FAULT ACCOMMODATION BY INVERSE SIMULATION THROUGH SOLVING A DIFFERENTIAL ALGEBRAIC SYSTEM OBTAINED FROM A BOND GRAPH

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ABSTRACT

When an abrupt parametric fault occurs in a system, active fault tolerant control (FTC) aims at reconstructing the system input after the fault has been isolated and estimated so that the fault is compensated and the system output follows a desired trajectory despite the fault.

Input reconstruction by *inverse simulation* is a quite general approach to active FTC which can be supported by existing sophisticated simulation software for the solution of DAE systems. Desired outputs are obtained by computing a forward model of the healthy system. Values of the reconstructed input into the faulty systems needed at the next time instant are obtained by computing the DAE system of the inverse model *simultaneously*.

The proposed approach assumes a single fault hypothesis and is illustrated by application to averaged models of simple examples of power electronic systems. Differentiation of the inverse model equations with respect to time results in a semi-explicit ordinary differential equation (ODE) system.

Keywords: Active fault tolerant control, fault isolation and estimation, fault accommodation, input reconstruction, inverse simulation, power electronic systems, averaged bond graph models.

1. INTRODUCTION

The increasing equipment of mechatronic systems with communicating smart sensors, actuators, embedded digital circuitry and software renders them into what is sometimes called intelligent systems and enables them to operate autonomously to some extent. This includes to detect and isolate faults and to recover from faults by accommodating them autonomously. Approaches to fault detection and isolation (FDI) can be roughly categorised into data-driven and model-based methods Borutzky (2015). Among the latter ones are techniques based on bond graphs that derive analytical redundancy relations (ARRs) from a bond graph (BG) and check their residuals against thresholds Djeziri et al. (2007); Samantaray and Ould Bouamama (2008); Y. Touati et al. (2012); Wang et al. (2013); Borutzky (2015). As to fault tolerant control (FTC), passive and active techniques are known. Passive approaches use a controller with a fixed control law that ensures stability and the control objectives in the presence of faults of a certain class of anticipated faults. In an active approach, the control law is changed without changing the controlled system after a fault has been detected and isolated so that an acceptable system performance in the presence of a persisting fault can be maintained. Changing the control law and the system is often termed reconfiguration. An elaborated presentation of fault tolerant control may be found in Blanke et al. (2006). A bibliographical review on reconfigurable fault-tolerant control systems is given in Y. Zhang and J. Jiang (2008). Bond graph model-based approaches to passive as well as to active FTC have been presented in Nacusse, M. and Junco, S.J. (2011); Samantaray and Ould Bouamama (2008); Loureiro, R. (2012); Allous, M. and Zanzouri, N. (2014). In active FTC, changing the controller law after a fault has occurred requires system inversion, i.e. to find an input so that the faulty system produces a desired output. One way to decide whether a model is invertible and to determine an input required to produce a desired output is to assign bicausalities to a BG Gawthrop (1995); Ngwompo et al. (2001a,b); Loureiro, R. (2012).

In this paper, input reconstruction is based on the solution of the DAE system of a forward model of the healthy system deduced from a BG and considered as a system for the required input into the faulty system. This way, existing sophisticated software for the solution of DAEs such as OpenModelica OpenModelica Consortium (nd) can be used for nonlinear inverse simulation, though, in general, the solution of a DAE system is not guaranteed and stability issues may arise.

The paper is organised in the following manner. The next section briefly addresses the problem of isolating and estimating a fault as this step is a prerequisite for FTC. In Section the construction of an input that compensates for a fault is confined to linear time-invariant multiple input multiple output (MIMO) systems as forward models and assumes a single fault hypothesis. The output of the healthy system is considered the desired output for which an input must reconstructed after a fault has happened. Differentiation of the inverse model equations with respect to time results in a semi-explicit ODE system for the required input. An example demonstrates that the matrix pre-multiplying the state vector of this semi-explicit ODE system is not necessarily a matrix with constants coefficients in contrast to the forward model.

The procedure is applied to a boost converter and to a buck converter driven DC motor as two simple examples of power electronic systems. The dynamic behaviour of the healthy systems is represented by a BG model with variables averaged over the switching cycle time. In Section , a case study considers as a fault scenario the leakage of the capacitor in the buck converter driven DC motor sytem and the recovery from this fault. Simulation runs have been performed by means of the open source software Scilab Scilab Enterprises (nd).

2. PARAMETRIC FAULT ISOLATION AND ESTIMATION

In bond graph model-based fault detection and isolation (FDI), analytical redundancy relations (ARRs) are deduced from a diagnostic bond graph (DBG). Their evaluation gives so-called ARR residuals that are close to zero in the case of a healthy system and exceed given thresholds in the case a fault has happened. If nonlinear constitutive component equations permit to eliminate unknown variables so that ARRs can be obtained in closed form, their structure is usually captured by a structural fault signature matrix (FSM). If a single fault hypothesis can be adopted, and if the fault can be mapped onto a faulty parameter, and if the parameter has a unique component fault signature, then the fault can be isolated by just inspecting the FSM.

In case two components with parameters Θ_1 and θ_2 respectively have got the same structural component fault signature in the FSM, then, given the single fault hypothesis still holds, parameter sensitivities of ARRs may be used to isolate the fault Y. Touati et al. (2012).

Furthermore, multiple faults may happen simultaneously but the ARR residuals sensitive to them may not be structured, i.e. the component fault signatures are not linear independent. As a result, faults cannot be isolated by inspection of the FSM. In this case, parametric faults can be identified by comparing actual parameters obtained by parameter estimation with their nominal values (Samantaray and Ould Bouamama, 2008; Wang et al., 2013; Borutzky, 2015). However, iterative parameter estimation takes time which may be an issue in online fault diagnosis. Solving a nonlinear least squares problem by means of a gradient based algorithm may be faster than an algorithm that uses function evaluations only but convergence is not guaranteed.

Finally, even if the system itself stays healthy, sensor and actuator faults may degrade its control and thus the performance of the closed loop system. Given a single fault hypothesis, sensor and actuator faults assumed to be additive can be easily isolated and estimated. ARR residuals can be used to determine their size. In (Y. Touati et al., 2012), bicausality assigned to a DBG is used for the estimation of isolatable faults. Alternatively, the magnitude of a, say sensor fault, can be obtained directly form the ARR established for the sensor junction.

3. INPUT RECONTRUCTION

Assume that an abnormality in the dynamic behaviour of a multiple input, multiple output (MIMO) system is observed at time instant t_1 and that it is due to a single parametric fault. Once the fault has been isolated and its size has been estimated, it can be accommodated by designing a new input into the faulty system so that the fault is compensated and the system produces a desired output behaviour despite the fault. The required input can be determined by constructing an inverse model. There are various approaches to system inversion. The required input may be obtained by

- designing a feedback system,
- numerical solution of a DAE system,
- application of bicausality to a bond graph Gawthrop (1995); Ngwompo et al. (1996)

A review of inverse simulation methods may be found in (Murray-Smith, 2012, Chapter 4). All of them have their pros and cons.

3.1 Model inversion by numerical solution of a DAE system

In the following, an approach based on the solution of a DAE is considered. Assume that the dynamic behaviour of a healthy system may be captured by a linear time-invariant multiple input, multiple output (MIMO) forward model deduced from a BG.

$$\dot{\boldsymbol{x}}(t) = \mathbf{A}\boldsymbol{x}(t) + \mathbf{B}\boldsymbol{u}(t) \tag{1a}$$
$$\boldsymbol{y}(t) = \mathbf{C}\boldsymbol{x}(t) \tag{1b}$$

where $\boldsymbol{x}(t)$ denotes the vector of state variables, $\boldsymbol{u}(t)$ the input vector, and $\boldsymbol{y}(t)$ the output vector. The constant coefficient matrices $\mathbf{A}, \mathbf{B}, \mathbf{C}$ are of appropriate dimensions. Suppose that the single parametric fault that has occurred at t_1 is isolated and identified as of $t_2 > t_1$. Then for $t > t_2$ an input $\boldsymbol{u}_{\mathrm{req}}(t)$ is required that forces the faulty system to produce the output of the healthy system as the desired output in the presence of the fault, i.e. $\boldsymbol{y}_{\mathrm{des}}(t) = \boldsymbol{y}(t)$. The equations of the faulty system then read

$$
\begin{aligned}
\dot{\tilde{\boldsymbol{x}}}(t) &= \tilde{\mathbf{A}}\tilde{\boldsymbol{x}}(t) + \tilde{\mathbf{B}}\boldsymbol{u}_{\mathrm{req}}(t) && \text{(2a)}\\
\boldsymbol{y}_{\mathrm{des}}(t) &= \tilde{\mathbf{C}}\tilde{\boldsymbol{x}}(t) && \text{(2b)}
\end{aligned}
$$

where $\tilde{\boldsymbol{x}}(t)$ denotes the unknown state of the faulty system and $\boldsymbol{u}_{\mathrm{req}}(t)$ the input to be determined. As the faulty parameter may affect the coefficients of all matrices, the latter ones are distinguished from the ones of the healthy system by a tilde.

Differentiation of the algebraic constraint (2b) with respect to time and substitution of (2a) into the result gives the DAE system

$$
\begin{bmatrix} \boldsymbol{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}
\begin{bmatrix} \dot{\tilde{\boldsymbol{x}}} \\ \dot{\boldsymbol{u}}_{\mathrm{req}} \end{bmatrix}
=
\begin{bmatrix} \tilde{\mathbf{A}} & \tilde{\mathbf{B}} \\ \tilde{\mathbf{C}}\tilde{\mathbf{A}} & \tilde{\mathbf{C}}\tilde{\mathbf{B}} \end{bmatrix}
\begin{bmatrix} \tilde{\boldsymbol{x}} \\ \boldsymbol{u}_{\mathrm{req}} \end{bmatrix}
+
\begin{bmatrix} \mathbf{0} \\ -\dot{\boldsymbol{y}} \end{bmatrix}
\qquad (3)
$$

where $\mathbf{I}$ denotes the $n \times n$ identity matrix. DAE (3) is of index 1 if $(\tilde{\mathbf{C}}\tilde{\mathbf{B}})^{-1}$ exists and can be numerically computed by a BDF-based solver. The algebraic constraint in (3) can be solved for $\boldsymbol{u}_{\mathrm{req}}$ and substituted into the ODE for $\tilde{\boldsymbol{x}}$.

$$
\dot{\tilde{\boldsymbol{x}}}(t) = (\tilde{\mathbf{A}} - \tilde{\mathbf{B}}(\tilde{\mathbf{C}}\tilde{\mathbf{B}})^{-1}\tilde{\mathbf{C}}\tilde{\mathbf{A}})\tilde{\boldsymbol{x}}(t) + \tilde{\mathbf{B}}(\tilde{\mathbf{C}}\tilde{\mathbf{B}})^{-1}\dot{\boldsymbol{y}}(t) \qquad (4)
$$

Differentiation of the algebraic constraint in (3) with respect to time results in another DAE system.

$$
\begin{bmatrix} \boldsymbol{I} & \mathbf{0} \\ \tilde{\mathbf{C}}\tilde{\mathbf{A}} & \tilde{\mathbf{C}}\tilde{\mathbf{B}} \end{bmatrix}
\begin{bmatrix} \dot{\tilde{\boldsymbol{x}}} \\ \dot{\boldsymbol{u}}_{\mathrm{req}} \end{bmatrix}
=
\begin{bmatrix} \tilde{\mathbf{A}} & \tilde{\mathbf{B}} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}
\begin{bmatrix} \tilde{\boldsymbol{x}} \\ u_{\mathrm{req}} \end{bmatrix}
+
\begin{bmatrix} \mathbf{0} \\ \ddot{\mathbf{y}} \end{bmatrix}
\qquad (5)
$$

DAE (5) can be turned into an explicit ODE system if $(\tilde{\mathbf{C}}\tilde{\mathbf{B}})^{-1}$ exists. The input required to accommodate the fault reads

$$
\boldsymbol{u}_{\mathrm{req}}(t) = (\tilde{\mathbf{C}}\tilde{\mathbf{B}})^{-1}[-\tilde{\mathbf{C}}\tilde{\mathbf{A}}\tilde{\boldsymbol{x}}(t) + \dot{\boldsymbol{y}}(t)] \qquad (6)
$$

It may happen that some states of the faulty system can be expressed as a function of the remaining states, the input $\boldsymbol{u}_{\mathrm{req}}$ to be determined, the desired output $\boldsymbol{y}_{\mathrm{des}}$ and its time derivatives. The result is an inverse model of reduced order. This is illustrated by means of the example in the next section. See also the appendix.

3.2 Example: Buck-converter driven DC motor

Fig. 1 displays a circuit schematic of a buck-converter-driven DC motor and Fig. 2 shows a corresponding averaged forward BG model, where R_L denotes the resistance of the inductor, R_a and L_a the resistance and the inductance of the motor's armature winding, b a friction parameter and τ_{load} an external load torque. The transistor Q and the diode have been modelled as a non-ideal switch with an ON-resistance R_{on}, $m := 1 - d$, where d denotes the duty cycle of the signal $u(t)$

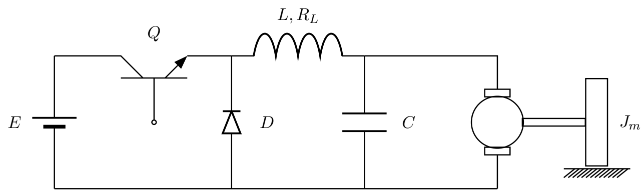

Figure 1: Schematic of a buck-converter-driven DC motor

controlling the transistor Q. The MTF and the Se source on the left side may be combined into a modulated source MSe.

From the BG in Fig. 2, the following equations of the healthy system are obtained.

$$
\begin{aligned}
1_1 : 0 &= mE - (R_{\mathrm{on}} + R_L)i_L - L\frac{di_L}{dt} - u_C && (7)\\
0_1 : 0 &= i_L - C\frac{du_C}{dt} - i_a && (8)\\
1_2 : 0 &= u_C - R_a i_a - L_a\frac{di_a}{dt} - k\,\omega && (9)\\
1_3 : 0 &= k\,i_a - b\,\omega - J_m\frac{d\omega}{dt} - \tau_{\mathrm{load}} && (10)\\
y &= \omega && (11)
\end{aligned}
$$

Let the identified parametric fault be the resistance of the motor inductance, $\tilde{R}_a := R_a + \Delta R_a$, and let $\tau_{\mathrm{load}} = 0$ for simplicity. If the load torque is different from zero it must be measured or estimated. Accordingly, a load torque estimator would be required in a practical implementation. Furthermore, let $y_{\mathrm{des}} = \omega_{\mathrm{des}} = y = \omega$, $u := mE$, $u_{req} = \tilde{m}E = (1 - \tilde{d}(t))E$, $\tilde{x}_1 := \tilde{i}_L$, $\tilde{x}_2 := \tilde{u}_C$, $\tilde{x}_3 := \tilde{i}_a$, $\tilde{x}_4 := \tilde{\omega} = \omega$ and $R := R_{\mathrm{on}} + R_L$. Then the DAE system determining u_{req} reads

$$
\begin{aligned}
0 &= u_{\mathrm{req}} - R\tilde{x}_1 - L\dot{\tilde{x}}_1 - \tilde{x}_2 && (12)\\
0 &= \tilde{x}_1 - C\dot{\tilde{x}}_2 - \tilde{x}_3 && (13)\\
0 &= \tilde{x}_2 - \tilde{R}_a\tilde{x}_3 - L_a\dot{\tilde{x}}_3 - k\,\tilde{x}_4 && (14)\\
0 &= k\,\tilde{x}_3 - b\,\tilde{x}_4 - J_m\dot{\tilde{x}}_4 && (15)\\
y &= \tilde{x}_4 && (16)
\end{aligned}
$$

Substituting (16) into (15) gives

$$
k\tilde{x}_3 = by + J_m\dot{y} \qquad (17)
$$

Comparison with (10) yields $\tilde{x}_3 = x_3$. Accordingly,

$$
\begin{aligned}
\tilde{x}_2 &= \tilde{R}_a x_3 + L_a\dot{x}_3 + ky\\
&= (R_a + \Delta R_a)x_3 + L_a\dot{x}_3 + ky\\
&= R_a x_3 + L_a\dot{x}_3 + ky + (\Delta R_a)x_3\\
&= x_2 + (\Delta R_a)x_3 && (18)
\end{aligned}
$$

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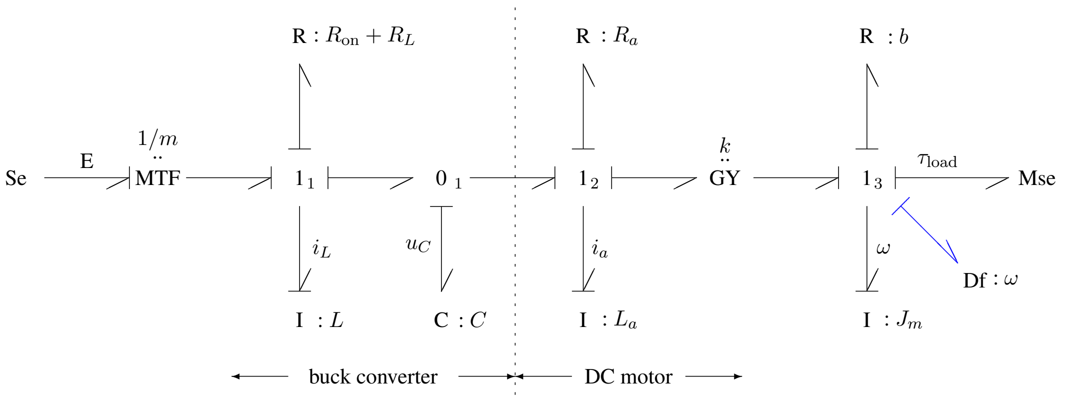

Figure 2: Averaged BG model of the healthy buck-converter-driven DC motor

Furthermore,

$$\begin{aligned} \tilde{x}_1 &= C\dot{\tilde{x}}_2 + \tilde{x}_3 \\ &= C\dot{x}_2 + C(\Delta R_a)\dot{x}_3 + x_3 \\ &= x_1 + C(\Delta R_a)x_3 \end{aligned} \tag{19}$$

Finally,

$$\begin{aligned} u_{\text{req}} &= R(x_1 + C(\Delta R_a)\dot{x}_3) + L(\dot{x}_1 + C(\Delta R_a)\ddot{x}_3 \\ &\quad + x_2 + (\Delta R_a)x_3 \\ &= \underbrace{R\dot{x}_1 + L\dot{x}_1 + x_2}_{u} \\ &\quad + (x_3 + RC\dot{x}_3 + LC\ddot{x}_3)(\Delta R_a) \\ &= u + \frac{1}{k}(by + J_m\dot{y} + RC(\dot{y} + J_m\ddot{y}) \\ &\quad + LC(b\ddot{y} + J_m y^{(3)}))\Delta R_a \end{aligned} \tag{20}$$

As a result, in this example, all unknown states of the faulty system, $\tilde{\boldsymbol{x}}(t)$, can be eliminated. The required input u_{req} is a function of the initial input u, the desired output y and its derivatives. In the case of no fault ΔR_a, the required input equals the initial input.

The result obtained in this example may be confirmed by assigning bicausalities to the BG, by following the propagation of bicausality from the output source-sensor element SS to the input SS element and by deducing equations from the bicausal inverse BG model depicted in Fig. 3. As there is a unique causal path from the output $y_{\text{des}} = \omega$ to the input $u_{\text{req}} = \tilde{m}E$, the forward model is structurally invertible Ngwompo et al. (2001a). Note that bicausalities force all storage elements into derivative causality. Hence, the inverse model will be of order zero.

From the bicausal inverse BG in Fig. 3, the following equations may be deduced and solved for the required input $u_{req} = \tilde{m}E$.

$$1_3: \quad e_2 = b\,\omega_{\text{des}} + J_m\dot{\omega}_{\text{des}} + \tau_{\text{load}} \tag{21}$$

$$GY: \quad e_3 = k\,\omega_{\text{des}} \tag{22}$$

$$\tilde{i}_a = f_3 = \frac{1}{k}e_2 \tag{23}$$

$$1_2: \tilde{u}_C = e_4 = \tilde{R}_a\tilde{i}_a + L_a\frac{d\tilde{i}_a}{dt} + e_3 \tag{24}$$

$$0_1: \tilde{i}_L = f_5 = f_3 + C\dot{e}_4 \tag{25}$$

$$1_1: \quad e_6 = R f_5 + L\frac{df_5}{dt} + e_5 \tag{26}$$

$$MTF: \quad e_6 = \tilde{m}E \tag{27}$$

3.3 Scheme of the active FTC procedure

Fig. 4 displays the scheme of the active FTC procedure based on system inversion by solving a DAE system. The real system may be subject to a fault as of some time instant t_1. Its input $\boldsymbol{u}(t)$ and measured quantities are delivered into a DBG model. It computation generates ARR residuals. On the basis of these ARR residuals a diagnosis module decides whether a fault has happened. If a parametric fault has occurred, it is isolated and quantified. At time $t_2 > t_1$, the faulty parameter $\tilde{p}$ is provided into an input reconstruction module that takes the output $\boldsymbol{y}(t)$ of a forward model of the healthy system as the desired output $\boldsymbol{y}_{\text{des}}(t)$ and determines an input $\tilde{\boldsymbol{u}}(t)$ into the faulty system by solving a DAE system so that the output of the faulty system recovers after the fault and matches the one of the healthy system.

3.4 Nonlinear problems

In the previous section, it was assumed that a power electronic system may be represented by an averaged linear time-invariant model. For model inversion, a semi-linear DAE system with a constant coefficient matrix premultiplying the descriptor vector could be turned into an explicit ODE sys-

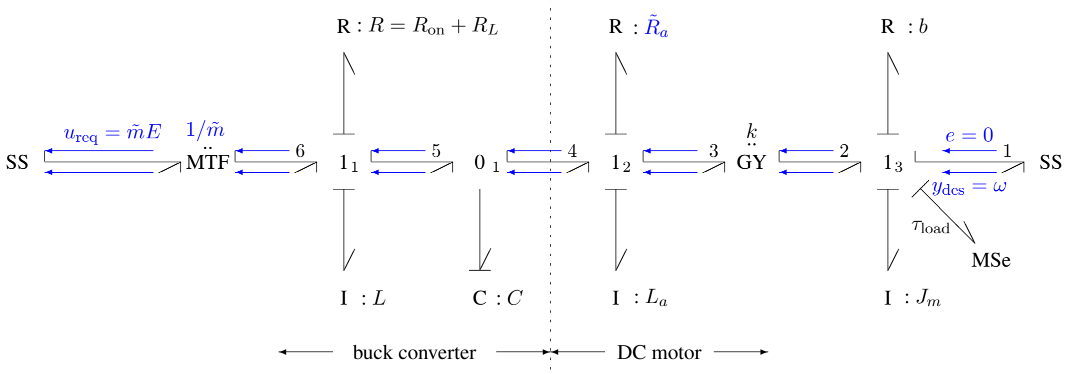

Figure 3: Inverse BG model of the buck-converter-driven DC motor

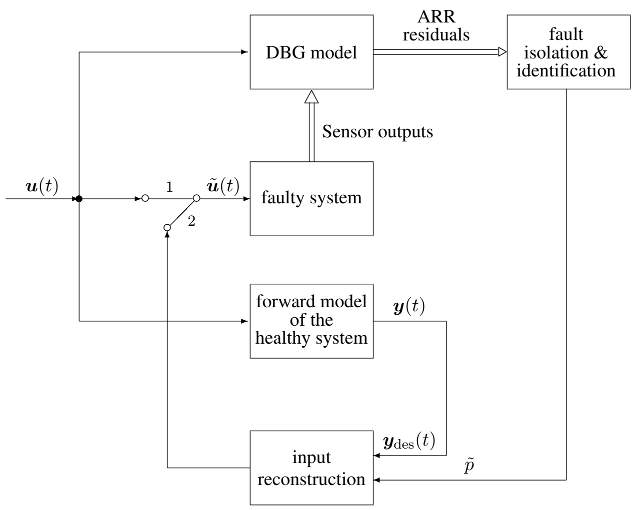

Figure 4: Scheme of the active FTC procedure

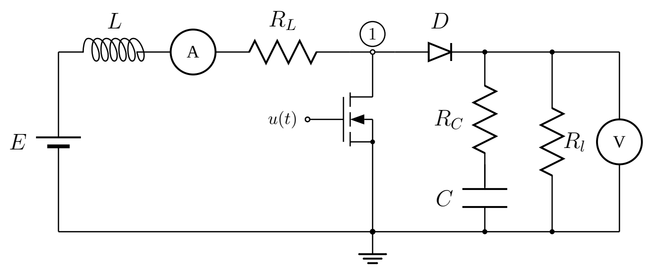

Figure 5: Circuit schematic of a boost converter

tem given $(\tilde{\mathbf{C}}\tilde{\mathbf{B}})^{-1}$ exists. For systems of slightly different structure, however, the forward model may be linear, while model inversion leads to a nonlinear problem. This is illustrated by the simple example of a boost-converter. Fig. 5 depicts its circuit schematic and Fig. 6 shows an averaged linear time-invariant forward BG model. The transistor and the diode have been considered as non-linear switches with ON-resistance R_{on} that commutate oppositely in a healthy system. Furthermore, the equivalent series resistance of the capacitor has been neglected.

Let $R := R_L + R_{\mathrm{on}}$. The equations of the averaged linear forward model deduced from the BG in Fig. 6 read

$$L\frac{di_L}{dt} = -Ri_L - mu_C + E \qquad (28a)$$

$$C\dot{u}_C = mi_L - \frac{u_C}{R_l} \qquad (28b)$$

$$y = u_C \qquad (28c)$$

Now, suppose that the resistance, R_L, of the inductor becomes faulty as of some time instant t_1. Given a constant voltage supply, E, the question then is how to change the duty cycle, d, of the signal $u(t)$ controlling the MOSFET transistor so that the output voltage $\tilde{V}(t) = \tilde{u}_C(t)$ of the faulty systems follows the output voltage $V(t) = u_C(t)$ of the healthy system despite the faulty resistance $\tilde{R}_L$. Changing the duty cycle, d, into a time dependent variable, $\tilde{d}(t)$ as of some time instant turns the signal $u(t)$ into a pulse width modulated (PWM) signal.

Let $\tilde{x}_1 := \tilde{i}_L$, $\tilde{x}_2 := \tilde{u}_C$, $y_{\mathrm{des}} := y$ and $\tilde{R} := \tilde{R}_L + R_{\mathrm{on}}$. Then the nonlinear DAE system that determines $m_{\mathrm{req}}(t) := 1 - \tilde{d}(t)$ read

$$L\dot{\tilde{x}}_1 = -\tilde{R}\tilde{x}_1 - m_{\mathrm{req}}y + E \qquad (29)$$

$$C\dot{y} = m_{\mathrm{req}}\tilde{x}_1 - \frac{1}{R_l}y \qquad (30)$$

$$y = \tilde{x}_2 \qquad (31)$$

Differentiation of the algebraic constraint (30) with respect to time and (29) give the semi-explicit ODE system

$$\begin{bmatrix} L & 0 \\ m_{\mathrm{req}} & \tilde{x}_1 \end{bmatrix} \begin{bmatrix} \dot{\tilde{x}}_1 \\ \dot{m}_{\mathrm{req}} \end{bmatrix} = \begin{bmatrix} -\tilde{R} & -y \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ m_{\mathrm{req}} \end{bmatrix} + \begin{bmatrix} E \\ \dfrac{\dot{y}}{R_l} + C\ddot{y} \end{bmatrix} \qquad (32)$$

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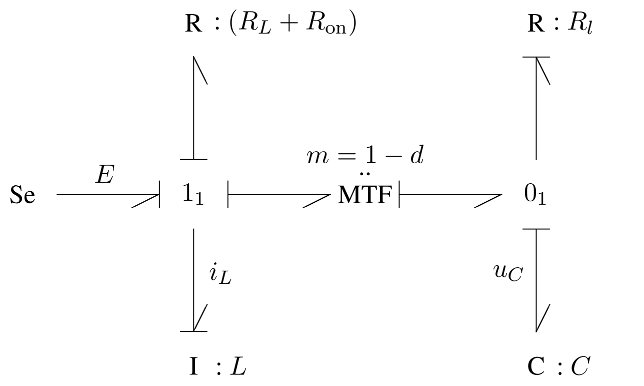


Figure 6: Averaged linear forward BG model of a boost converter

The unknown m_{req} can be eliminated from the first equation of the inverse model by means of the equations of the forward model. As $\tilde{u}_C = \tilde{x}_2 = y = u_C = x_2$, the following relation may be obtained.

$$m_{req}\tilde{x}_1 = C\dot{\tilde{x}}_2 + \frac{1}{R_l}\tilde{x}_2 = C\dot{x}_2 + \frac{1}{R_l}x_2 = mx_1 \tag{33}$$

Substituting $m_{req} = mx_1/\tilde{x}_1$ into the first ODE of the inverse model yields

$$\frac{1}{2}L2\tilde{x}_1\dot{\tilde{x}}_1 = -\tilde{R}\tilde{x}_1^2 - mx_1 y + E\tilde{x}_1 \tag{34}$$

Let $x := \tilde{x}_1^2$, then one obtains the explicit nonlinear ODE

$$\dot{x} = -\frac{2\tilde{R}}{L}x + \frac{2E}{L}\sqrt{|x|} - \frac{2mx_1 y}{L} \tag{35}$$

The last term on the right hand side of this ODE is known from a solution of the forward model. The values $x(t)$ can be used to determine $m_{req}(t)$.

Suppose that the fault in R_L occurred at $t = t_1$ and that it is isolated and quantified as of $t > t_2$. Then, the fault can be accommodated by switching from d to $\tilde{d}(t)$.

4. CASE STUDY

In this case study, the buck converter driven DC-motor (Fig. 1) is considered again. As a fault scenario, some leakage in the capacitor as of some time t_1 is assumed. As a result, the voltage across the capacitor C driving the motor will drop and so will the angular velocity ω. Given a constant voltage supply of the buck converter, the duty cycle of the signal controlling the transistor must be changed to keep up the angular velocity of the healthy system considered the desired output y_{des}.

4.1 Input Reconstruction

The leakage may be captured by a small resistance R_s in parallel to the capacitance that becomes effective as of some time t_1. Again, let $\tau_{load} = 0$. For $t > t_1$, the DAE system for the inverse model then reads

$$0 = u_{req} - R\tilde{i}_L - L\frac{d\tilde{i}_L}{dt} - \tilde{u}_C \tag{36}$$

$$0 = \tilde{i}_L - C\dot{\tilde{u}}_C - \frac{\tilde{u}_C}{R_s} - \tilde{i}_a \tag{37}$$

$$0 = \tilde{u}_C - R_a\tilde{i}_a - L_a\frac{d\tilde{i}_a}{dt} - ky \tag{38}$$

$$0 = k\tilde{i}_a - b\omega - J_m\dot{\omega} \tag{39}$$

$$y = \omega \tag{40}$$

where $u_{req} = \tilde{m}E$ is to be determined, $y(t)$ is known (measured) and $R = R_L + R_{on}$.

Let i_a and u_C denote the armature current and the capacitor voltage in the healthy system obtained by solving the forward model. Equation 39 then entails $\tilde{i}_a = i_a$ and $\tilde{u}_C = u_C$. Substitution of equations yields for the required input

$$\begin{aligned} u_{req} &= \tilde{m}E \\ &= R(C\dot{u}_C + \frac{u_C}{R_s} + Ri_a) \\ &\quad + L\frac{d}{dt}(C\dot{u}_C + \frac{u_C}{R_s} + i_a) + u_C \\ &= mE + \frac{R}{R_s}u_C + \frac{L}{R_s}\dot{u}_C \end{aligned} \tag{41}$$

In steady state, (41) reduces to

$$\begin{aligned} u_{req} = \tilde{m}E &= R\left[\frac{1}{R_s}(R_a\frac{b}{k} + k) + \frac{b}{k}\right]\omega \\ &\quad + (R_a\frac{b}{k} + k)\omega \end{aligned} \tag{42}$$

As long as the leakage in the capacitor has not occurred, $R_s \to \infty$ and $\tilde{m}E = mE$. This gives an equation the steady state value of the desired angular velocity must fulfill.

$$mE = \left[(R + R_a)\frac{b}{k} + k\right]\omega \tag{43}$$

If the fault is not accommodated by a changed input $\tilde{m}E$, then the angular velocity reaches a faulty steady state value $\tilde{\omega}$ that is given by the equation

$$\begin{aligned} mE &= R\left[\frac{1}{R_s}(R_a\frac{b}{k} + k) + \frac{b}{k}\right]\tilde{\omega} \\ &\quad + (R_a\frac{b}{k} + k)\tilde{\omega} \end{aligned} \tag{44}$$

The parameters listed in Table 1 give the steady state values $\omega = 69.63\, rad/s$ and $\tilde{\omega} = 35.57\, rad/s$.

Table 1: Parameters of the system buck converter - DC-motor

Parameter	Value	Units	Meaning
E	12.0	V	Voltage supply
L	20	mH	Inductance
R_L	0.1	Ω	Resistance of the coil
R_{on}	0.1	Ω	ON resistance (switch, diode)
d	0.5	–	Duty ratio
C	40	μF	Capacitance
L_a	2.6	mH	Armature inductance
R_a	2.0	Ω	Armature resistance
k	0.046	Vs/rad	Motor constant
J_m	$7.0 \cdot 10^{-4}$	kgm^2	Moment of inertia
b	$8.4 \cdot 10^{-4}$	Nms/rad	Friction coefficient
τ_{load}	0	Nm	Load moment
R_s	0.2	Ω	Resistance accounting for capacitor leakage

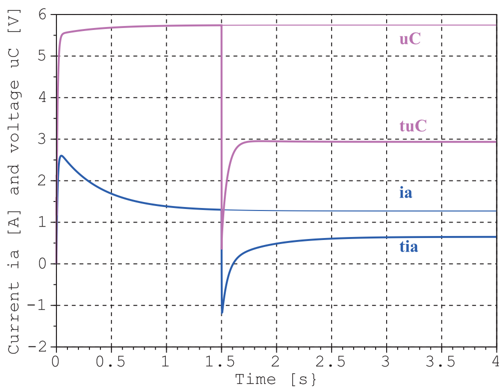


Figure 7: Time evolution of the armature current and the output voltage of the buck converter

4.2 Simulation of the fault

Simulation runs have been performed by means of the open source software Scilab Scilab Enterprises (nd) and have used the parameters given in Table 1.

Fig. 7 shows the time evolution of the armature current and the output voltage of the buck converter if the capacitor leakage is not compensated by a reconstructed input. The analytically computed steady state values for the healthy system read $i_a = 1.27A$, $u_C = 5.75V$ and for the faulty not accommodated system $\tilde{i}_a = 0.65A$, $\tilde{u}_C = 3.0V$. In Fig. 7, the tilde is indicated by the prefixed letter `t`.

Fig. 8 depicts the time history of the desired angular velocity, ω_{des}, the faulty velocity $\tilde{\omega}$ (denoted as `tw` in the figure) and the accommodated faulty velocity ω_{acc}. Again, the simulation run confirms the analytically computed steady state

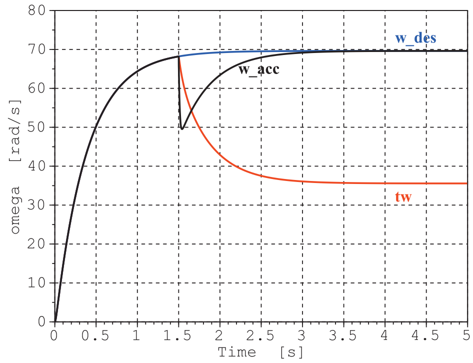


Figure 8: Time histories of the desired, the faulty and the accommodated faulty angular velocity

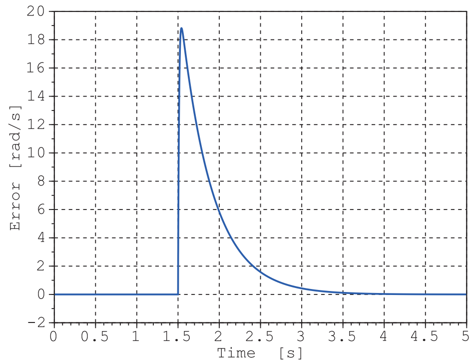


Figure 9: Time history of the error between the desired, the faulty angular velocity

values.

As of $t_1 = 1.5\,s$, a small resistance of $R_s = 0.2\,\Omega$ in parallel to the capacitor becomes effective. As a result of the capacitor leakage modelled this way, the capacitor voltage drops sharply and so does the angular velocity. If this fault is not compensated, the faulty steady state value is roughly half of the desired one.

It is assumed that detection and isolation of this fault takes about $0.02\,s$ so that fault accommodation can start at $t_2 = 1.52\,s$. Computing the reconstructed input by numerically solving the DAE system of the inverse model on a multiprocessor, multi-core computer also takes some time. As can be seen, after the leakage of the capacitor has happened, the reconstructed input $u_{\text{req}}(t)$, in fact, forces the faulty angular velocity, $\tilde{\omega}(t)$, to follow the desired velocity $\omega_{\text{des}}(t)$. The dynamics of the recovery depends on how much the capacitor voltage has dropped and on the parameters of the systems. Given the parameters in this case study, it takes about $1.5\,s$ to recover from this sharp drop of the angular velocity (Fig. 9). Finally, Fig. 10(a) shows that the armature current i_a in the accommodated system, apart from a peak at $t = 1.5\,s$ caused by the abrupt leakage of the capacitor, in fact, remains unchanged as it is determined by the desired angular velocity.

Fig. 10(b) indicates that the voltage u_C in the accommodated system does not drop to the steady state value of $3.0V$ but is forced in a very short time to regain the value of the healthy system so that the motor continues operating with the desired angular velocity despite the leakage of the capacitor. As a result, the inductor current i_L increases significantly (Fig. 10(c)). The steady state value rises from $1.27A$ to $30.07A$.

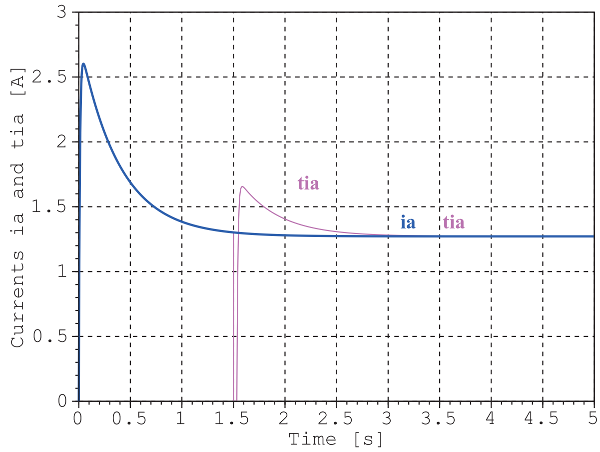
CONCLUSION

Once an abrupt parametric fault that has occurred in a system at some time instant is isolated and its magnitude is estimated, the system input can be reconstructed so that the system returns to a desired output after some delay. In this paper, a forward model deduced from a BG is considered a DAE system of the inverse model that may be solved by inverse simulation for the unknown required input that forces the faulty system to produce the output of the healthy one. The inverse simulation uses values of output variables that are provided by numerical computation of the forward model of the healthy system.

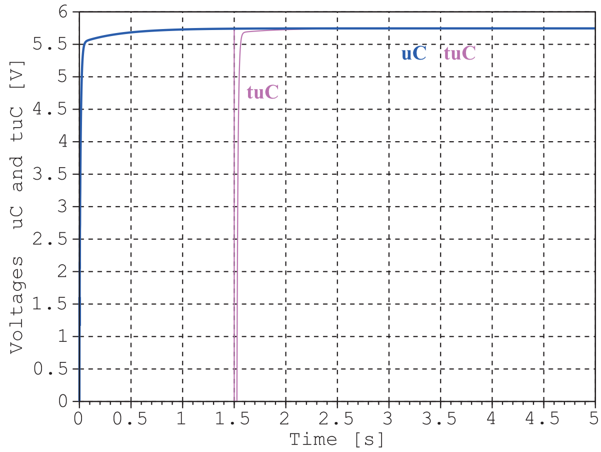
The analysis has been confined to linear time-invariant forward models. It is shown that after differentiation of the algebraic output equation a semi-explicit ODE system can be obtained that determines the required input. The approach to active FTC is applied to two simple examples of power electronic systems which have been represented by an averaged BG model. In a case study, a fault scenario in a buck converter driven DC motor has been considered.

The reconstruction of the system input in response to abrupt faults by inverse simulation is quite general. Existing sophisticated software for solving nonlinear DAE systems can be used. Analytical determination of the reconstructed system input is not necessary if possible at all. For complex systems with fast dynamics, the time needed for computing a reconstructed input after a fault has occurred may become an issue. This time adds to the time needed to detect and to isolate a fault and increases the time delay until the reconstructed input becomes effective. Exploitation of parallelism in the system model and computation on a multiprocessor system may reduce the time delay.

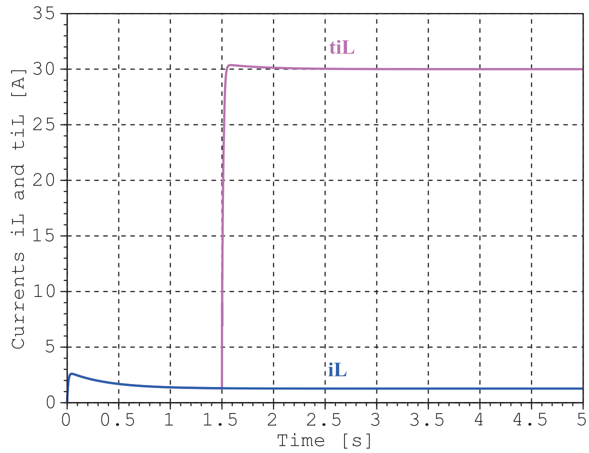
An advantage of input reconstruction by means of inverse simulation is that it is applicable to cases in which an analytical determination of the input required to accommodate a fault is not possible. Subjects of further research may be the application to systems with nonlinearities and to hybrid systems other than switched power electronic systems. Hybrid systems operate in various modes and their dynamic behaviour in one mode may be quite different from that in an-



(a) Time history of the currents i_a and $\tilde{i}_a$



(b) Time history of the voltages u_C and $\tilde{u}_C$



(c) Time history of the currents i_L and $\tilde{i}_L$

Figure 10: Time history of the currents i_a, i_L and the voltage u_C in the accommodated system

other mode. This affects FDI as well as input reconstruction for active FTC.

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APPENDIX

In order to keep the presentation concise consider a linear time-invariant single input single output (SISO) system with three states. Let u denote the input and y the output.

$$\dot{x}_1 = a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + b_1 u \tag{45}$$

$$\dot{x}_2 = a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + b_2 u \tag{46}$$

$$\dot{x}_3 = a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + b_3 u \tag{47}$$

$$= f_3(x_1, x_2, x_3, u) \tag{48}$$

$$y = c_1 x_1 + c_2 x_2 + c_3 x_3 \tag{49}$$

Let $c_1 \neq 0$ without loss of generality. Then (49) may be solved for x_1.

$$x_1 = \frac{1}{c_1}(-c_2 x_2 - c_3 x_3 + y) = f_1(x_2, x_3, y) \tag{50}$$

Differentiation of (49) with respect to time gives

$$\dot{y} = c_1\dot{x}_1 + c_2\dot{x}_2 + c_3\dot{x}_3 \tag{51}$$

$$\ddot{y} = c_1\ddot{x}_1 + c_2\ddot{x}_2 + c_3\ddot{x}_3 \tag{52}$$

Substitution of (45) – (47) into (51) and (52) results in two equations that may be written as

$$\begin{aligned} \dot{y} &= f_2(x_1, x_2, x_3, u) && (53) \\ \bar{b}\dot{u} &= f_4(x_1, x_2, x_3, u, \ddot{y}) && (54) \end{aligned}$$

where $\bar{b} = c_1 b_1 + c_2 b_2 + c_3 b_3$.

Substitute (50) into (53) and assume that the resulting linear equation can be solved for x_2, i.e. the coefficient premultiplying x_2 does not vanish.

$$x_2 = \bar{f}_2(x_3, u, y, \dot{y}) \qquad (55)$$

Substituting x_1 and x_2 into (47) and (54) then yields

$$\begin{aligned} \dot{x}_3 &= \bar{f}_3(x_3, u, y, \dot{y}) && (56) \\ \bar{b}\dot{u} &= \bar{f}_4(x_3, u, y, \dot{y}, \ddot{y}) && (57) \end{aligned}$$

As a result, the required input is a function of a reduced number of states, the desired output and its derivatives.