H∞ CONTROL WITH TIME DOMAIN SPECIFICATIONS APPLIED TO A HEAT EXCHANGER NETWORK

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ABSTRACT
Multivariable control techniques have been applied to chemical plants, ranging from multi-loop control up to predictive controls. This work intends to present a study developed using a control solution based on the H∞ design with time domains specifications applied on a heat exchanger network (HEN) with bypasses. The HEN is frequently used in chemical processes to promote energy transfer between hot/cold streams, reducing the utility consumption as its main objective. The H∞ control is used here to ensure a prescribed time domain response by means of a solution of model matching problem so that the closed loop dynamics are approximately the same as the ones of the reference model. The simulations results have demonstrated that the proposed control leads to a good performance with process variables decoupled, null offset and output responses with the prescribed dynamics.

Keywords: heat exchanger, heat exchanger network, H∞ control, process control

1. INTRODUCTION

The heat exchanger (HE) is an important equipment widely used in chemical plants in order to promote an suitable energy exchange between hot/cold streams, minimizing the utilities consumption (cold water and high pressure steam) necessaries for the manufacturing processes. The reducing on utilities consumption are always desirable because it reflects directly on the industries expenditure, minimizing the usage of electrical and fuel (gas, oil) energies to cool / heat the process streams.

This reduction is achieved by using some HE's, where this arrangement is named as heat exchanger network (HEN) that is an indispensable equipment to achieve the reducing on utilities consumption. The HEN is responsible to promote the energy integration of the chemical plant by interchanging some hot and cold streams (Hewitt *et al*., 1993)

The major problem faced by the process engineers on this energy integration is to choose which streams (hot/cold) will perform the energy integration. One of the current techniques to develop this study that helps engineers to achieve the desirable integration on the industries is the Pinch technology (Hewitt *et al*., 1993).

Nowadays, the most of the industrial HEN applications uses the shell and tube HE. Independently of the HE model used, is very important that the stream outlet temperatures are in a specified range (Hewitt *et al*., 1993). This specified range of temperatures could be achieved by using an efficient control system.

There are some different control techniques that could be used in industrial processes. Normally, the methodologies that are such complex demands a high engineered design but they present a superior performance. On the other hand, there are methodologies that are easy to design but normally not capable to lead to a desirable performance. As an example, the PID is the most common controller due to its simple design procedure (such as the heuristic tuning methods) and implementation, widely used on chemical plants.

However, on chemical process that presents several inputs/outputs, the PID designs will demand an additional engineering effort to tune and design the controllers. In this situation, the DMC Predictive controller may be considered a suitable control strategy to be used. It may deals with many inputs/outputs, complex dynamics, dead time or inverse output response (Hu and Sun, 2002), but its drawback is that it needs much engineering contribution to be designed (Gonzalez *et al*., 2006).

Based on the literature, the most common applied control techniques to perform the closed loop control on a HEN are the predictive control, the neural network and the feed forward PID (Ogunnaike, 1994). In the range of suitable controllers to be applied on a HEN, one may include the H∞ control. The H∞ design is typically performed on frequency domain and it is quite powerful in the sense that it allows a clear way of dealing with wide spectrum tradeoffs of the project. Nevertheless, its design usually requires a deeper engineering background and the time domains specifications must be first converted to equivalent frequency domain specifications.

The H∞ control problem can be expressed as an optimization problem in which a controller must be found to satisfy the requirements of robustness, performance and control effort. The most common way to design the H∞ is the so called mixed sensitivity formulation that involves the use of weighting matrices, responsible for adjusting the performance, control effort and stability robustness. To take the most of this technique and also allowing time domain specifications, this work proposes a control solution for a HEN based on a H∞ model matching control, where the time domain specifications are converted in a reference model. In order to perform this requirement, the H∞ mixed sensitivity formulation is adapted to solve the model matching and the H∞ controller found is capable to force the transfer matrix of the closed loop system to be close to the reference matrix with a prescribed precision (Leonardi, 2002). Note that H∞ control applied to the HEN is not commonly found on the scientific literature (Delatore *et al*., 2009).

2. THE HE MATHEMATICAL MODEL AND HEN ARRANGEMENT

Before presenting the studies about the control design and its performance applied on a HEN, it is important to define a physical arrangement of the exchangers, determining the manipulated, the controllable and the disturbances variables of the HEN. The shell and tube HE, as mentioned before, is the most common one used in chemical plants, mainly due to its low cost, uncomplicated repairing/maintenance and it can also be built with a lot of different exchanger area, in order to be suitable to almost all chemical processes. For this reason, the shell and tube HE were chosen to be used on this work.

The configured HEN was obtained by interconnecting adequately the inputs and the outputs of the HE model (Novazzi, 2006), which results on a HEN with two hot streams and one cold stream, two output controllable variables and two manipulated variables. Figure 1 shows the arrangement adopted of the HEN considered on this work.

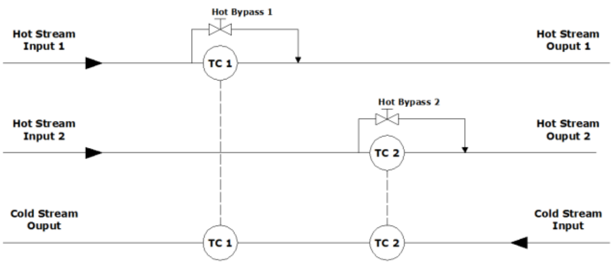

Figure 1. The HEN structure.

The variables related to the HEN are:

- Controllable variables: $\boldsymbol{TH_{OUT\ 1}}$ (hot stream 1 output) and $\boldsymbol{TH_{OUT\ 2}}$ (hot stream 2 output);
- Manipulated variables: $\boldsymbol{fh_{I\ 1}}$ (bypass valve position, hot stream 1) and $\boldsymbol{fh_{I\ 2}}$ (bypass valve position, hot stream 2);
- Disturbance variables: $\boldsymbol{TH_{IN\ 1}}$ (hot stream 1 input), $\boldsymbol{TH_{IN\ 2}}$ (hot stream 2 input) and $\boldsymbol{TC_{IN}}$ (cold stream input).

The mathematical model of the HE used to obtain the HEN arrangement proposed on Figure 1 was obtained through stream energy balance, as represented in Equations (1), (2), (3) and (4) (Novazzi, 2006):

$$\frac{dT_{H,i}}{dt} = (a_1 - a_2)T_{H,i-1} - (a_1 + a_2)T_{H,i} + a_2 T_{C,n-i} + a_2 T_{C,n-i+1} \quad (1)$$

$$\frac{dT_{C,n-i+1}}{dt} = a_4 T_{H,i-1} + a_4 T_{H,i} + (a_3 - a_4)T_{C,n-i} - (a_3 + a_4)T_{C,n-i+1} \quad (2)$$

$$\frac{dT_{Hby,i}}{dt} = a_5 T_{Hby,i-1} - a_5 T_{Hby,i} \quad (3)$$

$$\frac{dT_{Cby,n-i+1}}{dt} = a_6 T_{Cby,n-i} - a_6 T_{Cby,n-i+1} \quad (4)$$

where T is the stream temperature, m is the mass flow rate, ρ is the density, v is the relationship between volume and length of the exchanger, t is the time, z is the axial position, A is the heat transfer area, CP is the specific heat, V is the volume and U is the global heat transfer coefficient. The indices c and h refers to the variables related to the *cold* and to the *hot* stream. The indices *by* refers to the bypass and the parameters a_1 up to a_6 equals to

$$a_1 = n m_H (1 - f_H)/(\rho_H V_H), \qquad a_2 = UA/(2\rho_H V_H C_{p,H}),$$

$$a_3 = n m_C (1 - f_C)/(\rho_C V_C), \qquad a_4 = UA/(2\rho_C V_C C_{p,C}),$$

$$a_5 = n m_H f_H /(\rho_H V_{Hby}), \qquad a_6 = n m_C f_C /(\rho_C V_{Cby}),$$

Equations (2) up to (5) were discretized and solved by finite difference method, in Matlab (Novazzi, 2006). Note that the HE model is nonlinear due to the product between the a_3 parameter and T_C on Equation 2. The bypasses positions $\boldsymbol{f_H}$ and $\boldsymbol{f_C}$ are the manipulated variables that affect the controlled variables TH_{OUT} and TC_{OUT}. The simulations developed by using the HE equations demonstrated that the dynamics involved an be basically represented by first-order transfer functions with variable loop interaction. At least, the nominal variables, the physical dimensions of the HEN and the fluid characteristics are described in Table 1.

Table 1. Nominal values of the HEN

Variable	Description	Value
ρ	Fluid Density	Cold, Hot streams: 1000 kg.m^{-3}
CP	Specific Heat	Cold, Hot streams: 1000 J·kg^{-1}·ºC^{-1}
T	Input temperature	Cold stream: $TC_{IN\ 1}$ = 24 ºC Cold stream: $TC_{IN\ 2}$ = 28 ºC Hot stream: $TH_{IN\ 1}$ = 55 ºC Hot stream: $TH_{IN\ 2}$ = 55 ºC
U	Global coefficient of Heat Exchanger	U = 190 W·m^{-2}·ºC^{-1}
A	Heat Exchanger area	A = 0.1 m^2
n	Number of cells	*n=20*

3. THE H∞ CONTROLLER

During the development of a control system, the engineer aims to primarily stabilize the plant. Besides, it is also supposed to obtain a particular transient response, ensure the rejection of measurement noise, the improvement in the steady-state error and the robustness to variations in parameters of the plant model (Doyle and Stein, 1981).

For the development of these designs, one can apply classical control techniques which generally involve the study of plants with single input and single output (SISO), using analytical methods (Laplace transform, Routh criteria, etc.) and graphics (Nyquist, Bode, etc.). However, when the system has multiple inputs and multiple outputs (MIMO), the designer may find it difficult to apply classical control techniques (Doyle and Stein, 1981).

Several of the current techniques for multivariable linear control systems uses a plant model representation in state space and one of the most powerful of these techniques is the H∞ control. It allows dealing explicitly with robustness issues and the design specifications are usually presented in the frequency domain (Williams, 1991). The technique most commonly used for designing H∞ controllers is the so-called mixed sensitivity problem in which a sub-optimal problem is solved.

In this work the H∞ compensator is designed using the mixed sensitivity technique and the performance specifications refer to force the output temperature to follow set-point step changes with null steady state errors with a dynamic approximately to a first order system with a predetermined time constant and also that the closed loop system becomes approximately uncoupled. Note that, since the performance specifications are given in the time domain, they must be converted to equivalent specifications in the frequency domain. This work shows that a convenient way of doing this is solving a model matching problem using the mixed sensitivity technique.

The H∞ control problem can be expressed as an optimization problem in which the controller must be able to meet the requirements of robustness, performance and control effort. In the mixed sensitivity procedure this is done by molding the loop gain. Figure 2 show a typical open loop shape that was molded by means of the matrices $W_1(s)$ and $W_3(s)$ that are part of the formulation of the optimization problem.

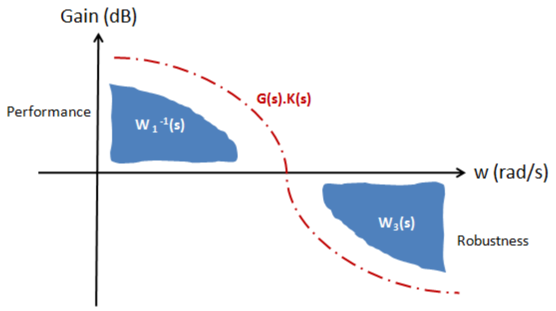

Figure 2. H∞ loop shaping.

The H∞ mixed sensitivity problem can be schematized by Figure 3 where $G(s)$ represents the plant and $K(s)$ the controller which stabilizes the plant and has the same number of states of it.

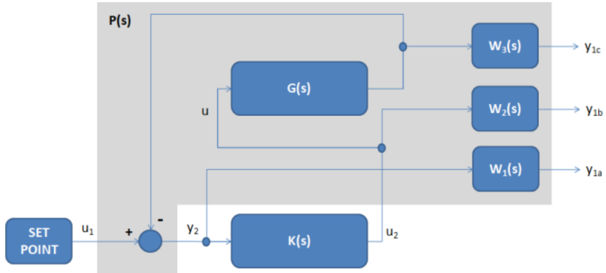

Figure 3. H∞ control with the W1(s), W2(s) and W3(s).

The augmented plant is obtained by inserting the requirements of robustness, performance and control effort by means of the weighting functions $W_1(s)$, $W_2(s)$ and $W_3(s)$, which penalize the error $E(s)$, the control effort $U(s)$ and the output $Y(s)$, respectively, where

$$Y(s) = \begin{bmatrix} Y_{1a}(s) \\ Y_{1b}(s) \\ Y_{1c}(s) \end{bmatrix}.$$

With the specifications previously defined, the design will find a H∞ controller $K(s)$ satisfying the inequality

$$\left\| \begin{matrix} W_1(s)S(s) \\ W_2(s)R(s) \\ W_3(s)T(s) \end{matrix} \right\|_\infty < 1, \qquad (5)$$

where $T(s)$ matrix is the complementary sensitivity function between the outputs and the input, $S(s)$ is the sensitivity function, and $R(s)$ is a function related to the control effort.

In addition to this traditional design in which the performance specifications are expressed indirectly by the matrix $W_1(s)$, it is possible that the controller $K(s)$ is obtained by a more direct performance specification. Figure 4 shows how to adapt the H∞ problem for this purpose through the matrix $G_{REF}(s)$ which is a reference matrix containing the expected dynamics for the closed loop. The control structure of Figure 4 is known as model matching (LEONARDI, 2002).

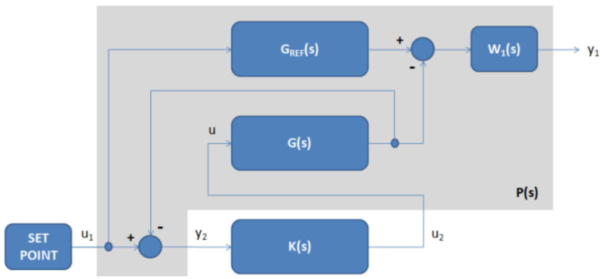

Figure 4. H∞ model matching.

The $W_1(s)$ matrix, in this case, is the penalty of the difference between the signals $Y_{REF}(s)$ and $Y(s)$, and because of the nature of $G_{REF}(s)$, the matrix $W_1(s)$ is often chosen as a merely constant matrix. The solution of the H∞ problem given by (6) solves the problem of determining the model matching controller $K(s)$ capable of making $T(s)$ close to $G_{REF}(s)$ with an accuracy given by the $W_1(s)$ matrix.

$$\left\| W_1(s)T(s) \right\|_\infty < 1. \qquad (6)$$

4. METHODOLOGY

This work, as pointed out before, intends to present a H∞ control solution for a HEN using a time domain specifications for the control design. The model matching problem ensures that the H∞ controller imposes approximately the same dynamics present on the reference model (Deshpande, 1989). If necessary, the control design can be improved by adding the $W_2(s)$ and $W_3(s)$ matrices to deal with control effort issues and robustness.

Then it is shown how the methodology was applied to control the HEN. The H∞ model matching design begins by defining the reference matrix

$$G_{REF}(s) = \begin{bmatrix} \dfrac{1}{110s+1} & 0 \\ 0 & \dfrac{1}{110s+1} \end{bmatrix}. \qquad (7)$$

$G_{REF}(s)$ represents the desirable closed loop dynamics, using a suitable time constant (110 seconds) for this experiment. The null values on $G_{REF}(s)$ matrix represent the desired decoupling between the two channels and the unity gains came from the null steady state error in tracking reference step signals. On the controller implementation, an anti-windup architecture (Aström *et al.*, 1988) was also used, resulting on the control structure shown in Figure 5.

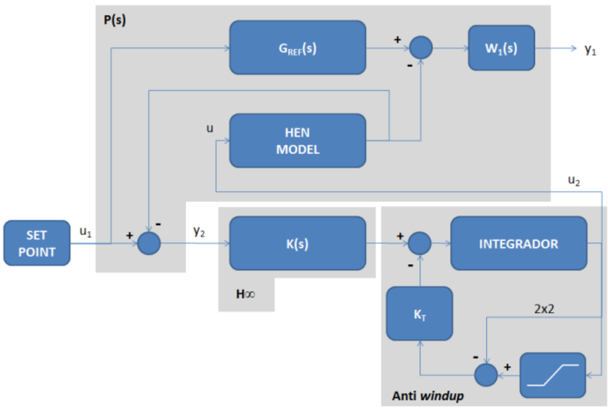

Figure 5. H∞ *model matching with Anti-windup.*

The HEN model $G(s)$ on Figure 4, was defined by linearizing the model proposed on section 2. The linearization was performed by using the Matlab, resulting on a 50th order model. A reduction order was applied, resulting on a 15th order model. The weighting matrix $W_1(s)$ = 50, constant over the entire frequency band, representing a 2% matching error and the $K(s)$ matrix obtained on the H∞ design is presented as follows, using the Matlab as support.

$$A_K = \begin{bmatrix} -0.603 & -16.620 & -0.714 & -4.870E^{-4} & 0 & 0 \\ 16.620 & -0.649 & -2.832E^{-3} & 0.646 & 0 & 0 \\ 0.714 & -2.836E^{-3} & -0.600 & -15.210 & 0 & 0 \\ 1.812E^{-4} & -0.646 & 15.210 & -0.651 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B_K = \begin{bmatrix} -1.173 & -2.633 \\ 1.050 & 2.682 \\ 2.572 & -0.993 \\ -2.517 & 1.121 \\ 0.897 & 0.467 \\ 0.465 & -0.896 \end{bmatrix}$$

$$C_K = \begin{bmatrix} -7.840E^{-3} & 0.125 & -2.754 & -2.756 & 0 & 0 \\ 2.882 & 2.878 & 0.132 & -6.697E^{-3} & 0 & 0 \end{bmatrix}$$

$$D_K = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

5. METHODOLOGY

The following results were obtained by numerical simulation and show the controller performance in view of performance specifications.

5.1. Cold mass flow (mc) disturbance rejection

On this first simulation, the disturbance was applied on the HEN at t = 500 sec. by a modification on the nominal cold mass flow (m_C) to m_C = 0,135kg.s^{-1}. The simulation time was 1500 sec. Figure 6 show the $TC_{OUT\,1}$ and $TC_{OUT\,2}$ plots and Figure 7 show the control effort demanded.

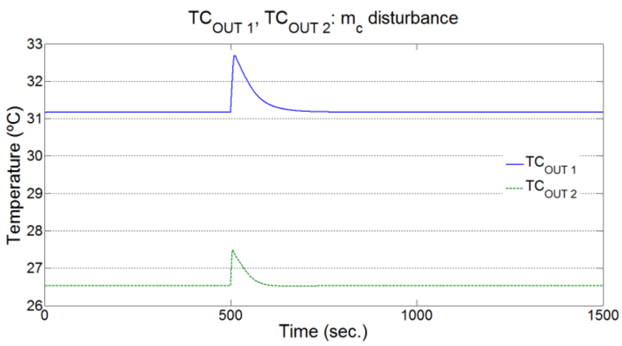

Figure 6. TC_{OUT1} e TC_{OUT2}: m_C disturbance.

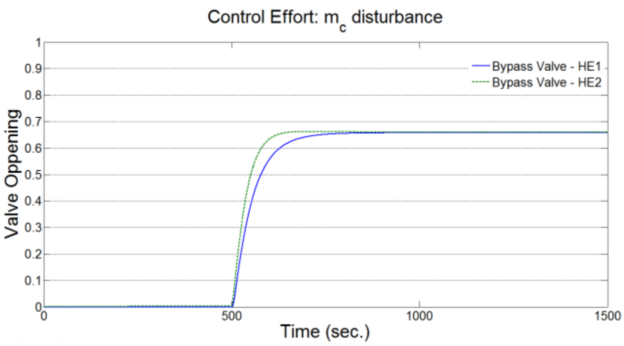

Figure 7. Control effort: m_C disturbance.

Analyzing the Figures 6 and 7, some conclusions could be attended about the HEN performance with the H∞ controller on cold flow mass disturbance:

- It is possible to notice that the controller rejected the disturbance in a suitable time, returning the output temperatures values $TC_{OUT\,1}$ and $TC_{OUT\,2}$ to its original setpoint value;
- To keep unchanged the cold output temperatures values, the bypass valves open diverting more cold fluid to the HE outlet, while a smaller fraction continues to flow internally to the exchanger.

5.2. Hot input temperature (THIN) disturbance rejection

The second simulation verified the controller performance on reject the TH_{IN} disturbance, by modifying the nominal (+4% step variation) values of the heat streams. The disturbance happened on the instants t = 500sec. and t = 1500 sec. and the simulation time was adjusted to 2500 sec. Figure 8 show the $TC_{OUT\,1}$ and $TC_{OUT\,2}$ plots and Figure 9 show the control effort demanded.

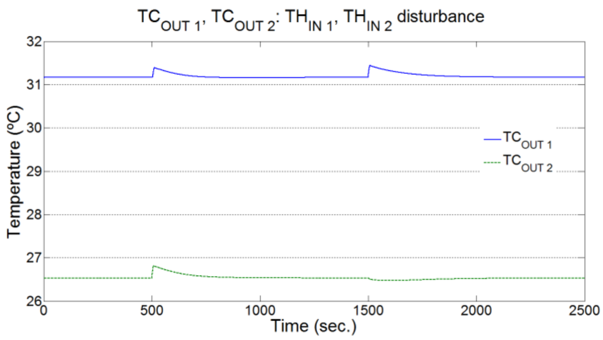

Figure 8. TC_{OUT1} e TC_{OUT2}: TH_{IN} disturbance.

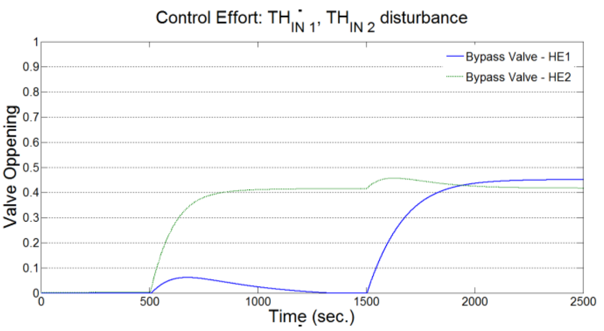

Figure 9. Control effort: TH_{IN} disturbance.

Analyzing the Figures 8 and 9, some conclusions could be attended about the HEN performance with the H∞ controller on hot input temperature disturbance:

- It is possible to notice that the controller rejected the disturbance in a suitable time, returning the output temperatures values $TC_{OUT\,1}$ and $TC_{OUT\,2}$ to its original setpoint value;
- To keep unchanged the cold output temperatures values, the bypass valves open diverting more cold fluid to the HE outlet, while a smaller fraction continues to flow internally to the exchanger.

5.3. Setpoint change

The third and last simulation demonstrated the controller performance on setpoint variation. The setpoint was changed on the instant t = 500 sec. and the simulation time was 1500 sec. The TC_{OUT} setpoints were adjusted to 31,3ºC and 26,8ºC to TC_{OUT1} and to TC_{OUT2}, respectively. Figure 10 show the $TC_{OUT\,1}$ and $TC_{OUT\,2}$ plots and Figure 11 show the control effort demanded.

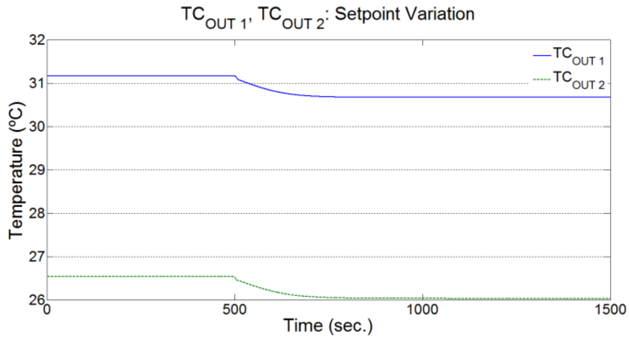

Figure 10. TC_{OUT1} e TC_{OUT2}: setpoint variation.

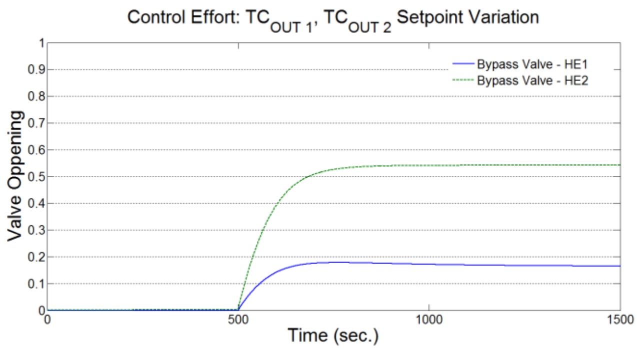

Figure 11. Control effort: setpoint variation.

Analyzing the Figures 10 and 11, some conclusions could be attended about the HEN performance with the H∞ controller on setpoint variation:

- The new requested setpoint values designed to TCOUT1 and to TCOUT2 were successfully achieved;
- The bypasses valves kept between its maximum and minimum values. No saturation was noticed.

6. CONCLUSIONS

In this work we have studied the use of H∞ control to provide a specified time domain performance for a heat exchanger network.

It was shown here how time domain specifications can be converted into an equivalent reference model and how the H∞ mixed sensitivity control technique can be adapted for the controller design.

Observing the H∞ model matching closed loop simulations responses applied to the HEN (Figures 6 up to 11), the following conclusions can be pointed out:

- The H∞ design was developed by using a non-traditional method. Normally, the specifications and the requirements for the traditional H∞ project are expressed on a frequency domain. The model matching technique, adopted on this paper, gives the possibility to design the H∞ controller by indirectly using time domain specifications;

- The H∞ model matching controller obtained to implement the closed loop control for the HEN presents a small order since we previously reduced the plant model order;
- In spite of H∞ not being a new control methodology, the results obtained suggested that it can be an interesting alternative when compared to other traditional techniques, such as PID. The H∞ presented an accurate performance to decouple the variables, associated with a short time to establish the outputs, as notice on Figures 6 up to 11;
- New studies with the HEN will be performed by including robustness issues on the H∞ design.

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