VOLTAGE ENVELOPE, NOISE AND HILBERT TRANSFORM

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ABSTRACT
The fluctuation of voltage is one of the important power quality events. Different methods have been proposed for estimating the voltage envelope, but the presence of noise is, in general, not considered. A method for estimating the envelope in presence of noise, based on the Hilbert transform and a low-pass filter, is presented. The results obtained from a real signal measured from an arc furnace are shown.

Keywords: Hilbert transform, flicker, voltage fluctuation

1. INTRODUCTION

Voltage fluctuations can be described as systematic variations or random variations in the voltage envelope. The fluctuation of voltage is one of the important power quality events due to the effects of electronic and control systems, and in the light flicker. There are several sources of voltage flickers as arc furnaces, fans, pumps, lifts, switching of powers factor capacitors, large motors (IEC 60038; Arrillaga, Watson, and Chen 2000).

Different methods have been proposed for estimating the magnitude and frequency of flicker. The IEC 61000-4-15 and IEEE 1453 standards recommend the square demodulation, a method used in demodulation of AM signals, which consists in tracking the flicker envelope by squaring the input voltage signal. Other methods proposed are Fast Fourier Transform (Schauder 1999), Least Absolute Value (Soliman, and El-Haway 2000), Kalman filters (Girgis, Stephens, and Makram 1995), Wavelet transform (Chen, and Meliopoulos 2000), Teager Energy Operator (Abdel-Galil, El-Saadany, and Salama 2002). Hilbert transform is also used (Abdel-Galil, El-Saadany, and Salama 2004; Su, and Wang 2008; Li, Zhao, and Han 2005; Marei, Abdel-Galil, and El-Saadany 2005), and, in particular, using Prony analysis and Hilbert Transform (Feilat 2006).

Recently, the performance of several flicker detecting methods were compared (Chen, Jia, and Zhao 2009). The core of flicker analysis is to track the envelope of voltage signal, that is, the instantaneous amplitude. Then, an important characteristic of the algorithms proposed is their on-line behavior; the faster is the estimation of the voltage envelope values, the better is the on-line behavior.

Another aspect of the problem is the presence of noise. In Tong, Yuan, Li, and Song 2008 this problem is pointed out and solved using the Hilbert transform for estimating the flicker envelope and the wavelet transform for extracting other noises contained in the voltage flicker. A signal obtained from an arc furnace shows that the high frequency noise can not be ignored. However, the influence of noise in the methods previously cited is not considered.

This paper is organized as follows: the use of the Hilbert transform for an efficient estimation of the voltage envelope is explained in Section 2, in Section 3 we describe the signal used to test the method, and finally Section 4 presents the numerical results.

2. ENVELOPE ESTIMATION USING THE HILBERT TRANSFORM

2.1. Estimating the envelope

In this section we review some results concerning the estimation of the envelope of a discrete signal.

The Hilbert transform is used in signal processing to derive the analytic representation of a signal $x[n]$. The analytic representation of a signal is well known for continuous-time signals (Carmona, Hwang, and Torresani 1998) and it is also defined for discrete signals as

$$z[n] = x[n] + i\, x_h[n]$$

Where $x_h[n] = \mathrm{H}\{x[n]\}$ denotes the discrete Hilbert transform of the sequence $x[n]$ (Li, Li, and Qian 2010). This representation allows a straightforward identification of the envelope of an amplitude modulated signal. An amplitude modulated signal is modeled by:

$$x[n] = a[n]\cos(\omega n) \qquad (1)$$

Where the frequency content of $a[n]$ has an upperbound less than ω. In this conditions, Bedrosian theorem for discrete signals (Li, Li, and Qian 2010) states that:

$$\mathrm{H}\{x[n]\} = a[n]\mathrm{H}\{\cos(\omega n)\},$$

which turns into $\mathrm{H}\{x[n]\} = a[n]\sin(\omega n)$. Now, the analytic representation of the signal takes the simple form

$$z[n] = a[n]e^{i\omega n},$$

and the amplitude (or the envelope) $a[n]$ is easily obtained from $a[n] = |z[n]|$.

For the sake of completeness, we include Bedrosian theorem as stated in (Li, Li, and Qian 2010):

Theorem 1: Suppose that $z_1[n]$ and $z_2[n]$ are complex sequences with discrete-time Fourier transforms $Z_1(e^{i\omega})$ and $Z_2(e^{i\omega})$. Then

$$\mathrm{H}\{z_1[n]z_2[n]\} = z_1[n]\mathrm{H}\{z_2[n]\}$$

if there exists a nonnegative number $\sigma < \pi$ such that

$$Z_1(e^{i\omega}) = 0, \text{ for } 0 < \sigma < |\omega| < \pi, \text{and}$$
$$Z_2(e^{i\mu}) = 0, \text{ for } 0 < |\mu| \le \sigma < \pi.$$

2.2. Hilbert filter

In this section we describe the Hilbert filter in more detail.

The discrete Hilbert transform $\mathrm{H}\{x[n]\}$ of the sequence $x[n]$ is defined in the frequency domain as (Hahn 1996)

$$X_h(\omega) = F\{\mathrm{H}\{x[n]\}\} = -i\,\mathrm{sgn}(\omega)X(\omega) \quad (2)$$

where $X(\omega)$ is the discrete Fourier transform of x:

$$X(\omega) = F\{x[n]\} = \sum_{n=-\infty}^{\infty} x[n]e^{-i\omega n}$$

From equation (2), the transfer function of the Hilbert transform for discrete signals is

$$H(\omega) = \begin{cases} -i, & 0 < \omega < \pi \\ 0, & |\omega| = \pi \\ i, & -\pi < \omega < 0 \end{cases}$$

$$H(\omega) = -i\,\mathrm{sgn}[\sin(\omega)] = G(\omega)e^{i\frac{\pi}{2}}$$

with $G(\omega) = -\mathrm{sgn}[\sin(\omega)]$. The discrete time representation of the Hilbert filter is easily obtained from this expression. In fact, $G(\omega)$ is an odd function whose Fourier transform reads

$$G(\omega) = \frac{4}{\pi}\sum_{m=0}^{\infty}\frac{1}{2m+1}\sin[(2m+1)\,\omega]$$

Denoting $h(k) = F^{-1}\{H(\omega)\}$ the inverse Fourier transform of $H(\omega)$, we have

$$h(k) = \frac{2}{\pi k}\sin k\frac{\pi}{2},\ k \ge 0, \text{ and } h(-k) = -h(k)$$

Figure 1 shows the impulse response $h[k]$ of a Hilbert filter of order 38 and Figure 2 shows the magnitude response $|H(\omega)|$.

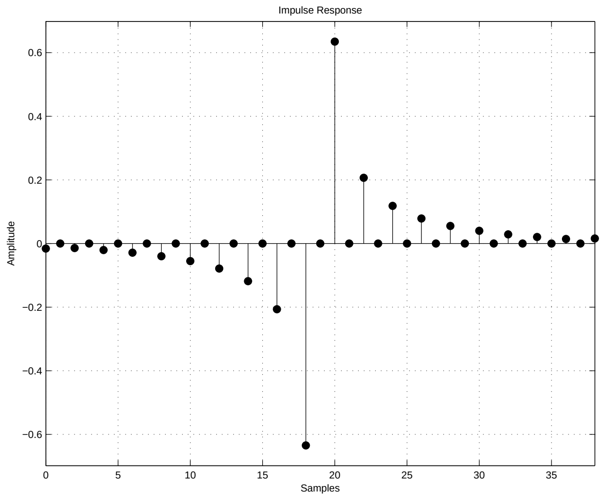

Figure 1: Impulse response

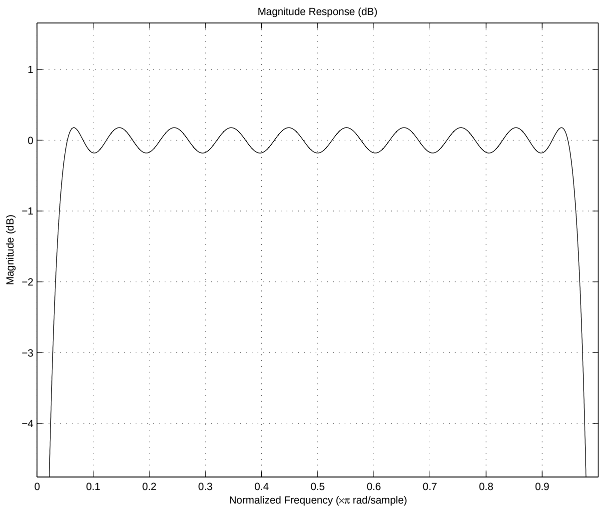

Figure 2: Magnitude response

2.3. Implementation of the Hilbert filter

The Hilbert transform can be easily implemented in the discrete domain by an FIR filter. This kind of discrete filter allows obtaining constant group delay and constant phase over the entire bandwidth. Moreover, the nature of the filter makes unnecessary the stability analysis.

Moreover, if the filter order is even, it behaves as a band pass filter which has zeros at 0 Hz and at the Nyquist frequency. So that its impulse response is similar to that shown in Figure 1, in which the odd coefficients are zero. However, if the filter order is odd, the zero in the Nyquist frequency disappears and the odd coefficients are no longer zero.

For this reason, the Hilbert filter of even order is easier to implement than the filter of odd order. This is because the zero coefficients can be omitted. Therefore fewer multiplications and additions are required (see figure 3).

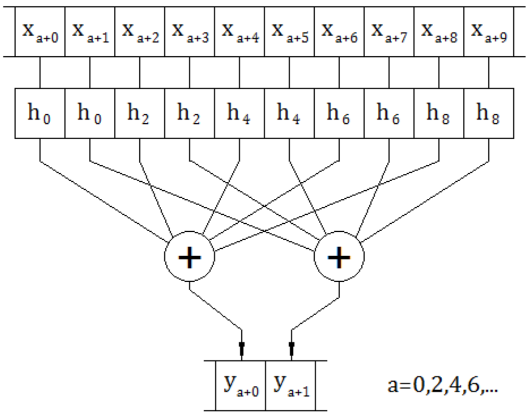

Figure 3: Convolution strategy

In this paper, the calculation of the coefficients is performed with the "Filter Design & Analysis Tool" in MATLAB

3. A SIGNAL FROM AN ARC FURNACE

In this paper, the performance of the proposed model on tracking the voltage flicker signal envelope is examined with a signal coming from real measurements. It is a typical AC arc furnace application in a steel plant. This arc furnace is served from a 13.8 kV bus. The measured signal is one of the phase voltages and it was sampled at a sampling frequency of 1000 Hz.

Since this signal is distorted by noise that comes from making physical measurements, a serious issue is the robustness of the method for estimating flicker. Previous works hardly consider this problem.

As an example, we make some comments on Prony algorithm which has been used in this problematic (Feilat 2006). Prony algorithm is good at system identification provided that the available samples come from a signal completely predictable and free of any randomness. Under these conditions, it is a good alternative for obtaining mathematical models in the form of damped complex exponentials from a small number of samples. This type of representation allows a straightforward calculation of the Hilbert transform of the signal (Feilat 2006). However, when the signals have some degree of randomness like signals immersed in noise, Prony algorithm is very unstable and it requires a large amount of samples to achieve an acceptable approximation. This method has some other drawbacks: it is very difficult to estimate the optimal number of exponentials to use in the approximation, the calculation involves two pseudo-inverse matrices whose systems are poorly conditioned, which increases the instability, and finally it has a high computational cost.

4. NUMERICAL RESULTS

We present numerical results on the estimation of voltage envelope of the signal from an arc furnace which was described in the previous section.

The method is implemented in Simulink from MATLAB (see Figure 4).

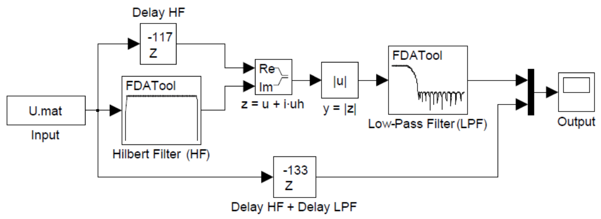

Figure 4: Block diagram of the estimator

From the block diagram Figure 4 we can make the following analysis:

1. The Hilbert filter is a FIR, all zeros filter of order 234, linear phase, whose magnitude response are shown in Figure 5.

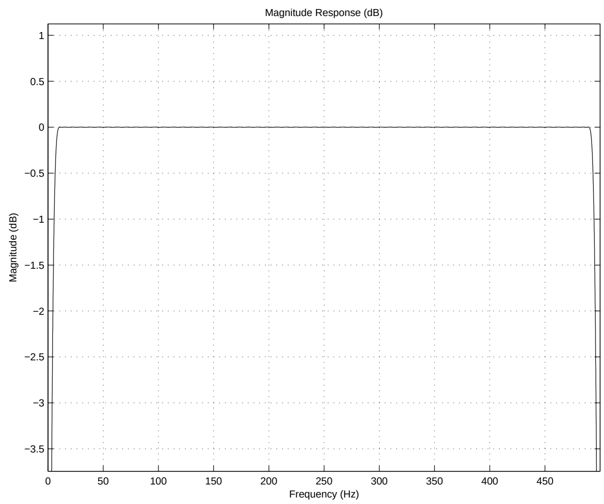

Figure 5: Magnitude response of Hilbert filter

2. The envelope is estimated, then a low-pass filter is applied to minimize the influence of noise. This is a FIR equiripple filter of order 32, with cutoff frequency 120 Hz.
3. Because of the fact that the signal passes through two FIR filters, there is a delay time in the tracking processes. However, as the two filters have a linear phase response, these have a constant group delay response. Therefore, this time delay is constant for all frequencies and it can be calculated. In this case, it resulted in a total delay of 133 samples, and it represents 0.133 sec at a sampling frequency of

1000Hz. It represents approximately 8 cycles of the fundamental frequency (60Hz).

In Figure 6 and 7 are shown estimations of the voltage envelope corresponding to the diagram of Figure 4 using the measurement of the arc furnace voltage. There are two intervals of 0.5 sec: [18.5,19] and [19,19.5].

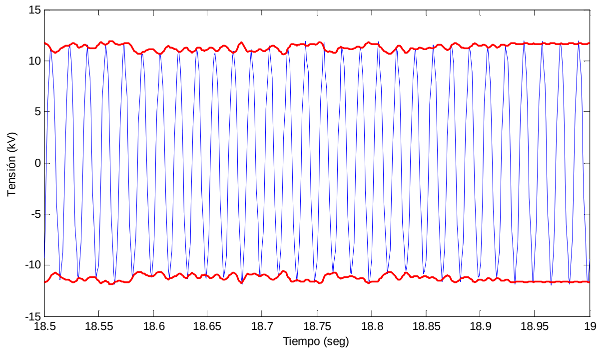

Figure 6: The signal and its estimated envelope.

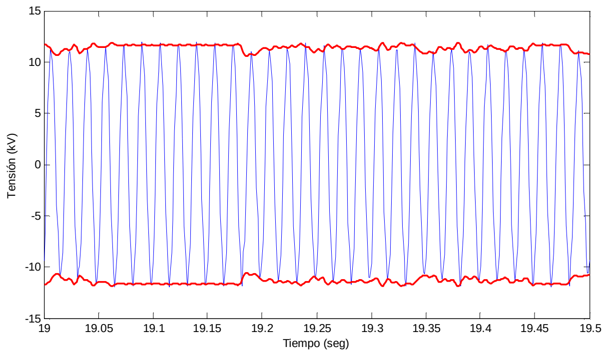

Figure 7: The signal and its estimated envelope.

5. CONCLUSIONS

We have presented a method for estimating signal flicker in presence of noise using the Hilbert transform. The characteristics of both filters (the Hilbert filter and the low pass filter) are described in Section 4. The proposed algorithm was implemented in a DSP Blackfin EZ-537. We tested this technique on a real world signal produced by an arc-furnace. In contrast to the method presented in (Feilat 2006), this algorithm does not have instabilities, as was analyzed in section 4.

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REFERENCES

International Standard IEC 60038:2009. IEC standard voltages

International Standard IEC 61000-4-15:1999. Flickermeter: Functional and Design Specifications.

IEEE Standard 1453:2004. IEEE Recommended Practice for Measurement and Limits of Voltage Fluctuations and Associated Light Flicker on AC Power Systems.

Abdel-Galil, T.K., El-Saadany, E.F., and Salama, M.M., 2002. Energy operator for on-line tracking of voltage flicker levels. *Proceeding of IEEE/PES Winter Meeting, 3:1153–1157.* January 27-31, New York (New York, USA).

Abdel-Galil, T.K., El-Saadany, E.F., and Salama, M.M., 2004. On-line tracking of voltage flicker utilizing energy operator and Hilbert transform. *IEEE Transactions on Power Delivery* 19:861–867.

Arrillaga, J., Watson, N.R., and Chen, S., 2000. *Power System Quality Assessment*. Ney York: Wiley.

Carmona, R., Hwang, W-L., and Torresani, B., 1998. *Practical Time-Frequency Analysis, Gabor and Wavelet Transforms with an Implementation in S.* Academic Press.

Chen, M.T., and Meliopoulos, A.P.S., 2000. Wavelet-based algorithm for voltage flicker analysis. *Proceedings of the IEEE 9th International Conference on Harmonics and Quality of Power*, 2:732–738. October 1-4, Orlando (Florida, USA).

Chen, Q., Jia, X., and Zhao, C., 2009. Analysis on Measuring Performance of Three Flicker Detecting Methods. *Proceeding of IEEE/PES General Meeting*, 1–7. July 26-30, Calgary (Alberta, Canada)

Feilat, E.A., 2006. Detection of voltage envelope using Prony analysis-Hilbert transform method. *IEEE Transactions on Power Delivery* 21(4):2091–2093

Girgis, A.A., Stephens, J.W., and Makram, E.B., 1995. Measurement and prediction of voltage flicker magnitud and frequency. *IEEE Transactions on Power Delivery* 10(3):1600–1605.

Hahn, S.L., 1996. *Hilbert Transform in Signal Processing*. Artech House.

Li, H., Li, L., and Qian, T., 2010. Discrete-time analytic signals and bedrosian product theorems. *Digital Signal Processing* 20:982–990.

Li, T., Zhao, Y., and Han, Y., 2005. Application of Hilbert-Huang transform method in detection of harmonic and voltage flicker. *Power System Technology* 29:74–77.

Marei, M.I., Abdel-Galil, T.K., and El-Saadany, E.F., 2005. Hilbert transform based control algorithm of the DG interface for voltage flicker mitigation. *IEEE Transactions on Power Delivery* 20(2):1129–1133.

Schauder, C., 1999. STATCOMfor compensation of large electric arc furnace instalations. *Proceeding of IEEE/PES Summer Meeting*, 2:1109–1112. July 18-22, Edmonton (Alberta, Canada).

Soliman, S.A., and El-Haway, M.E., 2000. Measurements of power system voltage and flicker levels for power quality analysis: a static LAV state estimation based algorithm. *International Journal of Electrical Power and Energy Systems* 22:447–450.

Su, H., and Wang, Y., 2008. Voltage flicker detection method based on mathematical morphology filter and Hilbert transform. *Proceeding of Canadian Society for Ecology and Evolution*, 28:111–114. May 11-14, Vancouver (British Columbia, Canada).

Tong, W., Yuan, S., Li, Z., and Song, X., 2008. Detection of Voltage Flicker Based on Hilbert Transform and Wavelet Denoising. *Proceeding of the 3th International Conference on Electric Utility Deregulation and Restructuring and Power Technologies*, 2286–2289. April 6-9, Nanjing (Jiangsu, China).