator, the electric power generator and the rotor dynamics. The simulations showed that this method can provide a good and robust dynamic response.

The work of Hamarasheed, Haris and Nopiah (2012) presents an adaptive multiple control technique. The controllers used are the LQG/LTR and the PI, optimized to meet the plant requirements. From the results obtained, the LQG/LTR control showed good performance for smooth changes of disturbance, but exhibiting oscillating response in situations of sudden changes. The PI controller, in turn, showed good performance for both, i.e., for smoothly or sudden disturbances.

The work of Singh, Naresh and Gupta (2013) proposes the use of genetic algorithms to determine the tuning parameters of a controller for compensation of the temporary droop in the regulation of hydroelectric turbines. The authors tested four different performance indices. The goals were to minimize the variation in speed, in face of two different steps sizes. After the tests, the ITAE performance index was chosen as the best option, so its associated parameters were adopted.

Anbo, Xiangang and Hao (2013) proposed a distributed multi-agent genetic algorithm applied in optimizing parameters of a self-adaptive PID, to be used in world’s largest hydroelectric power plant located in China. The results show that the proposed simulation performed better when compared to the conventional genetic optimization algorithm, and furthermore reduced significantly the optimization time.

This work proposes an alternative solution to the control problem found in many installations of hydroelectric plants by means of an optimization strategy called Direct Transcription involving the parameters of a PID controller.
2. METHOD

The parametric optimal control problem of a hydraulic turbine was formulated as a nonlinear optimization problem and solved with the technique of direct transcription.

2.1. Dynamical Model

The hydroelectric plant facilities may have different settings related to the characteristics of each situation in which they are designed. In general, hydroelectric plants are composed of the items shown in Figure 1.

For this work it was considered a plant without intake tunnel and surge tank. Based on the diagram of Jaeger et al. (1994), the block diagram of Figure 2 shows the typical control system for this kind of hydroelectric plant.

The actuator is driven by a servo system which acts on the wicket gates ring of the turbine and its position determines the flow of water through the turbine. Based on the work of Qian, Yi and Liu (2011), the actuator can be expressed as a first order transfer function (1), disregarding the effects of the pilot valve and the dead-zone.

\[ \frac{g}{u} = \frac{1}{1 + sT_G} \]  

The block diagram in Figure 3 represent the hydraulic actuator transfer function of Equation (1). From that diagram, it is immediate to write Equation (7) to represent it in state space.

![Figure 3: Block diagram of the hydraulic actuator.](image)

For a Francis turbine connected to the reservoir via a single penstock without intake tunnel, the model suggested by Kundur (1993) and Machowski, Bialek and Bumby (2008) is given by

\[ \frac{P_m}{g} = \frac{1 - sT_W}{1 + sT_W/2} \]  

that is, a first order transfer function with a non-minimum-phase zero. The zero here models the inverse response, typical on maneuvers with hydraulic turbines.

The block diagram in Figure 4 also represent the turbine transfer function of Equation (2). The auxiliary variable \( x \) was chosen at output of the integrator so that the turbine can be fully represented in state space by means of Equations (9) and (12).

![Figure 4: Detailed block diagram of the turbine.](image)

As described by Kundur (1993), the generator receives the mechanical torque from the turbine by the rotor shaft and produces an electromagnetic torque. In a simplified manner, the dynamics of the generator is of first order

\[ \frac{\omega}{(M_m - M_e)} = \frac{1}{K_D + sT_M} \]  

where \( M_m \) is the mechanical torque and \( M_e \) is the equivalent torque related to the electrical load. For studies of frequency and load, it is preferred to express the equation (3) in terms of mechanical and electrical power, rather than torque. Based on the development Kundur (1993) and Machowski, Bialek and Bumby (2008), the torque deviation \( (M_m - M_e) \) is equivalent to the power deviation \( (P_m - P_e) \). Thus, the model of the generator can be represented by the block diagram in Figure 5 so that its model in state space results in Equation (10).
Figure 5: Detailed block diagram of the generator.

In real applications, the controller is typically a PID compensator whose transfer function is

\[ u / (\omega_f - \omega) = K_p + K_i / s + K_d s / (T_d s + 1), \] (4)

The block diagram in Figure 6 also represents the PID transfer function of Equation (4) in a detailed way. The auxiliary variables \( z \) and \( v \) were chosen at outputs of the integrators so that the control law can be fully represented in state space by means of Equations (5), (6), and (11).

Figure 6: Detailed block diagram of the PID controller.

The model of the closed loop control system of the Figure 2 has five states when the controller is a PID with filtering of the derivative. Equations (5) to (12) summarize the system model.

\[ \ddot{z} = K_i (\omega_f - \omega) \] (5)
\[ \dot{v} = 1 / T_d (\omega_f - \omega) - v \] (6)
\[ \dot{g} = 1 / T_o (u - g) \] (7)
\[ 2P_m + T_d \dot{P_m} = 2 (g - T_e \dot{g}) \] (8)
\[ \dot{x} = 1 / T_e (g - P_m) \] (9)
\[ \dot{\omega} = 1 / T_{\omega} (P_m - P_f) - K_\omega \omega \] (10)
\[ u = K_p (\omega_f - \omega) + (z) + K_d (\dot{v}) \] (11)
\[ P_m = 2 (x - g) \] (12)

2.2. Optimal Control

The numerical solution of an optimal control problem can be classified into two methods, the indirect and the direct one. The indirect method, in practice, is often inefficient, as in the case described by Murty (2008) where an application of the optimal control was designed to control the frequency of a hydraulic turbine constituted by one generator unit. The system was represented in the state space and it was solved a linear regulator problem by means of the algebraic matrix Riccati. However, the author revels few concerns about the application effectiveness.

In the direct method it is necessary to transform the original optimal control problem into a nonlinear programming problem by means of the discretization of the state and control, and then solving the resulting nonlinear programming problem. Based on the discretization of the state and control, direct methods can be classified into two different types:

- Discretization of both the state variables and control and then the resulting model is solved by an algorithm for nonlinear programming problems. A variety of these direct methods was developed and applied, including the method of direct transcription. We highlight the work of Subchan and Zbikowsky (2009) and Betts (2010). It can be pointed out that this method has a large domain of convergence (Stryk and Schlemmer, 1994).

- Discretization of control so that the states and the performance index can be obtained by numerical integration of the state equations. This approach is known as Shooting technique. Although this method may suffer from convergence issues it provides normally a highly accurate solution (Stryk and Schlemmer, 1994).

The controller design can therefore be done by means of a nonlinear optimization problem in which the system dynamic appear as constraint. We argue that the use of direct transcription technique is most suitable for this application.

In this work it was used PROPT (Rutquist and Edval, 2010) which automates the conversion of an optimal control problem into an equivalent problem of parametric optimization to be solved by Sparse Nonlinear Optimization (SNOPT) that is an appropriate solver for nonlinear systems with sparse matrices. It is worth mentioning that sparsity is an intrinsic characteristic in Direct Transcription problem caused by model discretization in time.

Considering a quadratic objective function, the optimal control problem we are interested in solving, can be written as follows:
\[
\min J = \int_{0}^{\tau} \left( (\omega - \omega_0)^2 + \lambda (g)^2 \right) dt
\]

This objective function is used to try to make the rotor speed \(\omega\) follow the reference \(\omega_0\) where \(\lambda\) represents the balance between control and speed error.

The system is subject to the dynamic constraints of equations (5) to (10), the algebraic constraints of equations (11) and (12), and also subject to the operational constraints (14) and (15). Note that equations (5) to (12) are inequalities constraints whereas (14) and (15) are inequalities ones of the nonlinear optimization problem. Since the objective function is quadratic and all constraints are linear, this problem is classified as a quadratic optimization problem. However, even being a convex problem, the success finding the global minimum depends mainly on the features of the solver.

\[ g_{\text{min}} \leq (g) \leq g_{\text{max}} \quad (14) \]

\[ \dot{g}_{\text{min}} \leq (\dot{g}) \leq \dot{g}_{\text{max}} \quad (15) \]

The operational constraints (14) and (15) refer primarily to the actuator. The actuator is driven by a servo system which acts on the wicket gates ring of the turbine and its position determines the flow of water through the turbine. Based on the work of Qian, Yi and Liu (2011), the actuator can be expressed as a first order transfer function (1), disregarding the effects of the pilot valve and the dead-zone.

The minimum and maximum aperture constraints are physical limits. The speed constraint is the also physical limitation that the actuator can impose during the transient response, but can also represent a resource one may use to mitigate the effect of inverse response of the turbine. The inverse response of the turbine appears by the presence of non-minimum-phase zero in its transfer function. The consequence of this is that the rapidly changing the position of the wicket gates, the flow does not change immediately because of the inertia of the moving fluid, but the pressure changes rapidly. After that the forces are balanced and the pressure goes back to the steady state value and the change in the flow promotes the change of mechanical power and, as a result, changing the rotation of the generator. Therefore, making a slow variation of the wicket gates can help reduce the effect of inverse response.

Note that other operating constraints can be easily incorporated into the optimization problem. Some of them that are relevant to the problem are related to the time domain performance during a maneuver of changing the operating condition such as maximum overshoot, settling time, maximum peak of the inverse response, etc.

Notice that the constraints (5) and (11) of the optimization problem are nonlinear because in them the controller gains appear multiplying the state \(\omega\) or the state \(v\), that are also parameters of the problem. However, although there are nonlinear constraints, the partial derivatives of these constraints on optimization parameters (Jacobian) are linear. This is because the nonlinearity is due to the product of optimization parameters. As a result, the optimization problem lies in the quadratic programming class that is easier to solve than other problems with more stringent nonlinearity. However, even quadratic programming problems can suffer numerical problems as is often happens with integration method of the dynamic equations (Shooting). In the Shooting method the nonlinearity of the dynamic model does not appear in constraints as in the Direct Transcription but affect the integration of the model that will then allow to calculate the objective function and also its gradient and constraints. That is, the objective function and constraints are a nonlinear function of the controller parameters. Note that a small change in the initial value of a parameter may involve a high cumulative error at the end of integration, which can compromise the convergence of optimization.

3. RESULTS

The objective in this work is to evaluate the suitability of direct transcription method as an alternative to the design of a hydraulic turbine control system for power generation since usual shooting method is often very troublesome regarding convergence. To test the ability of the method, typical maneuvers with power generation systems are considered. They refer to the change of the angular rotor speed and load disturbance rejection caused by the variation in power consumption.

Based on the work of Kundur (1993), normally the typical values of \(T_W\) are within the range of 0.5 to 4.0 s, and values \(T_D\) are close to 0.5 s. The \(T_M\) values are within in the range of 5.0 to 10.0 s as can be seen in Qian, Yi and Liu (2011), Fang and Shen (2005). The nominal values of the plant parameters used in this study were selected based on the work of Kundur (1993) and are presented in the Table 1. The constant \(T_D = 0.1\) s was chosen as suggested by Fang and Shen (2005).

<table>
<thead>
<tr>
<th>(T_G)</th>
<th>(T_W)</th>
<th>(T_M)</th>
<th>(K_D)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5 s</td>
<td>2.0 s</td>
<td>10 s</td>
<td>1</td>
</tr>
</tbody>
</table>

Although the design method proposed in this work is not similar to the tuning method proposed by Hagihara et al. (1979), the initial estimates for the controller gains are based on this method to avoid a trivial estimate. The suggested values are:

\[ K_p = \frac{0.8T_M}{T_W} \quad (16) \]

\[ K_i = \frac{0.24T_M}{T_D} \quad (17) \]

\[ K_d = 0.27T_M \quad (18) \]
Since the control effort is explicitly being considered by operational constraints, it was adopted $\lambda = 0$ in all the situations discussed in this paper.

3.1. Nominal Constraints

In the first simulation, the values considered arise from operational restrictions and are similar to those used by Sansevero (2006):

$$-0.5 \leq g \leq 0.5 \text{ pu}$$  \hspace{1cm} (19)

$$-0.2 \leq \dot{g} \leq 0.2 \text{ pu/s}$$  \hspace{1cm} (20)

The constraint (19) associated with the opening of the wicket gates reflects the assumption that the linear model for deviations was obtained for about 50% of the opening of the wicket gates, so that it can be changed by +/- 50%. The constraint (20) implies that the opening speed of the wicket gates can be changed up to 20% from nominal opening.

The optimization problem was solved by the direct transcription method using 200 nodes (discretization). The model has five states and without an explicit input variable, resulting 5 parameters per node, that is, 1000 parameters to be optimized, in addition to the three gains of the PID controller. The Jacobian results in a sparse Jacobian and the used solver (SNOPT) disregard the large number of null elements. It would probably be impractical to solve this problem of direct transcription with a dense matrix solver.

Solving the optimization problem, the gains for the PID controller are $K_p = 1.444$, $K_i = 0.226$, $K_d = -0.109$ and the cost function resulted in $J = 0.124$.

The Figure 7 illustrates the performance obtained with the system during two maneuvers: the first maneuver consists in the reduction of rotation $\omega$ of 0.1 pu at $t = 0$ s and, the second maneuver consists of a disturbance rejection to a load power variation $P_e$ of 0.2 pu at $t = 50$ s.

The graph (a) of the Figure 7 shows the performance of the maneuver. The steady value of the rotation speed was reached after about 30 seconds and with an overshoot of approximately 25% with an inverse response of 0.013 pu. In the graph (b), during the first maneuver, it can be observed that the mechanical power exhibited a maximum variation of roughly the same percentage value of the corresponding opening of the wicket gates, but exhibited an inverse response during about 2 s reaching 0.125 pu. This percentage value is much larger than the inverse response of power and, moreover, is significant, since the change in the power was only 0.1 pu. In the maneuver of disturbance rejection, the inverse response of the power exhibited a maximum deviation of 0.026 pu for a variation of 0.2 pu, that is, with a very low inverse response. The graph (c) shows that the opening of the wicket gates had a maximum variation of about 0.23 pu considering both maneuvers, i.e., far below of the operating limit.

![Figure 7: Time Response with Optimal PID.](image)

Finally, the graph (d) shows that the wicket gates speed variation had operational constraint active only at the beginning of the maneuver because of the abrupt change in the wicket gates caused by the change with the reference step. In summary, the control system design via optimization produced responses satisfying all performance specifications.

3.2. Milder Constraints

Assuming now that the hydraulic actuator is such that the rate of change of position of the wicket gates can be increased in twice, that is

$$-0.4 \leq \dot{g} \leq 0.4 \text{ pu/s}$$  \hspace{1cm} (21)

Solving the original optimization problem with this less restrictive bond, the optimal gains are $K_p = 2.54$, $K_i = 0.24$, $K_d = 0.06$ and the cost function resulted $J = 0.10$.

The new controller represents a more aggressive system with values of gain higher than original values. The graphs of the Figure 8 confirm what was expected, that is, the system responds more quickly with less settling time compared to the original system, but with a larger control effort and higher overshoot in the response. Similarly, to the original case, the actuator speed limit was only active during the beginning of the first maneuver with the application of the reference step.
3.4. Design via Shooting

Here it was utilized the optimization method of integration of differential equations (Shooting) for comparison purpose. The only change was the inclusion of a state variable $\alpha$ in the dynamic model, so, the cost function can be rewritten as

$$\min_{\rho} J = \alpha,$$

such that

$$\alpha = (\omega_r - \omega)^2.$$

Compared with the direct transcription, which was implemented with 200 nodes and about 1000 parameters to be optimized, in the Shooting there are only 3 parameters, that is, only the PID controller gains. However, in opposition to this apparent advantage, this method presents a high sensitivity, because a small change in the parameters at the beginning of the solution may result in large differences at the end of integration. This feature can generate difficulties of convergence as seen in different scenarios of this work.

As expected, the Shooting method was quite sensitive to the initial estimate and, only for values close of the optimal values, there was convergence in the response. Table 2 illustrates some of these cases, and in all cases that converged, the value of the cost function resulted $J = 0.1247$.

Unlike Shooting method, the direct transcription rarely presented convergence problems according to initial estimates, always converging to the global minimum whatever is the initial guess. It’s emphasized that the direct transcription had always the same good performance despite of its 1008 parameters in contrast to the 3 parameters of Shooting method.

<table>
<thead>
<tr>
<th>Initial Estimate</th>
<th>Convergency</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K_p$</td>
<td>$K_i$</td>
</tr>
<tr>
<td>1.44</td>
<td>0.23</td>
</tr>
<tr>
<td>1.83</td>
<td>-0.06</td>
</tr>
<tr>
<td>1.40</td>
<td>0.20</td>
</tr>
<tr>
<td>-0.43</td>
<td>0.62</td>
</tr>
<tr>
<td>10</td>
<td>10</td>
</tr>
<tr>
<td>-10</td>
<td>-10</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>1.86</td>
<td>-1.37</td>
</tr>
</tbody>
</table>

In the work presented by Stryk and Schlemmer, (1994), the authors combine the two methods, Direct Transcription and Shooting, joining the advantages of each method: large domain of convergence and highly accurate solutions. Might be that this hybrid approach could be a good choice for this application to be investigated next.
4. CONCLUSIONS
This work analyzed the optimal control technique Direct Transcription in order to optimize the tuning parameters of a controller used in the regulation of speed and power of hydroelectric turbines. It was elect a PID controller because of its large acceptance in industrial application although the method can be adapted to other types of controllers.

Some numerical examples were considered by changing the operational constraints related to the opening bound of the wicket gates and also to the maximum opening speed. The proposed technique was compared with the usual technique, the Shooting. The findings were compatible, however, the Shooting technique proved to be very sensitive to initial estimates, presenting convergence problems, even processing a much smaller number of parameters than those processed by direct transcription. The numerical results suggest that direct transcription technique is well suited to the control system design for hydroelectric turbines.

As a proposal for further study an alternative is to investigate the performance of the control system considering improvements in the plant model, such as nonlinearities; considering water compressibility phenomena; including tunnel and expanding tanks. Also multiple generating units in parallel can be considered.

It is proposed to investigate the extent of the project to investigate the problem of modeling errors, producing a robust controller.

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