ROBUST MODEL MATCHING CONTROL APPLIED TO A CRANE

Eduardo Luiz Lozano de Campos[a], Fabrizio Leonardi[b]

[a][b] Centro Universitário da FEI, São Bernardo do Campo, Brazil

[a] llcampos@uol.com.br, [b] fabrizio@fei.edu.br

ABSTRACT
This paper discusses the robust closed loop control design subject to parametric uncertainties applied to a crane during a maneuver. Usually crane trajectories are generated by formulating a minimum time optimal control in open loop. However, the optimality of the solution is not maintained due to variations in the plant over time. This work proposes the use of a model matching structure to reduce the problems related to model uncertainties thus trying to preserve the trajectory optimality. The robust compensator minimizes explicitly the matching error between the real plant and the reference plant. In this application the main uncertain parameter is the pendulum length and plays the role of the load lifting. To illustrate the application experiments were done using a lab scale equipment. The results observed are very close to those obtained from numerical simulation.

Keywords: Robust Control, Optimal Control, Model Matching

1. INTRODUCTION
Cranes are common in various industry segments for transporting loads. These systems are considered efficient for their safety in transportation, mechanical robustness and reliability of loading and unloading tasks.

Even a crane manually operated need to be controlled accordingly but one should not expect great performance since human action can be inaccurate and not always the best path will be produced, which can generate, for example, swings that may endanger the operation, products, equipment, etc. When an automated system is used, it is reasonable to expect for better performance. In this case, the system is responsible for controlling all the variables subjected to the physical constraints, seeking minimum time, lower power consumption and minimal oscillation. (Puglia, Leonardi and Ackermann, 2011; Da Cruz and Leonardi, 2012).

As proposed by Sorensen, Singhose and Dickerson (2007), the control schemes of cranes may be grouped into three categories: time-optimal control, command shaping and feedback control. His own publication can be considered in the category of command shaping with a forward action used to determine the appropriate command signal in order to reduce the swing during a maneuver. The command shaping approach was also used by Lee and Choi (2001), who developed a way to determine the trajectory of the crane based on Lyapunov stability theorem. Another example is the work of Chen, Hein and Wörn (2007), where it is proposed an open-loop control with the trajectory defined based on the principle of acceleration compensation. In the work of Lau and Pao (2001), he discusses about the equivalence of minimum time optimal control and command shaping for flexible systems.

Normally the minimum time problem is treated in open loop, but due to modeling errors and disturbances is necessary to use a closed loop control strategy to maintain the optimal trajectory with a certain precision. An example of this is the work of Hičár and Ritók (2006) which uses the pole allocation method by means of the Ackermann formulae to provide a robust control to a crane.

Different closed loop control structures, such as IMC, multiloop, model matching, etc., are somehow equivalent to each other in the sense that it is usually possible to represent the same control law on different topologies. However, the choice of a particular structure can make easy the analysis, design or even its implementation. This paper proposes the use of a model matching structure to perform the closed loop control law for a crane which must robustly maintain the optimal trajectory of both minimum time and minimum control effort. The optimal maneuver begins with the crane at rest and carries the load at a fixed distance, reaching the destination also at rest. The optimal trajectory does not consider the lift during the maneuver, and therefore, if this occurs, the trajectory is no longer optimal. The purpose of using the model matching is precisely for overcome that problem. The controller must make a closed loop to behave as the plant that was used to obtain the optimal trajectory, that is, without lifting. Thus, the controller must be robust to variations in crane cable length.

The optimal trajectory was generated in the similar way as in Puglia, Leonardi and Ackermann (2011). It takes into account physical constraints of the system, such as the maximum control effort. Notice that the design of the robust model matching controller must also take into consideration those constraints.

To easily incorporate constraints in the robust controller design, we have chosen to conduct the project in the time domain by means of a parametric optimization of the controller coefficients.

A cart-pendulum lab system was used to illustrate the proposed approach.

2. METHOD

The proposed methodology is based on a model matching control structure and its design is discussed in the sequence. This structure is used to robustly maintain the optimal trajectory of a crane during a maneuver.

2.1. Model Matching

Consider the standard closed loop control system diagram in the Fig. 1, where x(t) is the reference signal, y(t) is the controlled output, and d(t) an auxiliary exogenous signal. Respectively, F(s) and P(s) represent the transfer functions of the controller and the plant nominal model.

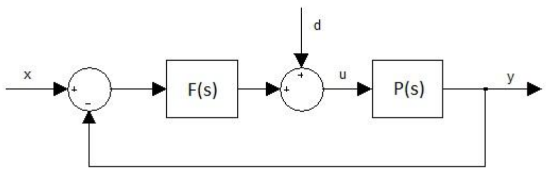

Figure 1: Closed Loop Control.

In general, the goal of the control system is to have y(t) closely following x(t). If the nominal plant model is known and the inverse of its transfer function $M(s) = 1/P(s)$ exists, one can use the input d(t) as a feed forward action, as shown in Fig. 2.

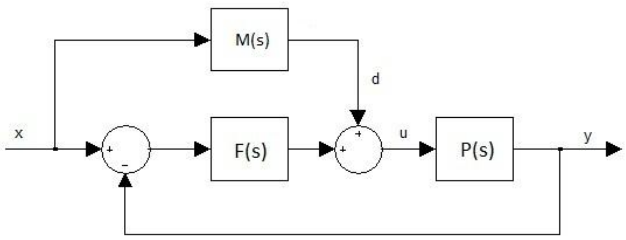

Figure 2: Closed Loop with Feed Forward Action.

In the absence of plant modeling errors the control system is reduced to an open loop, since y(t) = x(t). In the presence of modeling errors the controller F(s) need to compensate for the difference x(t)-y(t) by correcting the value of u(t). Even disregarding the modeling errors, often the inverse of the plant model may have issues for a practical application. For those cases M(s) can be taken as an approximation of the 1/P(s), and thus the controller F(s) will probably be required to compensate for higher deviations. When the plant exhibits constant gain at low frequency it is common to use a static N(s).

In an optimal control problem, typically, both the optimal trajectory y = y*(t) and the optimal control u = u*(t), are available. If it is necessary to keep the optimal trajectory with a closed loop control we can use the structure of the Fig. 2. However, one can achieve feed forward compensation without the use of M(s) explicitly, by simply using d(t) = u*(t) and x(t) = y*(t). In this scenario, the feed forward control action diagram can be redrawn in the equivalent form of the Fig. 3 where the plant model appears explicitly $N(s) = P(s)$.

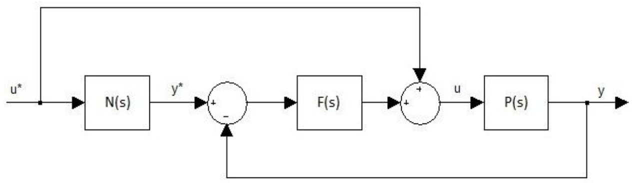

Figure 3: Closed Loop Optimal Control.

In a more general scenario the diagram of Fig. 3 can be used even when $N(s) \neq P(s)$ and may be applied in an attempt to make the closed loop transfer function $T(s) = y(s)/x(s)$ aproach N(s). In fact, this appeal is more easily seen by drawing the diagram in its equivalent form of Fig. 4.

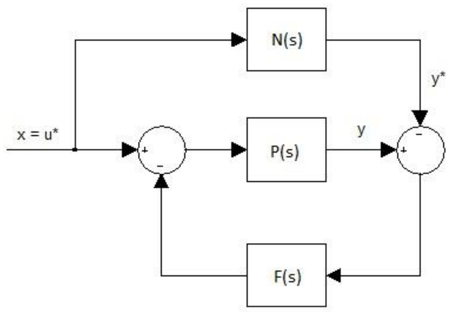

Figure 4: Model Matching Structure.

The structure in Fig. 4 is known in the technical literature and has been used in some applications like the one by Jonckheere (1999) for controlling a crippled aircraft. Note that the error signal in this diagram is, in fact, the difference between the response of the plant P(s) and the response of the reference model N(s). If the controller F(s) can make this error small enough, then the response of the plant is about the same response of the reference model. This effect has been called approximate model matching.

The model matching control structure is used in this work in order to make the plant response following an optimal trajectory. The reference model N(s) is the model used to generate the optimal trajectory and P(s) represents all real plants. The differences between N(s) and P(s) may be due to variations in the plant over time which may even be deliberate. In this work this feature is used to compensate eventual load hoisting and lowering during a maneuver.

2.2. Mechanical Model

A scheme of the cart-pendulum system used is shown in Fig. 5, where m is the load mass, x the cart position and φ the load angle.

The equations of motion describing the dynamics of the cart-pendulum model can be, for instance, derived using the Newton-Euler formalism as described in Schiehlen (1997).

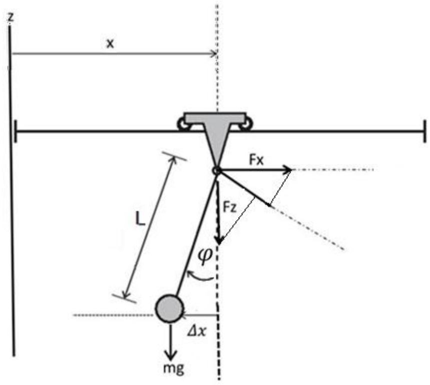

Figure 5: Cart-pendulum Scheme.

The resulting nonlinear model can be presented in the form of the following differential equation.

$$L\,\frac{d^2(\varphi(t))}{dt^2} + g\,sen(\varphi(t)) = \frac{d^2(x(t))}{dt^2}\cos(\varphi(t)) \qquad (1)$$

where g is the gravity acceleration and L de pendulum length.

Considering that the kinematics of the cart can be imposed arbitrarily, we define the manipulated variable

$$u(t) = x(t) \qquad (2)$$

That allows also defining

$$a(t) = \frac{d^2(x(t))}{dt^2}\ . \qquad (3)$$

In handling anti-oscillatory problems, it is expected that the maximum oscillation angle be small. This condition leads to the approximations $sen\varphi \approx \varphi$ and $\cos\varphi \approx 1$. These approximations simplify the equations of motion to

$$L\,\frac{d^2(\varphi(t))}{dt^2} + g\,\varphi(t) = \frac{d^2(x(t))}{dt^2} \qquad (4)$$

Mapping it to the Laplace domain and taking null initial conditions, one obtains the transfer function

$$\frac{Y(s)}{X(s)} = \frac{s^2}{Ls^2 + g} \qquad (5)$$

where

$$y(t) = \varphi(t) \qquad (6)$$

to be consistent with the notation used in section 2.1.

The model does not incorporate the Coulomb friction. However, it can be easily included as an additional torque in the equation (1). In such cases, the design of optimal control signal should take this into account or the closed loop control must be robust in the presence of this modeling error.

2.3. Optimality

The model matching control system proposed in this work should be able to closely keep the solution of the optimization problem proposed by Puglia, Leonardi and Ackermann (2011). This problem is defined by the objective function (7) and the constraints (8). That is, should minimize the sum of the absolute control a (acceleration) in each sampling time (1,.., n), subjected to the plant dynamics N(s), initial state $w(t_0)$ and final state $w(t_f)$ of the maneuver, and the limits $max|v|$ of the control effort. Besides, the optimal control a^*(t) and the optimal trajectory y^*(t) generated by Puglia, Leonardi and Ackermann (2011) also includes the time minimizing in the same optimization problem. Notice that the overall acceleration is minimized but the designer could add penalties related to each sampling time or even limit each value as an explicit constraint.

The optimal signal is used in the model matching control structure of Fig. 4 which is supposed to maintain y(t) close to the y^*(t).

$$\min_a J_1 = |a_1| + |a_2| + \cdots + |a_n| \qquad (7)$$

$$N(s),\ w(t_0),\ w(t_f),\ \max|a| \qquad (8)$$

We define here internal optimality of the model matching problem as the property of a(t), the input signal of the real Plant $P(s)$, be an optimum control signal in the sense of the equations (7) and (8). That is, the value of J_2 obtained by the solution of the optimization problem of equations (9) and (10) must be equal or less than J_1. Besides, since the problem also includes time minimizing, the control signal must also be optimal in this sense.

$$\min_a J_2 = |a_1| + |a_2| + \cdots + |a_n| \qquad (9)$$

$$P(s),\ w(t_0),\ w(t_f),\ \max|a| \qquad (10)$$

Note that typically P(s) ≠ N(s) and in general a(t) may not meet the requirements of internal optimality. That is, from the viewpoint of the real plant, the model matching structure can not preserve the optimality produced by Puglia, Leonardi and Ackermann (2011) since he applied the control to a plant P(s) = N(s).

Thus, it is defined here what we call external optimality. Since the transfer function y(s) to x(s) can match N(s) with a prescribed precision, so if we apply the optimal control signal

$$u^*(t) = \int\!\!\int^t a^*(t)\,dt \qquad (11)$$

to x(t), it sees a plant very close to N(s). Thus, the optimality of the original solution is preserved in an approximate way. With this external point of view, both kinematics constraints and control minimizing are aproximately preserved.

2.4. Control Effort

In the frequency domain it can be stated that the model matching problem is to find a compensator $F(s)$ such that the absolute value of transfer function $x(j\omega)$ to $e(j\omega) = y(j\omega) - y^*(j\omega)$ is below a certain prescribed value in the largest possible range of frequencies (Leonardi, 2006).

For the system shown in Fig. 4, the following equations apply

$$y(s) = P(s)\left(1 + F(s)P(s)\right)^{-1}\left(1 + F(s)N(s)\right) x(s) \qquad (12)$$

and

$$u(s) = \left(1 + F(s)P(s)\right)^{-1}\left(1 + F(s)N(s)\right) x(s) \qquad (13)$$

Since it is considered here that $N(s)$ is stable, stability of the system is determined solely by the closed loop which contains $P(s)$ and $F(s)$, wich is implicit assured once the performance is achieved.

Consider $\alpha > 0$ (typically $\alpha << 1$), a given number that expresses the desired precision associated to the model matching error in a certain range of frequencies, so that

$$\left|e(j\omega)\right| / \left|x(j\omega)\right| \leq \alpha \, . \qquad (14)$$

To ensure model matching we have the following sufficient condition,

$$\left|F(j\omega)P(j\omega)\right| \geq \frac{\left|P(j\omega) - N(j\omega)\right|}{\alpha} \qquad (15)$$

obtained from (12) to the typical case in which the loop gain and precision are respectively large, that is, for $|F(j\omega)\ P(j\omega)| >> 1$ and $\alpha << 1$. This condition shows that the loop gain increases with either increasing the distance between P and N as the inverse of α.

From equation (13) is immediate that

$$u(s) - x(s) = F(s)\left[1 + P(s)F(s)\right]^{-1}\left[N(s) - P(s)\right]x(s) \qquad (16)$$

Under approximate conditions given by $\alpha << 1$ and $|F(j\omega)\ P(j\omega)| >> 1$, then (16) leads to

$$u(j\omega) - x(j\omega) \cong P(j\omega)^{-1}\left[N(j\omega) - P(j\omega)\right] x(j\omega) \ , \qquad (17)$$

From equation (17) it follows immediately that

$$\frac{\left|u(j\omega) - x(j\omega)\right|}{\left|x(j\omega)\right|} \cong \left| P^{-1}(j\omega)\left[N(j\omega) - P(j\omega)\right]\right| \qquad (18)$$

This last equation shows that the relative increase in control effort is approximately the same as the relative difference between the plant and the reference model. Therefore, reference models that are distant from the plant model requires a high control effort to be followed. This is consistent with the condition (15) wich shows that the greater the distance between the plant and the reference model, the greater is the loop gain to ensure model matching.

2.5. Robustness

The modeling errors may be uncertainties in transfer function of the plant. However, classical margins of stability alone are unable to reveal the degree of robustness of a system because, even systems with favorable margins as [90º, ∞ dB], may have its corresponding Nyquist diagram close to -1 + 0j, and therefore, are not robust (Da Cruz, 1996).

Model uncertainties can be classified as structured and unstructured. Unstructured uncertainties are usually associated to unmodeled parts of the plant that are frequency dependent such as negletec dynamics. The structured uncertainties are associated with parametric uncertainties such as the one in this work.

The main parametric uncertainty of the plant model (5) is the distance L from the load to the cart. in fact this uncertainty is intentional and represents the changes of L over time required during the maneuver. If the performance of the control system is robust to this variation $L_{\min} \leq L \leq L_{\max}$, the external optimality is approximately preserved.

This paper proposes to use the parametric optimization of the controller in order to include constraints, beyond the problem of robustness to the variation of L. The optimization is performed on a time range suitable for the maneuver and uses as a reference the optimal signal obtained by Puglia, Leonardi and Ackermann (2011), but using the model matching control structure of Fig. 3 or Fig. 4.

To incorporate the problem of robustness in the formulation of parametric optimization, the objective function

$$J = \int_0^{tf} [e_1^2(t) + e_2^2(t) + \ldots + e_m^2(t)]dt \qquad (19)$$

includes the sum of square of the matching error between each of the considered m real plant $P_i(s)$, $i = \{1,.., m\}$, and the reference plant $N(s)$. That is, the transfer function of each $P_i(s)$ is considered here equal to $N(s)$ but with a distinct value of L in the range of $L_{\min} \leq L \leq L_{\max}$.

Since in this parametric optimization problem we can easily add several type of constraints, any physical restriction of the problem are conveniently incorporated. It should be noted that limiting the control effort has been considered in generating the signals $a^*(t)$ and $y^*(t)$, however they were generated for the open loop and in the absence of modeling errors. By using the model matching to keep the trajectory $y^*(t)$, there is no guarantee that the acceleration limit is still respected. Therefore, this restriction should be used again, now in the design of closed loop controller.

2.6. Controller Selection

Since the nominal plant model is open loop stable a wide class of controllers are candidate for the optimization problem. Although a purely proportional controller can stabilize the closed loop, it causes excessive noise amplification since the transfer function of the plant is just proper.

To allow attenuation for high frequency noise, the transfer function of the controller must be strictly proper. Thus, a good choice for the controller to this problem may be one with purely integral action.

The controller design methodology for robust model matching has been applied and the controller obtained has the following transfer function.

$$F(s) = \frac{12}{s} \tag{20}$$

3. RESULTS

The results presented in this section refer to the application of the robust model matching control using the controller of the equation (20) to a lab scale plant. The optimal signal was generated for a maneuver problem in minimum time and minimum control effort in a manner similar to the one that was obtained by Puglia, Leonardi and Ackermann (2011). The optimal control signal takes the pendulum from rest to the other end, away 0.25m from the start, also arriving at rest.

3.1. Testing Apparatus

The pendulum of the testing apparatus (see Fig. 6) consists of a 0.215kg mass connected to the cart by a rod. The mass can be fixed on the rod at different distances from the cart.

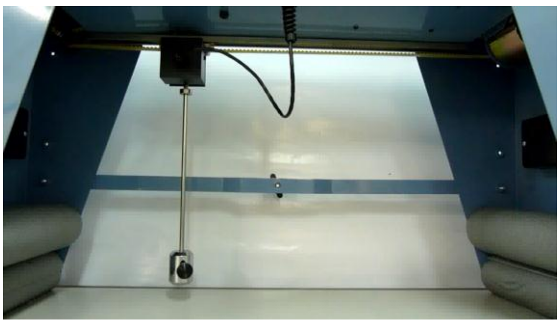

Figure 6: Bytronic Lab Equipment.

The cart driver has a built in position control with tachometer compensation, as indicated in Fig. 7. Since that control system is quite precise over the frequency range that matters in this problem, its dynamics can be reasonably neglected and thus the cart position is considered the manipulated variable as it was admitted in the methodology section.

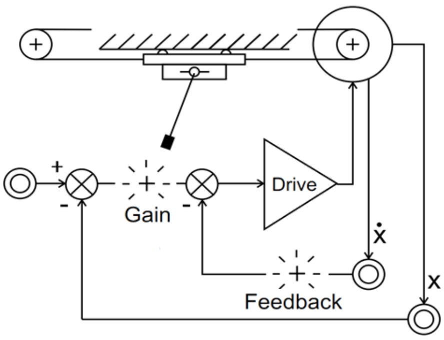

Figure 7: Schematic Diagram of the Equipment.

The robust controller used in this application has only an integral action. He was selected to be extremely simple and yet provide good robustness to parametric design, which in fact can be verified by the experimental results.

To implement the compensator it was used the Real-Time Target Windows™ (Mathworks, 2012) operating at a sampling frequency of 1KHz, the same rate used in the generation of the signals $a^*(t)$ and $y^*(t)$.

3.2. Experimental Results

The controller design was done by considering the variation in the length of the pendulum in the range $0.15\text{ m} \le L \le 0.25\text{ m}$, and being 0.25m the nominal reference value. That is, the reference plant is

$$\frac{Y(s)}{X(s)} = \frac{s^2}{0,25s^2 + 9.81} \tag{21}$$

The plots of Fig. 8 show the performance in time domain obtained with the controller. The maneuver begins at $t = 0$ *s* and ends at $tf = 1.3$ *s*.

The value of the length L in the reference model was kept fixed at L = 0.25 m and the L values of real equipment were changed within the range considered. Fig. 8 shows the worst case where the real length $L_{real} = 0.15$ m is most distant from the nominal $L = 0.25$ m. The figure contains two sets of plots. The first (a) shows the optimum position u^* (red) and the experimental value of u position (blue). Note that since the position is the manipulated variable of the control system, deviations of u from the value u^* represent the extra effort the control system needs to spend to perform the match.

In the second set of plots (b) it is shown the experimental angle (blue) that is expected to be close to the reference angle (red). To complete the analysis, it is also shown the angle behavior if the system is operating without control (black).

It can be seen that the integral controller, in fact, provides a robust performance to the control system. This is true since both the optimal control signal (position) and the optimal trajectory (angle), are close to their reference values even for huge variations of L.

In particular, note that in all plots the final cart position and the final angular position were achieved with small error.

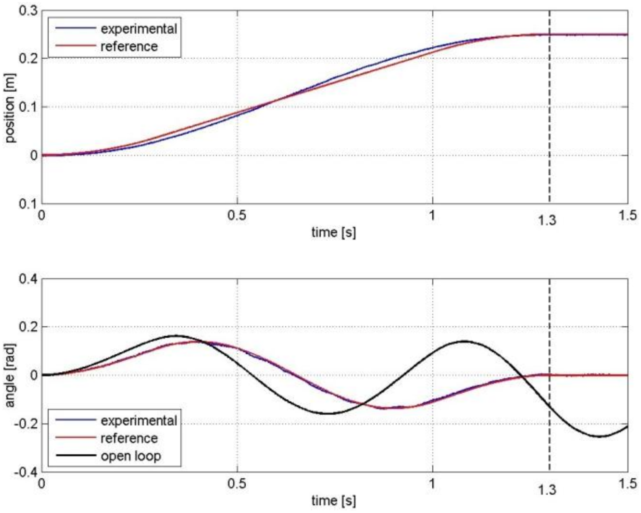

Figure 8: Performance of the Integral Controller for $L_{\text{real}} = 0.15$m.

The system begins at rest and reaches its destination in 1.3 s. It is clear from the plots that system remains at rest after 1.3 s

For the sake of comparison we also designed the controller using the H_2 mixed sensitivity. The performance perceived is similar, but the resulting compensator is of order 5th with two extra resonances to the loop which might be undesirable.

4. CONCLUSIONS

This paper discusses the use of a model matching structure for closed loop control of the optimal trajectory of a crane. We discussed the design of the compensator to reduce the problems related to parametric uncertainties of the plant, thus preserving the optimality of the initial solution. The project was conducted by means of parametric optimization of the compensator and the objective function includes the matching error of a number of plants with different values of the pendulum length.

The practical results were obtained applying the methodology to a cart-pendulum lab scale equipment. It was found that the designed controller gives robust performance even for a large parametric Plant variation as expected during the design.

The overall methodology was developed for a dedicated application. However, it might be applied for linear systems with few uncertain parameters and for modeling errors below 100%.

As a proposal for extending this work the following investigations are suggested.

This study did not investigate the problem of rejecting external disturbances caused, for instance, by wind. Adding a disturbance input to the model matching structure we obtain a control law that has two degrees of freedom. That is, it is possible to tune the robust controller in order to balance between the requirement to follow the reference signal and to reject the disturbance. A current research is examining this issue to propose a design methodology that takes this into consideration.

This study also did not investigate the problem of sensitivity of the response in the face of measurement noise and possible offset in its calibration. The mentioned above research is also investigating how the control structure can be altered to minimize this effect, mainly the one from residual offset. The investigation also includes the definition of artificial measurable variables and how the optimal control trajectory of the crane needs to be modified to do so. Preliminary results show that it is possible to find necessary and sufficient conditions for this mapping.

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