FAULT DIAGNOSIS IN NCS UNDER COMMUNICATION CONSTRAINTS: A QUADROTOR HELICOPTER APPLICATION

K. Chabir, M. A. Sid, D. Sauter

Nancy University, CRAN -CNRS UMR 7039 BP239, 54506 Vandoeuvre Cedex , France.

{karim.chabir, Mohamed-Amine.sid, dominique.sauter}@cran.uhp-nancy.fr

ABSTRACT
In this paper a method for fault diagnosis in quadrotor helicopter is presented. The proposed approach is composed of two stages. The first stage is the modelling of the system attitude dynamics taking into account the induced communication constraints. Then a robust fault detection and evaluation scheme is proposed using a post-filter designed under a particular design objective. This approach is compared with previous results based on the standard Kalman filter and gives better results for sensors fault diagnosis.

Keywords: Networked control systems, Diagnosis, generation residual, evaluation residual, Quadrator helicopter.

1. INTRODUCTION

Unmanned Aerial Vehicles (UAV) are receiving a great deal of attention during the last few years due to their high performance in several applications such as search and critical missions, surveillance tasks, geographic studies and various military and security applications. As an example of UAV systems, the quadrotor helicopter is relatively a simple, affordable and easy to fly system and thus it has been widely used to develop, implement and test-fly methods in control, fault diagnosis, fault tolerant control as well as multi-agent based technologies in formation flight. Navigation and guidance algorithms may be embedded on the onboard flight microcomputer/microcontroller or with the interference by a ground wirelesses/wired controller in others cases. In our setting the quadrotor is controlled over real time communication network with time-varying delays and therefore is considered as a Networked control system (NCS). In general NCS is composed of a large number of interconnected devices (system nodes) that exchange data through communication network. Recent research on NCS has received considerably attention in the automatic control community (Zhang, et al., 01; Tipsuwan and Chow, 03; Huajing et al., 07; Mirkin and Palmor, 05; Hespanha, et al., 07; Richard, 03). The major focus of the research activities are on system performance analysis regarding the technical properties of the network and on the controller design schemes for NCS.

However, the introduction of communication networks in the control loops makes the analysis and synthesis of NCS complex. There are several network-induced effects that arise when dealing with the NCS, such as time-delays (Niculescu, 00; Nilsson, et al., 98; Pan, et al., 06; Schollig, et al., 07; Dritsas, and Tzes, 07; Yi, et al., 06; Zhang, et al., 05; Behrooz, et al.,08), packet losses (Xiong, and Lam, 06; Sahebsara, et al., 07; Yu, et al., 04; Li, et al., 06) and quantization problems (Goodwin , et al., 04; Montestruque and Antsaklis, 07; Frank and Ding, 97). Because of the inherent complexity of such systems, the control issues of NCS have attracted attention of many researchers, particulary taking into account network-induced effects. Typical application of these systems ranges over various fields, such as automotive, mobile robotics, advanced aircraft.

The fault diagnosis has become an important subject in modern control theory (Frank and Ding, 97; Gertler, 98; Isermann, 06; Stoustrup, and Zhou, 08; Basseville, and Nikiforov, 93). The study of fault detection (FD) in NCS is a new research topic, which gained more attention in the past years. For instance, the results in (Sauter and Boukhobza, 06; Sauter, et al., 07 ; Llanos, et al., 07; Chabir, et al., 08; Chabir, et al., 09; Chabir, et al., 10; Al-Salami, et al., 08) are focus on networked-induced delays. The problem studied in (Zhang, et al., 04; Wang, et al., 06) is the analysis and design of FD systems in case of missing measurements. The fault detectability and isolability in NCS have been discussed in (Sauter, et al., 09; Chabir, et al., 09). The fault tolerant structure is studied in (Ding and Zhang, 07 ; Patton, et al., 07; Kambhampati, et al., 06).

Delays are known to degrade drastically the performances of a control systems, for this reason, many works aimed at reducucing the effects of induced network delays on NCS (Tipsuwan and Chow, 03; Yu, et al., 04; Li, et al., 06; Goodwin , et al., 04). In the majority of the studies concerning the stabilization of networked control systems, the delay is considered to be constant (Schollig, et al., 07) or bounded (Dritsas, and Tzes, 07), but the dynamics of the delay corresponding to the characterization of the network is not taken into account in general. Thus, it is interesting to estimate the delay, in order to generate an optimal control, as well as algorithms of faults detection that take into account the network characteristics. One approach is to consider the delay as a Markov chain (Yi, et al., 06; Zhang, et al., 05). In order to predict such a random delay, artificial neural networks can be used (Zhang, et al., 05). However, such a methods are considered to be not suitable for real time implementation (Behrooz, et al.,08).

The objective in this study is diagnosis of quadrotor attitude sensors fault under variable transmission delay. First, attitude dynamics model taking into account the variables transmission delay is presented. Then we propose a robust residual generation and evaluation scheme using a post-filter that verify a particular design objective. This approach is compared with previous results based on the standard Kalman filter and gives better results for sensors fault diagnosis.

The rest of the paper is organized as follows. In section 2, the quadrotor helicopter attitude dynamics is modeled and then controlled using LQR approach. Section 3, presents the first main result of this paper, which is related to the modeling of networked control systems. Finally, section 4 we present our second main result concerned with the residual generation and evaluation using an adaptive threshold. The paper is concluded in Section 5

2. DESCRIPTION OF QUADROTOR HELICOPTER DYNAMICS

The mini-helicopter under study has four fixed-pitch rotors mounted at the four ends of a simple cross frame Figure 1. The attitude is modeled with the Euler-angle representation which provides an easier expression for the linearized model. Moreover the Euler-angle representation is more intuitive. The inertial measurement unit model is given with the quaternion representation of the attitude. This choice is govern by the implementation of the attitude observer that will be easier with the quaternion parameterization of the attitude.

Figure 1: The quadrator mini-helicopte.

2.1. Quadrotor model

The quadrotor is a small aerial vehicle controlled by the rotational speed of four blades, driven by four electric motors (3) A quadrotor is considered a VTOL vehicle (Vertical Take Off and Landing) able to hover. Two frames are considered to describe the dynamic equations: the inertial frame $N(x_n, y_n, z_n)$ and the body frame $B(x_b, y_b, z_b)$ attached to the UAV with its origin at the centre of mass of the vehicle.

The quadrotor orientation can be parameterized by three rotation angles with respect to frame N: yaw (ψ), pitch (θ) and roll (Φ). $\omega \in \Re^3$ is the angular velocity of the quadrotor relative to N expressed in B. The quadrotor is controlled by independently varying the rotational speed ω_{mi}, $i = 1{:}4$, of each electric motor. The force f_i and the relative torque Q_i produced by motor i are proportional to ω_{mi}.

$$f_i = b\,\omega_{mi}^2 \tag{1}$$

$$Q_i = k\,\omega_{mi}^2 \tag{2}$$

where $k > 0$, $b > 0$ are two parameters depending on the density of air, the radius, the shape, the pitch angle of the blade and other factors.

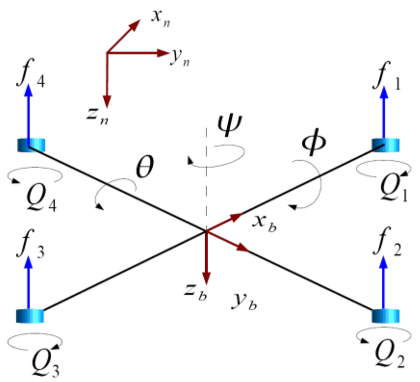

Figure 2: Quadrator mini-helicopte configuration: the inertial frame $N(x_n, y_n, z_n)$ and the body frame $B(x_b, y_b, z_b)$.

The three torques that constitute the control vector for the quadrotor are expressed in frame B as:

$$\tau_a^{\phi} = d\left(f_2 - f_4\right) \tag{3a}$$

$$\tau_a^{\theta} = d\left(f_1 - f_3\right) \tag{3b}$$

$$\tau_a^{\psi} = Q_1 + Q_3 - Q_2 - Q_4 \tag{3c}$$

d represents the distance from one rotor to the centre of mass of the quadrotor. From Newton-Euler approach, the kinematics and dynamic equations of the quadrotor are:

$$\left(\dot{\phi}, \dot{\theta}, \dot{\psi}\right)^T = M\,\omega \tag{4}$$

$$I_f\,\dot{\omega} = -\omega \times I_f\,\omega + \tau_a + G_a \tag{5}$$

where $I_f \in \Re^{3\times 3}$ represents the constant inertial matrix expressed in B (supposed to be $I_f = \mathrm{diag}(I_{fx}, I_{fy}, I_{fz})$) and $\times$ in (5) denotes the cross product. Matrix M is defined with

$$M = \begin{pmatrix} 1 & \tan\theta\sin\phi & \tan\theta\cos\phi \\ 0 & \cos\phi & -\sin\phi \\ 0 & \dfrac{\sin\phi}{\cos\theta} & \dfrac{\cos\phi}{\cos\theta} \end{pmatrix} \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix} \tag{6}$$

The gyroscopic torques G_a due to the combination of the rotation of the quadrotor and the four rotors, are modeled as:

$$G_a = \sum_{i=1}^{4} I_r(\omega \times e_z)(-1)^{i+1}\omega_{mi} \quad (7)$$

Ir is the inertia of the so-called rotor (composed of the motor rotor itself, of the shape and of the gears).

A linear control law that stabilizes around hover conditions the system described by the non-linear model (4) and (5) is established. Note that nonlinearities are second order, therefore it is reasonable to consider a linear approximation. From (4) and (5) and for hover condition ($\phi \approx \theta \approx \psi \approx 0$), it comes:

$$(\phi', \theta', \psi')^T = (\omega_1, \omega_2, \omega_3)^T \quad (8)$$

Then the dynamical model is obtained in terms of Euler angles

$$\phi'' = \theta'\psi'\left(\frac{I_{fy} - I_{fz}}{I_{fx}}\right) + \frac{\tau_a^\phi}{I_{fx}} \quad (9a)$$

$$\theta'' = \phi'\psi'\left(\frac{I_{fz} - I_{fx}}{I_{fy}}\right) + \frac{\tau_a^\theta}{I_{fy}} \quad (9b)$$

$$\psi'' = \phi'\theta'\left(\frac{I_{fx} - I_{fy}}{I_{fz}}\right) + \frac{\tau_a^\psi}{I_{fz}} \quad (9c)$$

The gyroscopic torques G_a are not considered for the design of the control law. However, they will be considered in simulations in order to analyze the robustness features.

2.2. Attitude control

In this section, the linearized model of (4) and (5) is first derived. Then a control law is briefly summarized. Note that this paper is not dedicated to the determination of a particular control law (see for instance (Guerrero-Castellanos, *et al.*, 07; Tayebi and McGilvray, 06). Therefore a LQ controller is implemented. In the third subsection, the estimation of the network induced delay with an Extended Kalman Filter is considered. This technique is then applied to the Network controlled quadrotor. Define the state variable:

$$x^T = (\phi, \phi', \theta, \theta', \psi, \psi')^T \quad (10)$$

The system (9) linearization around the hover conditions is:

$$\dot{x}(t) = Ax(t) + Bu(t) \quad (11)$$

where

$$A = \begin{pmatrix} A_0 & 0 & 0 \\ 0 & A_0 & 0 \\ 0 & 0 & A_0 \end{pmatrix}, \quad B = \begin{pmatrix} B_x & 0 & 0 \\ 0 & B_y & 0 \\ 0 & 0 & B_z \end{pmatrix}, \quad A_0 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

$$\text{and } B_i = \begin{pmatrix} 0 \\ 1/I_{fi} \end{pmatrix} \quad (12)$$

The attitude stabilization problem is to drive the quadrotor attitude from any initial condition to a desired constant orientation and maintain it thereafter. As a consequence, the angular velocity vector is also brought to zero and remains null once the desired attitude is reached, $x \to 0$, $t \to \infty$. The discrete linear controller is given by

$$u(kh) = -Lx(kh) \quad (13)$$

and the plant is modeled as:

$$x_{k+1} = \Phi x_k + \Gamma u_k \quad (14)$$

satisfy the system dynamics constraints:

$$J = \sum_{k=0}^{N-1}\left[x_k^T Q_d x_k + u_k^T R_d u_k\right] + x_N^T Q_0 x_N \quad (15)$$

where:

$$\Phi = e^{Ah}, \qquad \Gamma = \int_{kh}^{(k+1)h} e^{As} B ds,$$

$$Q_d = \int_{kh}^{(k+1)h} \Phi^T(s) Q \Phi(s) ds, \quad \text{and}$$

$$R_d = \int_{kh}^{(k+1)h} \left(\Gamma^T(s) Q \Gamma(s) + R\right) ds \quad (16)$$

where matrices Q_d, R_d and Q_0 are symmetric and positive definite. Furthermore, the following assumptions are done.

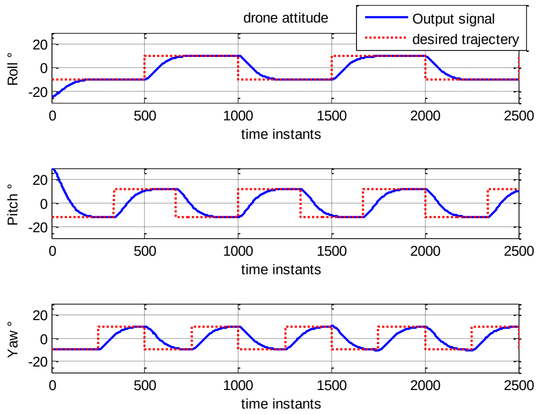

Figure 3: Quadrotor attitude (ϕ, θ, ψ) and reference.

Assumption 1: *The full state vector is available (angles and angular velocities). In practice, these variable states are obtained by merging the measurements of rate gyros, accelerometers and magnetometers using a dedicated attitude observer* (Guerrero-Castellanos, et al., 07).

Assumption 2: *A periodic sampling is used.*

Assumption 3: *The control signals remain constant between two updates.*

Proposition 1: *Consider the quadrotor rotational dynamics described by (9). Then, the discrete control* $u =$ *defined by:*

$$\begin{aligned} u(kh) &= \left[\tau_a^\phi(kh) \quad \tau_a^\theta(kh) \quad \tau_a^\psi(kh)\right]^T \\ &= -Lx(kh) \end{aligned} \quad (17)$$

which satisfies (14) while minimizing (15) locally stabilizes the quadrotor at $x = 0$.

Remark 1: The weighting matrices Q_d and R_d are chosen in order to obtain a suitable transient response, while only feasible control signals are applied to the actuators. Then for a sampling time h = 0.01s the matrix gain is.

$$L = \begin{bmatrix} 0.0352 & 0 & 0 \\ 0.0284 & 0 & 0 \\ 0 & 0.0352 & 0 \\ 0 & 0.0284 & 0 \\ 0 & 0 & 0.0352 \\ 0 & 0 & 0.0284 \end{bmatrix}^T \quad (18)$$

Here we simply present some results of the drone attitude simulation with a variable step response (Figure 3) and the LQ controller signal (Figure 4)

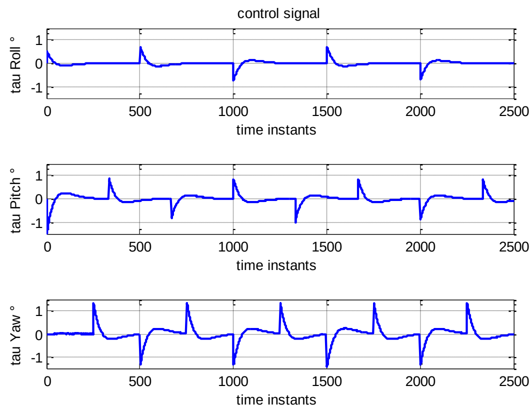

Figure 4: Control signal.

3. NCS MODEL AND TRANSFORMATION

Induced time delays in networked controlled systems can become a source of instability and degradation of control performance (Yi, *et al.*, 06; Zhang, *et al.*, 05; Behrooz, *et al.*, 08; Xiong and Lam, 06; Sahebsara, *el al.*, 07). When the system is controlled over a network, we have to take into account the sensor to controller delays and controller to actuator delays. Note that delays, in general, cannot be considered as constant and known. Network induced delays may vary, depending on the network traffic, medium access protocol and the hardware.

Assumption 4. *For data acquisition it is supposed that the sensor is time-driven and the sampling period is denoted by h. Both the controller and the actuator are event-driven. We mean that calculation of the new control or actuator signal is started as soon as the new control or actuator information arrives as illustrated in Fig. 5*

Assumption 5. *The unknown time-varying network induced delay at time step k is denoted by* τ_k *and* $\tau_k = \tau_k^{sc} + \tau_k^{ca}$ *is smaller than one sampling period* $\tau_k \le h$, τ_k^{sc} *and* τ_k^{ca} *are the sensor-to-controller delay and the controller-to-actuator delay, respectively. There is no packet dropout in the networks.*

Thus, the control input (zero-order hold assumed) over a sampling interval $[kh, (k+1)h]$ is:

$$u_t = \begin{cases} u_{k-1}, & t \in [kh,\ kh+\tau_k) \\ u_k, & t \in [kh+\tau_k,\ (k+1)h] \end{cases} \quad (19)$$

Let us first assume that the residual generation and evaluation algorithms are executed instantaneously at every sampling period k. Based on this assumption, if the control input is kept constant over each sampling interval h, and if we consider that fault inputs present slow dynamics, the discrete time system can be described by:

$$\begin{cases} x_{k+1} = \Phi x_k + \Gamma_{0,\tau_k} u_k + \Gamma_{1,\tau_k} u_{k-1} \\ y_k = C x_k \end{cases} \quad (20)$$

where

$$\Gamma_{0,\tau_k} = \int_0^{h-\tau_k} e^{As} B ds, \Gamma_{1,\tau_k} = \int_{h-\tau_k}^{h} e^{As} B ds \quad (21)$$

Like $\quad \Gamma = \int_0^h e^{As} \mathrm{B} ds = \Gamma_{0,\tau_k} + \Gamma_{1,\tau_k} \quad$ thus

$$\Gamma_{0,\tau_k} = \Gamma - \Gamma_{1,\tau_k} \quad (22)$$

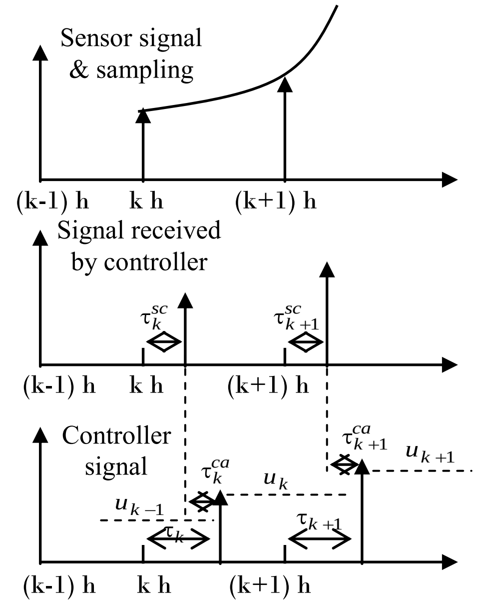

Figure 5: Timing diagram for data communication.

According to the property of definite integral, If we introduce the control increment $\Delta u_k = u_k - u_{k-1}$, let the plant (20) with unknown disturbance vector, d_k and fault vector, f_k which must be detected, is described by:

$$\begin{cases} x_{k+1} = \Phi x_k + \Gamma u_k + \Gamma_{1,\tau_k} \Delta u_k + \Xi_x d_k + \Psi_x f_k \\ y_k = C x_k + \Xi_y d_k + \Psi_y f_k \end{cases} \quad (23)$$

where $f_k \in \mathbb{R}^q$ the fault vector and $d_t \in \mathbb{R}^q$ the noise vector.

Suppose that the matrix A is called diagonalizable if P is invertible

$$\mathrm{A} = \mathrm{P}\Lambda\mathrm{P}^{-1} = \mathrm{P}diag\left(\lambda_1, \cdots, \lambda_n\right)\mathrm{P}^{-1} \tag{24}$$

where $\lambda_1, \cdots, \lambda_n$ are eigenvalues of matrix A, then there is:

$$\begin{aligned} e^{At} &= I + At + \cdots + \frac{1}{n!}A^n t^n \\ &= PP^{-1} + P\Lambda P^{-1}t + \cdots + \frac{1}{n!}\left(P\Lambda P^{-1}\right)^n t^n \\ &= P\left(I + \Lambda t + \cdots + \frac{1}{n!}\Lambda^n t^n \right)P^{-1} \\ &= Pe^{\Lambda t}P^{-1} \end{aligned} \tag{25}$$

Then, with (23), we have that:

$$\begin{aligned} \Gamma_{1,\tau_k}\Delta u_k &= \int_{h-\tau_k}^{h} Pe^{\Lambda t}P^{-1}Bds\ \Delta u_k \\ &= P\int_{h-\tau_k}^{h} e^{\Lambda t}dsP^{-1}B\Delta u_k \end{aligned}$$

$$= P\begin{bmatrix} \int_{h-\tau_k}^{h} e^{\lambda_1 t}ds & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \int_{h-\tau_k}^{h} e^{\lambda_n t}ds \end{bmatrix} P^{-1}B\Delta u_k \tag{26}$$

$$= P\begin{bmatrix} \frac{1}{\lambda_1}e^{\lambda_1 h} & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \frac{1}{\lambda_n}e^{\lambda_n h} \end{bmatrix} P^{-1}B\Delta u_k$$

$$-P\begin{bmatrix} \frac{1}{\lambda_1}e^{\lambda_1(h-\tau_k)} & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \frac{1}{\lambda_n}e^{\lambda_n(h-\tau_k)} \end{bmatrix} P^{-1}B\Delta u_k \tag{27}$$

$$\begin{aligned} &= \Gamma_\Delta \Delta u_k - Pdiag\left(\frac{1}{\lambda_{11}}, \frac{1}{\lambda_2}, \cdots \frac{1}{\lambda_n}\right) \\ &\times diag\left(\beta_k\right)\begin{bmatrix} e^{\lambda_1(h-\tau_k)} \\ e^{\lambda_2(h-\tau_k)} \\ \vdots \\ e^{\lambda_n(h-\tau_k)} \end{bmatrix} \end{aligned} \tag{28}$$

$$= \Gamma_\Delta \Delta u_k - \Gamma_{\Delta,k}\begin{bmatrix} e^{\lambda_1(h-\tau_k)} \\ e^{\lambda_2(h-\tau_k)} \\ \vdots \\ e^{\lambda_n(h-\tau_k)} \end{bmatrix} = \Gamma_\Delta \Delta u_k - \Gamma_{\Delta,k}d_{\tau_k} \tag{29}$$

where

$$\beta_k = \begin{bmatrix} \beta_k^1 & \beta_k^2 & \cdots & \beta_k^n \end{bmatrix}^T = P^{-1}B\Delta u_k \in \mathbb{R}^{n,1},$$

$$diag\left(\beta_k\right) = \begin{bmatrix} \beta_k^1 & 0 & \cdots & 0 \\ 0 & \beta_k^2 & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \beta_k^n \end{bmatrix} = diag\left(P^{-1}B\Delta u_k\right),$$

$$\Gamma_\Delta = P\begin{bmatrix} \frac{1}{\lambda_1}e^{\lambda_1 h} & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & \frac{1}{\lambda_n}e^{\lambda_n h} \end{bmatrix} P^{-1}B \quad \text{and}$$

$$\Gamma_{\Delta,k} = Pdiag\left(\frac{1}{\lambda_{11}}, \frac{1}{\lambda_2}, \cdots \frac{1}{\lambda_n}\right)diag\left(\beta_k\right)$$

According to (29) the model of Eq. (23) can also be rewritten as :

$$\begin{cases} x_{k+1} = \Phi x_k + \Gamma u_k + \Gamma_\Delta \Delta u_k \\ \qquad -\Gamma_{\Delta,k}d_{\tau_k} + \Xi_x d_k + \Psi_x f_k \\ y_k = Cx_k + \Xi_y d_k + \Psi_y f_k \end{cases} \tag{30}$$

By definition, $\Gamma_k^a = \begin{bmatrix}\Gamma & \Gamma_\Delta\end{bmatrix}$, $\mathrm{u}_k^a = \begin{bmatrix} u_k \\ \Delta u_k \end{bmatrix}$, $\Xi_{x,k}^a = \begin{bmatrix}\Xi_x & -\Gamma_{\Delta,k}\end{bmatrix}$ $\Xi_y^a = \begin{bmatrix}\Xi_y & 0\end{bmatrix}$ and $d_k^a = \begin{bmatrix} d_k \\ d_{\tau,k} \end{bmatrix}$ we get:

$$\begin{cases} x_{k+1} = \Phi x_k + \Gamma_k^a \mathrm{u}_k^a + \Xi_{x,k}^a d_k^a + \Psi_x f_k \\ y_k = Cx_k + \Xi_y^a d_k^a + \Psi_y f_k \end{cases} \tag{31}$$

Assuring the robustness of residual generators in practical situations against inevitable unknown input disturbances is commonly recognized as the main design problem for FDI schemes. In the case of structured types of uncertainties, the current literature proposes a variety of solutions for achieving robustness, see for instance (Chen and Patton, 99; Ding, 08). In the next section FDI is revisited, considering network effects.

Model based Fault detection relies on the generation of a residual which must be sensitive to failures and able to distinguish failures from other unknown disturbance inputs. The design must ensure that residuals are closed to zero in fault free situations while clearly deviating from zero in the presence of faults. In a first attempt, the idea is to consider a residual generator based on the state observer.

$$\begin{cases} \hat{x}_{k+1} = \Phi \hat{x}_k + \Gamma_k^a u_k^a + L\left(y_k - \hat{y}_k\right) \\ \hat{y}_k = C\hat{x}_k \end{cases} \tag{32}$$

and the residual generator:

$$r_k = T\left(y_k - \hat{y}_k\right) \tag{33}$$

where T and L are matrices that are designed in order to fulfill fault detection and isolation requirements. From (32) and (33), the estimation error $\varepsilon_k = x_k - \hat{x}_k$ and the output of the filter propagate as:

$$\begin{aligned} \varepsilon_{k+1} = (\Phi - LC)\varepsilon_k + \left(\Xi_{x,k}^a - L\Xi_y^a\right)d_k^a \\ + \left(\Psi_x - L\Psi_y\right)f_k \end{aligned} \tag{34}$$

where $\Phi - LC$ is a stable matrix, and L has to ensure a best estimate of the process states. It results that $\lim_{t\to\infty} \varepsilon_k = 0$, which leads (after z-transformation) to

$$\begin{aligned} r_z = T\left(C\left(zI - \Phi + LC\right)^{-1}\left(\Xi_{x,k}^a - L\Xi_y^a\right) + \Xi_y^a\right)d_k^a \\ + T\left(C\left(zI - \Phi + LC\right)^{-1}\left(\Psi_x - L\Psi_y\right) + \Psi_y\right)f_k \end{aligned} \tag{35}$$

The observer gain matrix L and T are determined such that the following requirements are guaranteed

1. Asymptotic stability under fault free conditions (i.e. $f_k = 0$);
2. Minimization of disturbance effects;
3. Maximization of fault effects;

Perfect fault detection, which means perfect decoupling from unknown inputs with:

$$T\left(C\left(zI - \Phi + LC\right)^{-1}\left(\Xi_{x,k}^a - L\Xi_y^a\right) + \Xi_y^a\right)d_k^a = 0 \tag{36a}$$

$$T\left(C\left(zI - \Phi + LC\right)^{-1}\left(\Psi_x - L\Psi_y\right) + \Psi_y\right)f_k \neq 0 \tag{36b}$$

Actually, there are various approaches (Gertler, 98; Chen and Patton, 99; Frank and Ding, 97; Ding, 08) to determine the gain matrices L and T, but we do not discuss this topic in the paper. If, it is now supposed that the system is controlled over a network, then we have to take into account the sensor to controller delays and controller to actuator delays.

For illustration purpose we consider a simulation of the system described by equations (11). It is supposed that the FD system based on the standard Kalman filtering is connected to the plant via a network.

In the simulations, the network delay is supposed to be Gaussian variable, the fault associated to the first attitude sensor "ϕ : Roll" occurs at time instant k= 1000 and the fault associated to the second attitude sensor "ψ : Yaw" occurs at time k= 1500.

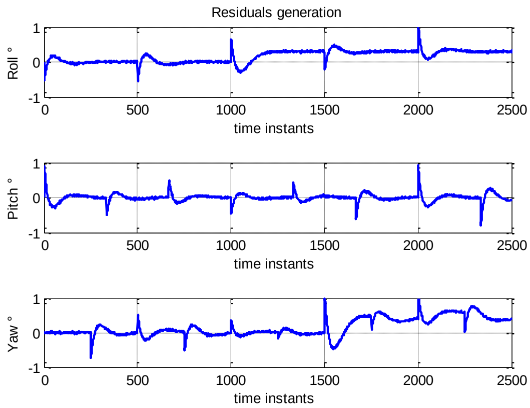

Figure 6: Residuals generation by standard kalman filter (IJAAC).

Result shown before doesn't allow (Fig.6.) to distinguish between the fault and the network variable delay effects. Hence, it appears that the robustness of the fault diagnosis system against network induced delays depend on the amplitude of the unknown term $\Gamma_{\Delta,k} d_{\tau,k}$.

Assuring the robustness of residual generators in practical situations against inevitable unknown input disturbances is commonly recognized as the main design problem for FDI schemes. In the case of structured types of uncertainties, the current literature proposes a variety of solutions for achieving robustness (Chen and Patton, 99; Ding, 08). In the next section FDI is revisited, considering network effects.

4. ROBUST RESIDUAL GENERATION AND EVALUATION

The objective of fault diagnosis is to perform two main decision tasks (Frank and Ding, 97): fault detection, consisting of deciding whether or not a fault has occurred, and fault isolation, consisting of deciding which element of the system has failed. The general procedure comprises the following two steps:

- Residual generation: the process of associating, with the pair model-observation, features that allow evaluating the difference with respect to normal operating conditions.

- Residual evaluation: the process of comparing residuals to some predefined thresholds according to a test and at a stage where symptoms are produced.

This implies designing residuals that are close to zero in fault-free situations while clearly deviating from zero in the presence of faults and that possess the ability to discriminate between all possible modes of faults, which explain the use of the term isolation.

Therefore, the objective here is to design a residual generator similar to the one described by equation (31) which in addition is robust against network delays influence. Several approaches have been proposed in the literature (Wang et al., 06; Sauter and Boukhobza, 06; Chabir, et al., 08)

4.1. Residual generation

A solution of the above mentioned problem towards the design of observer based residual generator will be derived. Under the following assumptions:

$$z_k = \begin{bmatrix} x_k \\ e_k \end{bmatrix} \tag{37}$$

The overall system dynamics, which includes the plant and the residual generator, can be expressed as

$$\begin{cases} z_{k+1} = \bar{A} z_k + \bar{B} u_k^a + \bar{\Xi}_{k,x} d_k^a + \bar{\Psi}_x f_k \\ r_k = T\bar{C} z_k + T\,\Xi_y^a d_k^a + T\,\Psi_y f_k \end{cases} \tag{38}$$

where $\bar{A} = \begin{bmatrix} \Phi & 0 \\ 0 & \Phi - K_k C \end{bmatrix}$, $\bar{C} = \begin{bmatrix} 0 & C \end{bmatrix}$, $\bar{B} = \begin{bmatrix} \bar{\Gamma}_k^a \\ 0 \end{bmatrix}$,

$\bar{\Xi}_{k,x} = \begin{bmatrix} \Xi_{x,k}^a \\ \Xi_{x,k}^a - L\Xi_y^a \end{bmatrix}$, and $\bar{\Psi}_x = \begin{bmatrix} \Psi_x \\ \Psi_x - L\Psi_y \end{bmatrix}$

It is assumed that the plant is mean square stable. Since the observer gain matrix L has no influence on the system in (38). The overall system dynamics (plant + residual generator) is mean square stable.

The post-filter T and the observer gain matrix L are the design parameters for the residual generator. The main objective of the design of the residual generator is to improve the sensitivity of the FD system to faults while keeping robustness against disturbances. Thus, the selection of the design parameters L, T can be formulated as an optimization problem such as:

$$Sup\, J = \sup_{L,T} \frac{\left\| G_z^{rf} \right\|_-}{\left\| G_z^{rd} \right\|_\infty} \tag{39}$$

where

$$G_z^{rd} = T\bar{C}\left(zI - \bar{A} + L\bar{C}\right)^{-1} \bar{\Xi}_{k,x} + T\,\Xi_y^a \tag{40a}$$

$$G_z^{rf} = T\bar{C}\left(zI - \bar{A} + L\bar{C}\right)^{-1} \bar{\Psi}_x + T\,\Psi_y \tag{40b}$$

4.2. Residual evaluation

The second step of the fault detection procedure is to evaluate the residual. Residual evaluation is an important step of model based FD approach, i.e. see for instance in (Ding, 08). This stage includes a calculation of the residual evaluation function and a determination of detection threshold. The decision for successful fault detection is finally made based on the comparison between the results obtained from the residual evaluation function and the determined threshold.

The following residual evaluation function is proposed :

$$J^e{}_k = \left\| r_k \right\|_{2,N} = \sqrt{\left(\frac{1}{N} \sum_{i=1}^{N} r_{k-i} \right)^T \left(\frac{1}{N} \sum_{i=1}^{N} r_{k-i} \right)} \tag{31}$$

Where N is the length of the evaluation window. The variance of the residual signal can be expressed as:

$$\sigma_{rk} = \mathrm{E}\left(\left(r_k - \bar{r}_k \right)^T \left(r_k - \bar{r}_k \right) \right) \tag{31}$$

Under the assumption that the unknown input and control input are L_2- bounded, the following theorem is given:

Theorem 1:

Given system (14) and the constants $\gamma_1 > 0, \gamma_2 > 0$. *The following equation holds true:*

$$\sigma_{rk} = \mathrm{E}\left(\left(r_k - \bar{r}_k \right)^T \left(r_k - \bar{r}_k \right) \right)$$

$$< \gamma_1 \sum_{j=0}^{k} \left(v_j^T v_j + \Delta u_j^T \Delta u_j \right) + \gamma_2 \left(v_k^T v_k + \Delta u_k^T \Delta u_k \right) \tag{31}$$

If there exist $P > 0$ so that:

$$\begin{bmatrix} -P & P\bar{A} & P\bar{B} & \bar{\bar{\Xi}}_{k,x} \\ \bar{A}^T P & -P & 0 & 0 \\ \bar{B}^T P & 0 & -I & 0 \\ \bar{\bar{\Xi}}_{k,x}^T P & 0 & 0 & -I \end{bmatrix} < 0 \tag{31}$$

$$\begin{bmatrix} -P & \bar{C} \\ \bar{C}^T & -\gamma_1 I \end{bmatrix} < 0 \tag{31}$$

$$\begin{bmatrix} -I & \Psi_y \\ \Psi_y^T & -\gamma_2 I \end{bmatrix} < 0 \tag{31}$$

where

$\bar{\bar{\Xi}}_{k,x} = \begin{bmatrix} \bar{\Xi}_{x,k}^a \\ \bar{\Xi}_{x,k}^a - L\Xi_y^a \end{bmatrix}$, $\bar{\Xi}_{x,k}^a = \begin{bmatrix} \Xi_x & -\bar{\Gamma}_{\Delta,k} \end{bmatrix}$, and

$\bar{\Gamma}_{\Delta,k}$ is calculated for $\Delta u = \max\left(\Delta u\right)$.

The proof is similar to the one mentioned in (Al-Salami, et al., 08), hence it is omitted. Note that Δu_k is set to the allowed upper bound of the control input $\max\left(\Delta u\right)$.

The threshold can set as : $J_k^{th} = \sqrt{\alpha_N \beta}$ (31)

Where $\beta = \sup \sigma_{rk}$

$$\beta = \gamma_1 \left(\delta_{d,2} + \sum_{j=0}^{k} \left(\Delta u_j^T \Delta u_j \right) \right) + \gamma_2 \left(\delta_{d,\infty} + \Delta u_k^T \Delta u_k \right)$$

where $\delta_{d,2} \geq \sum_{j=0}^{k} \left(v^T{}_j v_j \right), \delta_{d,\infty} \geq v^T{}_k v_k$.

are the L_2, L_∞ of the unknown input, respectively, and $0 < \alpha_N < 1$ is a constant value depends on the length of the evaluation window N .

The parameters γ_1, γ_2 are some constants which represents the bounds of the variance of the residual signal.

Note that because the residual signal is a white noise process, the threshold will depends on the statistical part of it (which means the variance of residual signal).

After the determination of a threshold, a decision has to performed, if a fault occurs. The Decision logic for the FD system can be defined as follows:

$J_k^e > J_k^{th} \Rightarrow$ fault

$J_k^e \leq J_k^{th} \Rightarrow$ no fault

The threshold $J_{th}(k)$ is adaptive and is influenced from Δu_k , which has to be calculated online.

In the next section simulations are performed in order to validate the results of the proposed residual evaluator.

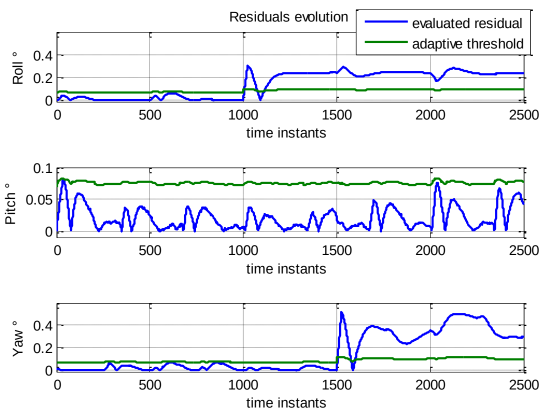

Figure 7: Evaluated residual.

The upper bounds on the unknown inputs are $\delta_{d,2} = 0.15, \delta_{d,\infty} = 0.28$. The length of the evaluation window is set to 50 and α_N is set to 0.3. The parameters of the Threshold (bounds on the variance of residual) are computed as $\gamma_1 = 0.0058, \gamma_2 = 0.05$. The threshold is then to determine (adaptive) on-line during the simulation.

From the result shown (fig. 7.) it is clear that the adaptive threshold allows fault detection and the liklyhood of the false alarm rate is extremely minimized.

5. CONCLUSIONS

In this paper the residual generation and evaluation issue is presented within the framework of networked control systems. The problems, addressed in this paper, are (i) robustness against network delays as well as noise (ii) reducing the false alarm rate. In this context, a quadrotor attitude sensors fault is detected by a post-filter and compared to an adaptive threshold. That considers the variation of control inputs as well as unknown inputs. The problem of threshold design is established in terms of linear matrix inequalities. Validation results show the effectiveness of the obtained results.

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AUTHORS BIOGRAPHY

KARIM CHABIR received his Master in Automatic and intelligent technique in 2006 from the National Engineering School of Gabes (Tunisia) and cotuttelle Ph.D. in Automatic Control, from University Henri Poincaré (France) and University of Gabes in 2011. His research works were carried out at the Research Centre for Automatic Control of Nancy (CRAN) and at the Research Unit of Modelling, Analysis and Control systems of the National Engineering School of Gabes, Tunisia. He was a member of Dependability and system diagnosis group (SURFDIAG). His current research interests are focused on model-based fault diagnosis and fault tolerant with emphasis on networked control systems. He was a secondary school teacher of Gabes, Tunisia, from 10/2003 to 09/2007. He was also an Assistant professor in Faculty of Science of Gabes, Tunisia, from 10/2007 to 08/2011. Temporary Teaching and Research (ATER) since 2011 at Faculty of Science and Technology of Nancy, France.

Mohamed Amine SID was born in 1986 in Algeria. He received his master degree in automatic control option from the department of electrical engineering at Sétif University, Algeria. Since 2010, he has been working towards a PhD in Automatic and Signal Processing at the Centre de Recherche en Automatique de Nancy (CRAN, CNRS). His main research interests are in Networked control Systems, Fault Detection.

Dominique Sauter received the D.Sc. degree (1991) from Henri Poincar´e University, Nancy 1 (now the University Lorraine), France. Since 1993 he has been a full professor at this university, where he teaches automatic control. He was the head of the Electrical Engineering Department for four years, and now he is a vicedean of the Faculty of Science and Technology. He is a member of the Research Center in Automatic Control of Nancy (CRAN) associated with the French National Center for Scientific Research (CNRS). He is also a member of the French-German Institute for Automatic Control and Robotics (IAR), where he has chaired a working group on intelligent control and fault diagnosis. His current research interests are focused on model-based fault diagnosis and fault tolerant control with emphasis on networked control systems. The results of his research works are published in over 50 articles in journals and book contributions as well as 150 conference papers.