STATE AND UNKNOWN INPUT OBSERVER: ANALYSIS AND DESIGN

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ABSTRACT

This paper presents a structural approach for the state and unknown input estimations of linear systems when the classical matching condition is not verified. At the analysis step, the finite and infinite structures of the model are studied from the bond graph representation. At the synthesis level, the observer is directly implemented from the initial model with some additives terms and can be represented by a bond graph model. An illustrative example which considers a real system is included.

keyword: Unknown Input Observer, bond graph, linear models, structural approach

1. INTRODUCTION

Consider a linear perturbed system described by a state space equation defined in (1), where $x \in \Re^n$ is the state vector, $z \in \Re^p$ is the vector set of measured variables (also output variables to be controlled in this paper). The input variables are divided into two sets $u \in \Re^m$ and $d(t) \in \Re^q$ which represent known and unknown input variables respectively.

$$\left\{ \begin{array}{l} \dot{x}(t) = Ax(t) + Bu(t) + Fd(t) \\ z(t) = Hx(t) \end{array} \right. \quad (1)$$

Generally, the state vector $x(t)$ cannot be entirely measured and the system is often subject to unknown inputs $d(t)$ (disturbance or failure...) which must be estimated. The unknown input and state observability problem (UIO) is a well known problem. Different approaches give solvability conditions and constructive solutions for this problem.

At the analysis step, before design, must of the approaches require the analysis of the structural invariants of the model which play a fundamental role in this problem. The infinite structure of the model is also often related to solvability conditions. They have been extensively studied in many papers and books [2], [24], [26], [18], [14], [19]. The knowledge of zeros is an important issue because zeros are directly related to stability conditions of the observer and of the controlled system.

For LTI models, constructive solutions with reduced order observers are first proposed with the geometric approach [15], [3], [2]. Constructive solutions based on generalized inverse matrices are given in [22] and then in [23] and [17]. Full order observers are then proposed in a similar way (based on generalized inverse matrices) in [8] and [7], but with some restriction on the infinite structure of the model (known as observer matching condition), which is a rather restrictive condition.

The algebraic approach is proposed in [29] and in [5] for continuous and discrete time systems, without restriction on the infinite structure of the model. When this condition (matching condition) is not satisfied [13] proposed unknown input sliding mode observers after implementing a procedure to get a canonical observable form of the model. New output variables are defined with some derivatives. In this approach, sliding mode observers combined with a high-gain approach are often proposed [20]. New developments are now proposed with an observer based approach for some classes of nonlinear systems with a fuzzy approach [30], fuzzy systems with time delays [28] or with uncertain systems [4].

This work makes the following contributions: extension of a previous work [31] when the classical matching condition is not verified and simple synthesis of an UIO with output variables' derivatives. In section 2, the previous UIO design is recalled with the different steps for design and synthesis. An extension without the matching condition is proposed in section 3. In that case, the UIO is a bond graph model, close to the bond graph model of the physical system. At the analysis step, a graphical approach is proposed and as for many estimation and control problems, the invariant zeros are of great interest in the analysis. An illustrative example which considers a real system is included in section 4, and we conclude in section 5.

2. UIO WITH MATCHING CONDITION

In the literature, the different proposed approaches con-sider first the finite structure of $\Sigma(H,A,F)$ and then its infinite structure. The finite structure gives some stability conditions on the UIO and the infinite structure some con-ditions on the existence of the UIO.

2.1. Finite and infinite structures of Σ(H,A,F)

The concepts of strong detectability, strong* detectability and strong observability have been proposed in [16] for sys-tems with unknown inputs $d(t)$. The strong detectability of system with only the unknown input vector $d(t)$ corresponds to the minimum-phase condition, directly related to the zeros of system $\Sigma(H,A,F)$ (finite structure). The system $\Sigma(H,A,F)$ in (1) is strongly detectable if and only if all its zeros s satisfy $Re(s) < 0$.

The infinite structure of multivariable linear models is characterized by different integer sets. $\{n'_i\}$ is the set of infinite zero orders of the global model $\Sigma(H,A,B)$ and $\{n_i\}$ is the set of row infinite zero orders of the row sub-systems $\Sigma(h_i,A,B)$. The infinite structure is well defined in case of LTI models [9] with a transfer matrix representation or with a graphical representation (structured approach), [10].

The row infinite zero order n_i verifies condition $n_i = \min\left\{k|h_iA^{(k-1)}B \neq 0\right\}$. n_i is equal to the number of derivations of the output variable $z_i(t)$ necessary for at least one of the input variables to appear explicitly. The global infinite zero orders [12] are equal to the minimal number of derivations of each output variable necessary so that the input variables appear explicitly and independently in the equations.

In order to solve the UIO problem for systems in (1), a necessary condition called *observer matching condition* for the existence of observers is often required (see [22]; [8]): rank$[HF]$ = rank$[F]$. For a SISO model, the infinite zero order of model $\Sigma(H,A,F)$ is equal to 1. When this condition is not satisfied [13] proposed unknown input sliding mode observers after implementing a procedure to get a canonical observable form of the model. This method can also be extended in the nonlinear case. Necessary and sufficient conditions are that system $\Sigma(H,A,F)$ is left invertible and minimum phase.

2.2. Bond graph models

In a bond graph model [21] and [25], causality and causal paths are useful for the study of properties, such as controllability, observability and systems poles/zeros. State space and transfer representations can be directly written from a bond graph model, thus properties of these mathematical representations can be derived before any calculus with a causal analysis. Bond graph models with integral causality assignment (BGI) can be used to determine reachability conditions and the number of invariant zeros by studying the infinite structure. The rank of the controllability matrix is derived from bond graph models with derivative causality (BGD).

A LTI bond graph model is controllable iff the two following conditions are verified [27]: first there is a causal path between each dynamical element and one of the input sources and secondly each dynamical element can have a derivative causality assignment in the bond graph model with a preferential derivative causality assignment (with a possible duality of input sources). The observability property can be studied in a similar way, but with output detectors. Systems invariant zeros are poles of inverse systems. Inverse systems can be constructed by bond graph models with bicausality (BGB) which are thus useful for the determination of invariant zeros.

The concept of causal path is used for the study of the infinite structure of the model. The causal path length between an input source and an output detector in the bond graph model is equal to the number of dynamical elements met in the path. Two paths are different if they have no dynamical element in common. The order of the infinite zero for the row sub-system $\Sigma(h_i,A,B)$ is equal to the length of the shortest causal path between the i^{th} output detector z_i and the set of input sources. The global infinite structure is defined with the concepts of different causal paths. The orders of the infinite zeros of a global invertible linear bond graph model are calculated according to equation (2), where L_k is the smallest sum of the lengths of the k different input-output causal paths.

$$\begin{cases} n'_1 = L_1 \\ n'_k = L_k - L_{k-1} \end{cases} \tag{2}$$

The number of invariant zeros is determined by the infinite structure of the BGI model. The number of invariant zeros associated to a controllable, observable, invertible and square bond graph model is equal to $n - \sum n'_i$.

2.3. UIO synthesis

The model $\Sigma(H,A,F)$ is supposed to be a SISO model in order to simplify the presentation. It can be easily extended to MIMO models using for example the same procedure as for the input-output decoupling problem with the concept of row and global infinite structures from a structural point of view in the analysis step. If a somewhat physical approach is proposed, some assumptions are also possible for the state space model deduced for example from a bond graph representation.

Asumption 1. It is supposed that the SISO system $\Sigma(H,A,F)$ defined in (1) is controllable/observable and that the state matrix A is invertible.

With Asumption 1, a derivative causality assignment is possible for bond graph models (physical model without null pole). The extension to models with non invertible state matrix is straight for bond graph models, because a graphical approach can be proposed in that case. It is not proposed in this paper.

The state equation (1) is now rewritten as (3).

$$\begin{cases} x(t) = A^{-1}\dot{x}(t) - A^{-1}Bu(t) - A^{-1}Fd(t) \\ z(t) = HA^{-1}\dot{x}(t) - HA^{-1}Bu(t) - HA^{-1}Fd(t) \end{cases} \tag{3}$$

If matrix $HA^{-1}F$ is invertible (Model $\Sigma(H,A,F)$ has no null invariant zero), the disturbance variable can be written in equation (4) and then the estimation of the disturbance variable can be written in equation (5). The extension to models with $HA^{-1}F = 0$ is straight and not proposed in this paper.

$$d(t) = -(HA^{-1}F)^{-1}[z(t) - HA^{-1}\dot{x}(t) + HA^{-1}Bu(t)] \tag{4}$$

$$\hat{d}(t) = -(HA^{-1}F)^{-1}[z(t) - HA^{-1}\dot{\hat{x}}(t) + HA^{-1}Bu(t)] \tag{5}$$

From the state equation (3), estimation for the state vector is defined in equation (6), which can also be written as (7), which is similar to a classical estimation, but with a difference in the last term. It needs the derivation of the measured variable. Matrix K is used for pole placement.

$$\hat{x}(t) = A^{-1}\dot{\hat{x}}(t) - A^{-1}Bu(t) - A^{-1}F\hat{d}(t) + K(\dot{z}(t) - \dot{\hat{z}}(t)) \quad (6)$$

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + F\hat{d}(t) - AK(\dot{z}(t) - \dot{\hat{z}}(t)) \quad (7)$$

In this approach [31], the state equations for the model and the observer are the same, with only an extra term for the observer. This observer is simple and take into account the control inputs, which is not always true in the literature.

The convergence of the disturbance variable can be verified with equation (8), obtained from (4) and (5).

$$d(t) - \hat{d}(t) = (HA^{-1}F)^{-1}HA^{-1}(\dot{x}(t) - \dot{\hat{x}}(t)) \quad (8)$$

The estimation of the disturbance variable converges to the disturbance variable only if $(\dot{x}(t) - \dot{\hat{x}}(t))$ converges asymptotically. Convergence of the state estimation must be proved with the study of the observer fixed poles.

Matrices N_{BO} and N_{BF} are introduced in (9), in order to simplify notations.

$$\begin{cases} N_{BO} = A^{-1} - A^{-1}F(HA^{-1}F)^{-1}HA^{-1} \\ N_{BF} = A^{-1} - A^{-1}F(HA^{-1}F)^{-1}HA^{-1} - KH \end{cases} \quad (9)$$

From previous equations, with $e(t) = x(t) - \hat{x}(t)$ it comes (10).

$$e(t) = N_{BF}\dot{e}(t) \quad (10)$$

This observer requires the matching condition defined in some well known approaches [16], [7] and in that case, fixed poles of the estimation error are all the invariant zeros of system $\Sigma(H,A,F)$ [31], which means that this system must be strong* detectable .

3. UIO EXTENSION

For many physical systems modeled by (1), the observer matching condition is not satisfied. To overcome the restriction imposed by this condition, an observer has been proposed in [13] using the infinite structure of model $\sum(H,A,F)$, and the derivatives of input and output variables.

An extension of the state and unknown input estimations is proposed in this paper without the observer matching condition in the SISO case. It can be easily extended to the MIMO case using the global infinite structure of model $\sum(H,A,F)$. Only some derivatives of the output variables are needed for this new observer.

3.1. UIO synthesis

Let r be the infinite zero order of the SISO model $\sum(H,A,F)$. It is the smallest positive integer such that $HA^{r-1}F \neq 0$.

The estimation of the disturbance variable is still written as in equation (5) and the estimation of the state vector is now written as (11). This new observer is very close to the previous one, with a matrix K used for pole placement multiplied by the r^{th} output variable derivative.

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + F\hat{d}(t) - AK(z^{(r)}(t) - \hat{z}^{(r)}(t)) \quad (11)$$

From (3) and (11), the state error estimation equation $e(t) = x(t) - \hat{x}(t)$ is given by (12), where N_{BF_r} is defined in (13) (Proof in Appendix).

$$e(t) = N_{BF_r}\dot{e}(t) \quad (12)$$

$$N_{BF_r} = A^{-1} - A^{-1}F(HA^{-1}F)^{-1}HA^{-1} - KHA^{r-1} \quad (13)$$

Matrix N_{BF_r} can be written in an easy way (obtained from $x(t) - \hat{x}(t)$). The difference between matrix N_{BF} defined in equation (9) and matrix N_{BF_r} defined in equation (13) is due to the output derivative and is just associated to the extra matrix A^{r-1}.

If the state equation (1) is written from a bond graph model, it is possible to draw a bond graph model for the state estimation defined in (11) because equation (11) is very close to the initial state equation. Some signal bonds must be added for the disturbance equation defined in (5). The structure of the observer is proposed in Fig. 1, with BGI for the bond graph model and BGO for the observer bond graph model.

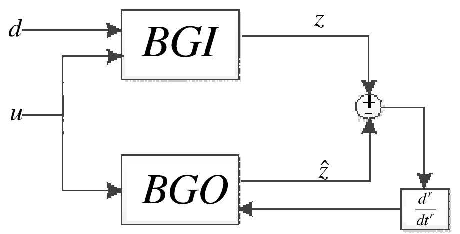

Figure 1: Structure of the observer based on bond graph model

With Fig. 1, it is possible to estimate unknown variables (disturbance inputs or actuators faults) in an easy way.

3.2. Properties of the observer

In equation (12), conditions for pole placement are studied.

If matrix N_{BF_r} is invertible, a classical pole placement is studied, and the error variable $e(t) = x(t) - \hat{x}(t)$ does not depend on the disturbance variable. The conditions for (12) to be an asymptotic state observer of $x(t)$ is that N_{BF_r} must be an Hurwitz matrix, i.e., has all its eigenvalues in the left-hand side of the complex plane. Properties of the observer are studied.

A necessary condition for the existence of the state estimator is proposed in Proposition 1.

Proposition 1: A necessary condition for matrix N_{BF_r} defined in (13) to be invertible is that $HA^{r-1}F \neq 0$.

Proof In Proposition 1, matrix $N_{BF_r}F$ is equal to $[A^{-1} - A^{-1}F(HA^{-1}F)^{-1}HA^{-1} - KHA^{r-1}]F$, thus it can be rewritten as $N_{BF}F = A^{-1}F - A^{-1}F(HA^{-1}F)^{-1}HA^{-1}F - KHA^{r-1}F =$

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$KHA^{r-1}F$. If condition $HA^{r-1}F \neq 0$ is not satisfied, the Kernel of matrix N_{BF_r} is not empty, which means that matrix N_{BF_r} is not invertible and that this matrix contains at least one null mode, thus pole placement is not possible (all its eigenvalues are not in the left-hand side of the complex plane).□

Condition defined in proposition 1 is an extension of the well-known matching condition defined in [16] and [7]. It means that the infinite zero order between the disturbance variable $d(t)$ and the measured variable $z(t)$ can be greatest than 1, equal to r with this observer.

It is now supposed that $HA^{r-1}F \neq 0$ is satisfied. Two properties are proved. First, it is proved that for matrix N_{BF_r}, r poles can be assigned and that the other poles (fixed poles) are the inverse of the invariant zeros of system $\Sigma(H,A,F)$.

Proposition 2: In matrix N_{BF_r} defined in (13), r poles can be chosen with matrix K.

Pole placement for matrix N_{BF_r} is equivalent to pole placement for system $\Sigma(HA^{r-1},N_{BO})$. The observability property of this system must be studied, and particularly the rank of the observability matrix which is equal to the number of poles which can be assigned.

The n rows of the observability matrix of system $\Sigma(HA^{r-1},N_{BO})$ are HA^{r-1}, $HA^{r-1}.N_{BO}$,...,$HA^{r-1}.N_{BO}^{n-1}$. Each row is calculated.

$$\begin{cases} HA^{r-1} \\ HA^{r-1}.N_{BO} = HA^{r-1}.(A^{-1} - A^{-1}F(HA^{-1}F)^{-1}HA^{-1}) \\ \quad = HA^{r-2} \\ HA^{r-1}.(N_{BO})^2 = HA^{r-3} \\ \vdots \\ HA^{r-1}.(N_{BO})^{r-2} = HA \\ HA^{r-1}.(N_{BO})^{r-1} = H \\ HA^{r-1}.(N_{BO})^{r} = 0 \\ \vdots \\ HA^{r-1}.(N_{BO})^{n-1} = 0 \end{cases} \tag{14}$$

The rank of this observability matrix is r because model $\Sigma(H,A)$ is observable and the non null rows calculated in (14) are thus linearly independent. This proved that r poles can be assigned in equation (12) and that the observer has $n-r$ fixed poles.□

Now it is proved that the fixed poles are the inverse of the invariant zeros of system $\Sigma(H,A,F)$.

Proposition 3: The eigenvalues of matrix N_{BO} defined in (9) are the inverse of the invariant zeros of system $\Sigma(H,A,F)$ ($n-r$ modes) plus r eigenvalues equal to 0.

Proof: Appendix□

Proposition 4: The fixed poles of the estimation equation error defined in (12) are the invariant zeros of system $\Sigma(H,A,F)$.

Proof From Proposition 3, the eigenvalues of matrix N_{BO} are the inverse of the invariant zeros of system $\Sigma(H,A,F)$ with r eigenvalues equal to 0, and since N_{BF} is invertible and only r poles can be chosen, all the fixed poles are the non null eigenvalues.□

4. EXAMPLE

The previous procedures are applied on a real hydraulic system modeled by bond graph. At the analysis step, proposed methods on bond graph models do not require the knowledge of the value of parameters, because intrinsic solvability conditions can be given and a formal calculus can be proposed at the synthesis level.

4.1. Bond graph model

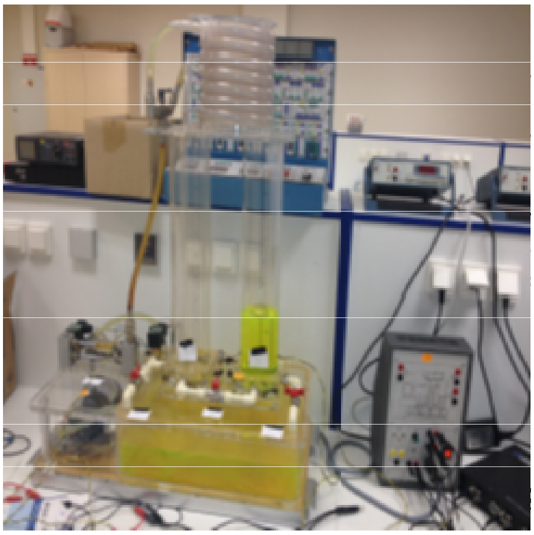

Figure 2: Hydraulic system with two tanks

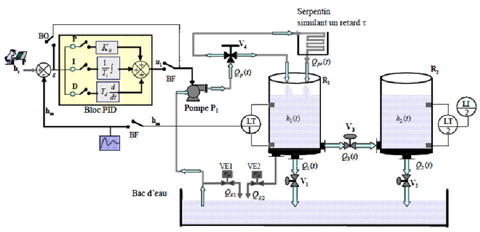

Figure 3: Process diagram to study

Consider for illustration the two coupled tanks depicted in figure 2 and 3. The aim of the two tanks is to provide a continuous water flow to a consumer via the valve V2. The process consists of two tanks R1 and R2 connected by a valve V3. The connection pipe between the tanks is placed at the bottom of the tanks. T1 is filled by a controlled pump P1 modeled as source of flow Q_p to keep water level constant. The pressures (image of levels) at the bottom of each tank are measured by sensors $P1_m$ and $P2_m$ respectively. The electro valves VE_1 and VE_2 can be used to simulate a leakage (as a disturbances). In the faultless mode VE_1 and VE_2 are closed. The three ways valve V_4 is used to connect inlet flow through the coil (to introduce a time delay) or directly to the tank R_1 in Fig. 2. The BGI model of the system with a disturbance signal is given in Fig. 4, and the state-space equations are presented in (15), with $(x_1,x_2)^t$ the state vector. It corresponds to the volume in the tanks.

$u = Q_p$ is the control input variable (flow) and $z_1 = h_2$ is the output vector. It corresponds to the height of liquid in second tank. In order to use classical bond graph rules, a parameter k is added before the output detector, Fig. 4, with $k = 0.0102 m^2 s^2 kg^{-1}$. d is the disturbance input variable (if the second valve V_2 is closed, $R_2 = \infty$. d is an unknown input if tap valve R_2 is open). The input $u(t)$ is a step function, i.e. $u(t) = 4.36 \cdot 10^{-4} m^3 s^{-1}$.

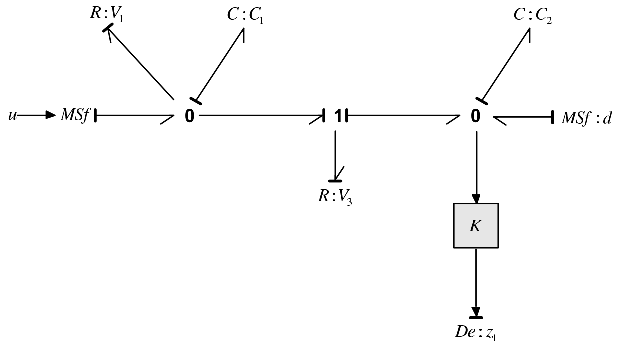

Figure 4: BGI model of the hydraulic system with the disturbance

$$\begin{cases} \dot{x}_1 = -\left(\frac{1}{C_1 R_3} + \frac{1}{C_1 R_1}\right) x_1 + \frac{1}{C_2 R_3} x_2 + u \\ \dot{x}_2 = \frac{1}{C_1 R_3} x_1 - \frac{1}{C_2 R_3} x_2 + d \\ z_1 = \frac{k}{C_2} x_2 \end{cases} \tag{15}$$

The bond graph model is controllable and observable (a derivative causality can be assigned). The numerical values of system parameters are shown in Table I. Simulations and control of this system are implemented with MATLAB®Simulink.

Table 1: Numerical values of system parameters

C_1	C_2	R_1	R_3
$7.78 \times 10^{-7} \frac{m^4 \cdot s^2}{kg}$	$8.01 \times 10^{-7} \frac{m^4 \cdot s^2}{kg}$	$4.21 \times 10^{7} \frac{pa \cdot s}{m^3}$	$5.78 \times 10^{7} \frac{pa \cdot s}{m^3}$

The valve V_2 is opened with start time $300s$ and end time $330s$. Then the disturbance variable d and it's estimate $\hat{d}$ and the estimation errors for the state variables are drawn.

4.2. Observer with matching condition

The design of the observer proposed in the previous section can thus be redesigned from a bond graph approach.

The causal path length between the output detector $De : z_1$ and the disturbance input $Sf : d$ is equal to 1, path $De : z_1 \to C : C_2 \to Sf : d$, thus the matching condition is verified, and there is an invariant zero in the system $\Sigma(H, A, F)$. After calculation, the invariant zero is $s = -\frac{1}{C_1}\left(\frac{1}{R_3} + \frac{1}{R_1}\right)$ which verifies the minimum phase condition. The bond graph representation of the observer is drawn in Fig. 5 in a general form without values for parameters.

For the considered hydraulic system, the two poles of the second order model are approximately equal to -0.064 and -0.0103. In the state estimation equation defined in (10), matrix $K = (k_1, k_2)^t$ is used for pole placement. The first

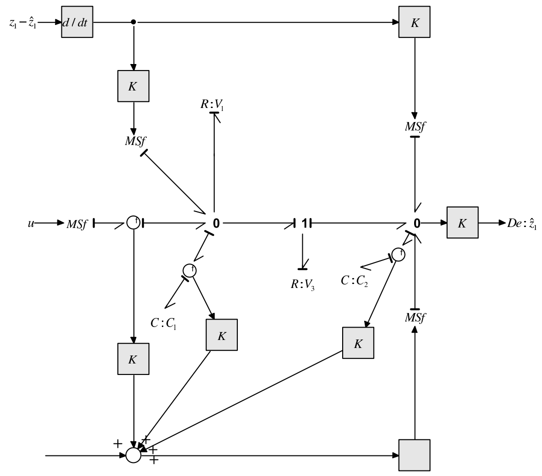

Figure 5: Observer with the bond graph representation

one is a fixed pole equal to $s_1 = -0.0527$. The second one is chosen at $s_2 = -0.6$, thus $k_2 = 1.3x10^{-6}$ because with some formal calculus, the second poles of matrix N_{BF}^{-1} defined in the state estimation error equation is $s = -\frac{C_2}{k_2}$.

The two estimated variables $\hat{h}_2$ and $\hat{d}$ are very close to the real variables, Fig. 6 and Fig. 7. The different figures prove the accuracy of this UIO.

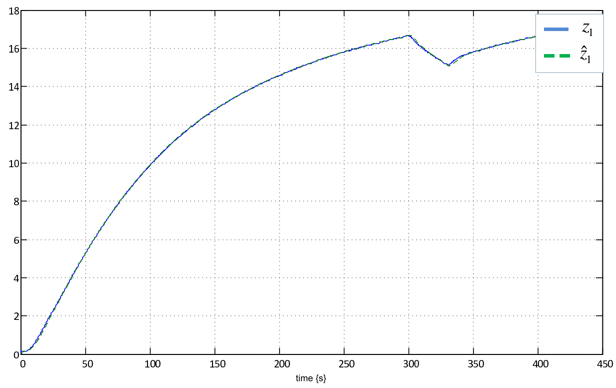

Figure 6: Water level in the second tank h2 and it's estimate hˆ2

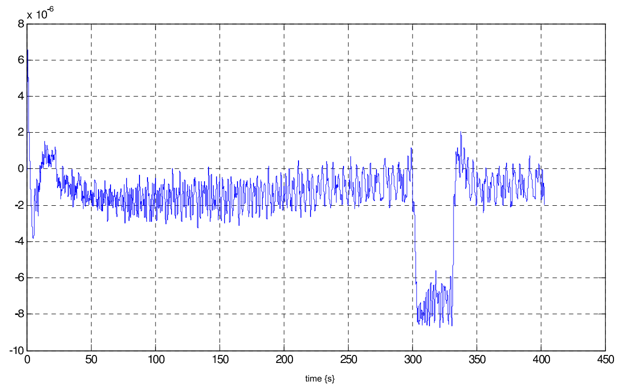

Figure 7: The disturbance variable estimate dˆ

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4.3. Observer without matching condition

In a second step, a new sensor z_2 is used to estimate the disturbance variable, as shown in Fig. 8. The state-space equations are presented in (16).

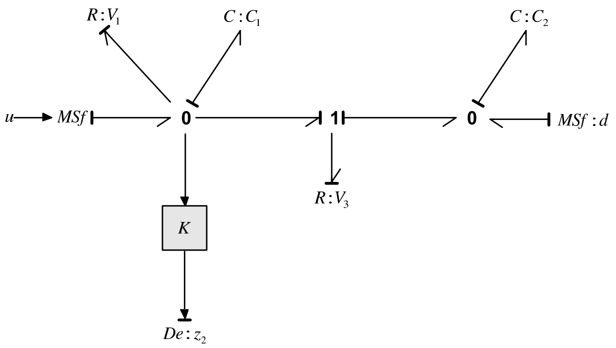

Figure 8: BGI model of the hydraulic system with the disturbance

$$\begin{cases} \dot{x}_1 = -\left(\frac{1}{C_1R_3} + \frac{1}{C_1R_1}\right)x_1 + \frac{1}{C_2R_3}x_2 + u \\ \dot{x}_2 = \frac{1}{C_1R_3}x_1 - \frac{1}{C_2R_3}x_2 + d \\ z_2 = \frac{k}{C_1}x_1 \end{cases} \tag{16}$$

In this case, the same parameters of the hydraulic system are used. The unknown input is a perturbation (opening of the valve R_2 with start time $720s$ and end time $750s$). The causal path length between the output detector $De : z_2$ and the disturbance input $Sf : d$ is equal to $r = 2$, the infinite zero order (path $De : z_2 \to C : C_1 \to R : R_3 \to C : C_2 \to Sf : d$). The classical matching condition is not verified. The extended observer is used, equation (17)

$$\dot{\hat{x}}(t) = A\hat{x}(t) + Bu(t) + F\hat{d}(t) - AK(\ddot{z}(t) - \ddot{\hat{z}}(t)) \tag{17}$$

The model order is equal to 2, thus there is not any invariant zero and all poles can be assigned in the error estimation equation. The state error estimation is given by (10) with $N_{BF_2} = A^{-1} - A^{-1}F(HA^{-1}F)^{-1}HA^{-1} - KHA$. The two poles are chosen at $s_1 = -0.5$ and $s_2 = -0.6$, thus the observer gains in matrix $K = [k_1, k_2]^t$ are $k_1 = 3.44 \cdot 10^{-6}$ and $k_2 = 3.44 \cdot 10^{-6}$.

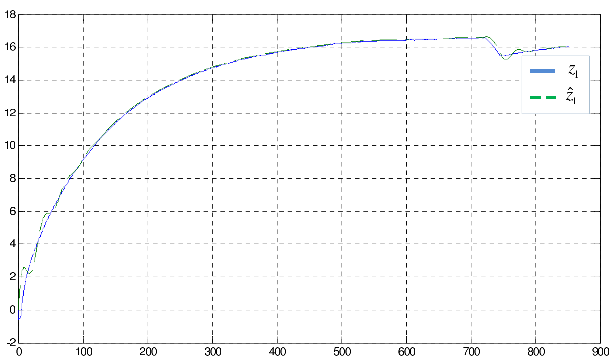

Figure 9: The water level in the first tank h_1 and it's estimate $\hat{h}_1$

Experimental results (fig. 9 - 10) show that the observer reproduces closely the output value of the water level in the tank $y = h_1$ and the unknown input. In this experimental system, the unknown input is not only a perturbation (opening of the valve R_2) but also noise, mostly at the beginning of the experiment, which can be pointed out at the beginning of the estimation, Fig. 10.

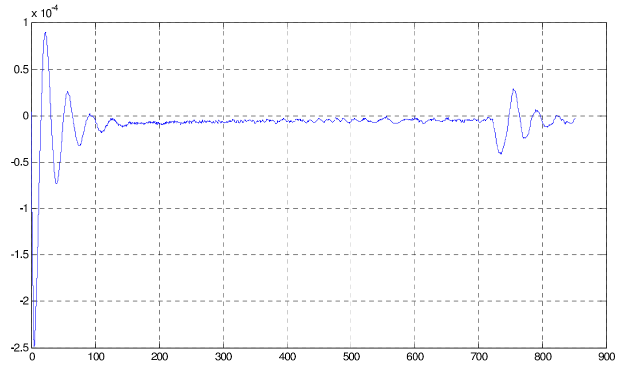

Figure 10: The disturbance variable estimate $\hat{d}$

Numerical differentiation of a signal is an old problem in automation and many problems have been solved (estimation, control...) with approximation of derivatives. Different approaches use an interpolation technique [11]. In other works, the authors apply a cubic spline interpolant method, [1] and [6].

In this case study, simulations are performed using the software Matlab-Simulink. For numerical differentiation of the real signal, a block "Discrete Derivative" is used. From the simulation results and experimental results presented in this paper, it should be noted that these differentiations are possible in this experiment.

5. CONCLUSION

In this paper, an extension of an unknown input observer is proposed when the classical matching condition is not verified. The necessary condition to obtain a stable solution is that invariant zeros belong to the left half complex plan (Hurwitz condition). A bond graph approach with classical graphical conditions is used. The approach is proposed for a real hydraulic linear system. Experiments have shown that the proposed observer is accurate. In the future this method will be extended to nonlinear systems.

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APPENDIX

Proof of equations (12) and (13)

Equations (12) and (13) are proved for $r = 2$. The extension for any integer r is simple are straight.

First, write $z = Hx$. A first order derivative is $\dot{z} = H\dot{x} = H(Ax + Bu + Fd) = HAx + HBu$ with $HFd = 0$. Thus $\ddot{z} = H\ddot{x} = HA\dot{x} + HB\dot{u}$. The same equation is written for $\hat{z}$, thus $\ddot{\hat{z}} = H\ddot{\hat{x}} = HA\dot{\hat{x}} + HB\dot{u}$. From these two expressions, a new one is written: $\ddot{z} - \ddot{\hat{z}} = HA\dot{x} + HB\dot{u} - (HA\dot{\hat{x}} + HB\dot{u}) = HA(\dot{x} - \dot{\hat{x}})$.

With an easy extension, it is proved that $z^{(r)} - \hat{z}^{(r)} = HA^{r-1}(\dot{x} - \dot{\hat{x}})$, which proves equations (12) and (13).

Proof proposition 3

First, the observability property of model $\Sigma(HA^{r-1}, N_{BO})$ is studied. The non observable poles are the roots of the invariant polynomials obtained from the Smith form of matrix $N(s)$ defined in (18). With matrix HA^{r-1}, only r modes of matrix N_{BO} can be assigned, because the rank of the observability matrix of system $\Sigma(HA^{r-1}, N_{BO})$ is equal to r. Structurally, with $s = 0$, the rank of matrix (18) degenerates. The non observable modes are all the null modes.

$$N(s) = \begin{pmatrix} sI - N_{BO} \\ HA^{r-1} \end{pmatrix} \tag{18}$$

The fixed poles of the state estimation error defined in (10) are thus the r null non observable poles of model $\Sigma(HA^{r-1}, N_{BO})$. Now, some equivalent transformations are proposed for the Smith matrix $S(s)$ of system $\Sigma(H, A, F)$ defined in (19).

$$S(s) = \begin{pmatrix} sI - A & -F \\ H & 0 \end{pmatrix} \tag{19}$$

$$S(s) \sim \begin{pmatrix} sA^{-1} - I & -A^{-1}F \\ H & 0 \end{pmatrix} \sim \begin{pmatrix} sA^{-1} - I & -A^{-1}F \\ H + sHA^{-1} - H & -HA^{-1}F \end{pmatrix} \tag{20}$$

$$\sim \begin{pmatrix} sA^{-1} - I & A^{-1}F(HA^{-1}F)^{-1} \\ sHA^{-1} & I \end{pmatrix} \tag{21}$$

$$\sim \begin{pmatrix} sA^{-1} - I - A^{-1}F(HA^{-1}F)^{-1}(-sHA^{-1}) & 0 \\ sHA^{-1} & I \end{pmatrix} \tag{22}$$

$$\sim \begin{pmatrix} sN_{BO} - I & 0 \\ 0 & I \end{pmatrix} \tag{23}$$

Since $\det(pI - N_{BO})$=$p^n\det(I - N_{BO}/p)$, with $s = 1/p$ it comes $\det(pI - N_{BO})$=$(-1)^n s^{-n}\det(sN_{BO} - I)$ which is a polynomial of degree n with variable p and of degree $-n$ with variable s. But, $\det S(s)$ is a polynomial of degree $n - r$, thus from a simple mathematical analysis it is proved that the polynomial $\det(pI - N_{BO})$ has r null roots and that the roots of the polynomial $\det(sN_{BO} - I)$ are the inverse of the non null roots of polynomial $\det(pI - N_{BO})$ since $(p = 1/s)$. In that case the non null poles of matrix N_{BO} are the inverses of the invariant zeros of model $\Sigma(H, A, F)$.

The non observable modes of system $\Sigma(HA^{r-1}, N_{BO})$ are thus all the inverse of the invariant zeros of system $\Sigma(H, A, F)$. They are the fixed modes of the state estimation error equation.